Reflectivity of steep-gradient plasmas in intense subpicosecond laser pulses

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The wave equation for laser light incident on a static, steep-gradient plasma has been solved numerically, with the inclusion of an intensity-dependent electron-ion collision rate, and incorporating spatial dispersion and wave-breaking effects. At moderate intensities ($\sim 10^{14}$ W/cm^2), the nonlinearity has little effect on the calculated reflectivity, but at higher intensities the reflectivity is greater than that predicted by a linear model. The discrepancy is small for a steplike plasma (with a density gradient scale length $L \sim 0.01\lambda_0$), but increases for longer scale lengths ($L \sim \lambda_0$). In the case of p-polarized light, the angular dependence of reflectivity is sharpened at higher intensities, and resonance effects play an important role even for very short scale lengths.

I. INTRODUCTION

Recent developments in laser technology have led to systems that can generate intense pulses of light of duration on the order of a picosecond or less [1–3]. Plasmas formed by focusing such intense femtosecond laser pulses onto solid targets are of great interest, and are fundamentally different from long-pulse plasmas produced in, for example, inertial confinement fusion experiments. Apart from possible applications as pulsed x-ray sources [4] and x-ray lasers [5], femtosecond-pulse plasmas also provide an opportunity to study hot, solid-density material, and to investigate a range of nonlinear, high-field effects.

In a femtosecond-pulse plasma, hydrodynamic expansion is limited, and the density gradient is very steep, with a scale length typically less than a laser wavelength. The lack of a large underdense region means that parametric laser-plasma coupling processes (for example, stimulated Raman scattering, stimulated Brillouin scattering, and two-plasmon decay) are greatly reduced [6]. Absorption of laser light occurs mainly through electron-ion collisions (inverse bremsstrahlung) or resonance absorption, and for a density gradient scale length much shorter than a laser wavelength, the light wave penetrates into solid-density material. The intense electric field drives various nonlinear processes, such as modifications to the electron-ion collision rate and the electron distribution function, multiphoton or field-induced ionization, and density profile modification.

Experiments have been performed to characterize a range of femtosecond-pulse plasmas in terms of temperature, density profile, and density gradient scale length [7-9]. In these cases the interpretation of the results was aided by comparing experimental data with predictions from a one-dimensional (1D) wave-propagation model. By fitting model calculations to the data, values for the scale length and collision rate were inferred. The intensities used in these experiments were in the range $10^{12}-10^{15}$ W/cm^2, where nonlinear modifications to absorption can be largely neglected.

The critical parameter for high-field effects in collisional absorption is the ratio of the quiver velocity of a free electron in the field, $v_{\rm osc} = eE_0/m\omega_0$, to the thermal velocity $v_{\rm th} = (T_e/m)^{1/2}$. Here, E_0 is the peak laser electric field, ω_0 is the laser angular frequency, T_e is the electron temperature (in energy units), and e and m are the electronic charge and mass, respectively. The ratio can be written

$$\frac{v_{\rm osc}}{v_{\rm th}} \simeq \left[\frac{I}{10^{16}\ {\rm W/cm^2}}\right]^{1/2} \left[\frac{\lambda_0}{249\ {\rm nm}}\right] \left[\frac{200\ {\rm eV}}{T_e}\right]^{1/2}, \tag{1}$$

where I is the laser intensity, and λ_0 is the laser wavelength.

At intensities such that $v_{\rm osc} \gtrsim v_{\rm th}$, collisional absorption is reduced below its low-field value [10–16]. Results from experiments performed in this high-field regime should be compared with predictions from a model which incorporates nonlinear effects.

In this paper we calculate the effect that a nonlinear reduction in the electron-ion collision rate has on the reflectivity of a static, steep-gradient plasma in the high-field regime ($I > 10^{15}$ W/cm^2). This reduction occurs in conjunction with other effects, such as modification to the electron distribution function and field-induced ionization, but we believe that the saturation of collisional absorption is important in its own right, and will remain even in a complete time-dependent treatment. In Sec. II we outline the nonlinear model, in particular the special treatment required for the case of p-polarized light. The results of the calculations are presented in Sec. III, and their relevance to femtosecond-pulse experiments is discussed in Sec. IV, where we also compare the importance of the nonlinear collision rate with other possible high-field effects. Finally, in Sec. V, we summarize the findings.

II. THE WAVE-PROPAGATION MODEL

The reflectivity of a fixed-profile plasma, assuming only collisional and resonance absorption, can be calculated by numerically solving the wave equation, for example, using a matrix method [17,18]. We use a rectangular coordinate system in which the plasma density gradient is in the x direction, and an electromagnetic plane wave is incident in the xy plane at an angle θ to the x axis. The plasma response is described by a local dielectric function, $\epsilon=\epsilon(x)$, and the plasma is assumed to be nonmagnetic, $\mu=1$. We describe potential nonlocal effects due to electron motion below. For s-polarized light, the equation for the only nonzero component of the electric field is

$$\frac{d^2E_z(x)}{dx^2}+k_0^2(\epsilon-\sin^2\theta)E_z(x)=0 , \quad (2)$$

where $E_z(x,y,t)=E_z(x)\exp(-i\omega_0 t+ik_0 y\sin\theta)$, and $k_0=\omega_0/c$ is the vacuum wave number. The corresponding equation for the nonzero component of the magnetic field in the case of p-polarized light is

$$\frac{d^2H_z(x)}{dx^2}-\frac{1}{\epsilon}\frac{d\epsilon}{dx}\frac{dH_z(x)}{dx}+k_0^2(\epsilon-\sin^2\theta)H_z(x)=0 . \quad (3)$$

In each case there are three nonzero components of E and H, but only two are linearly independent. Solving for the independent components generates a matrix equation, in which a 2×2 characteristic matrix contains all the information required for propagating an electromagnetic wave through the plasma. A continuously varying medium can be considered as the equivalent of a large number of very thin slices, and the overall characteristic matrix is then given by the product of the characteristic matrices for each slice. The electric and magnetic fields in the plasma can be calculated slice by slice, and the reflection coefficient for the medium as a whole is a function only of the overall characteristic matrix.

The dielectric function within each plasma slice is given by

$$\epsilon=1-\frac{n_e/n_{\rm cr}}{1+i\nu/\omega_0} , \quad (4)$$

where n_e is the electron density, $n_{\rm cr}=m\omega_0^2/4\pi e^2$ is the critical density, and ν is the electron-ion collision frequency. Since we do not attempt a full time-dependent treatment here, the density profile $n_e(x)$ has to be given some fixed form, for example, linear $[n_e=n_{\rm cr}(x/L)$ for $0\le x/L\le n_{\rm solid}/n_{\rm cr}$ and $n_e=n_{\rm solid}$ for $x/L>n_{\rm solid}/n_{\rm cr}]$, or exponential ($n_e=n_{\rm solid}\exp[(x-x_0)/L]$ for $0\le x\le x_0$, and $n_e=n_{\rm solid}$ for $x>x_0$, where x_0 is a parameter which cuts off the exponential tail at some large distance from the solid-density region).

For a fully ionized nondegenerate plasma, the collision rate ν would be given by the Spitzer formula [19],

$$\nu=\frac{4\sqrt{2\pi}}{3}\frac{Zn_e e^4\ln\Lambda}{m^{1/2}T_e^{3/2}} , \quad (5)$$

where Z is the degree of ionization, and $\ln\Lambda=\ln(\lambda_D/r_{\rm min})$ is the Coulomb logarithm. Here, the Debye wavelength has its usual definition, $\lambda_D=(T_e/4\pi n_e e^2)^{1/2}$, and $r_{\rm min}$ is the larger of the classical distance of closest approach, Ze^2/mv^2, and the de Broglie wavelength, $\hbar/2mv$.

For solid-density plasmas and low temperatures, strong-coupling effects and electron degeneracy become important, and the Spitzer formula is no longer strictly valid [20,21]. Numerical calculations [20] have shown that for temperatures on the order of 10 eV it may be in error by a large factor (~10). However, we will continue to use Eq. (5) as a semiempirical expression for the collision rate, because at the temperatures where we apply our model (>100 eV) the error in the Spitzer formula is relatively small, and we require an expression which is computationally straightforward.

If the laser intensity is such that $v_{\rm osc}\gtrsim v_{\rm th}$, then Eq. (5) needs modification to include the electron oscillatory motion. Nonlinear inverse bremsstrahlung has been extensively studied, but in most cases an analytic expression for the collision rate has only been obtained in the limiting cases where $v_{\rm osc}\ll v_{\rm th}$ or $v_{\rm osc}\gg v_{\rm th}$. However, Schlessinger and Wright [14] have shown that an approximate correction factor,

$$F=(1+\langle v_{\rm osc}^2\rangle/3v_{\rm th}^2)^{-3/2} , \quad (6)$$

is accurate to about 10% over a wide range of parameter values. Applying this correction factor to Eq. (5), we find that for $v_{\rm osc}\gg v_{\rm th}$, the collision rate decreases approximately as $I^{-3/2}$.

In a plasma with collisions, $\langle v_{\rm osc}^2\rangle$ can be determined by treating the electron motion as a damped, driven simple harmonic oscillator. This gives

$$\langle v_{\rm osc}^2\rangle=\frac{e^2E^2}{2m^2(\omega^2+\omega^2\nu^2)^{1/2}} , \quad (7)$$

where E is the local electric field strength. The collision rate, as calculated from Eqs. (5)–(7), thus has a complicated intensity dependence, and the wave equations (2) or (3) must be solved iteratively to obtain solutions for the fields.

In the case of p-polarized light the reduction in collision rate leads to a further complication. Since there is a component of the incident field in the direction of the density gradient, a resonant wave [22] can be driven in the plasma at the point where $n_e=n_{\rm cr}$. At low incident intensities, this resonance is strongly damped, but at higher intensities, the nonlinearity allows the resonance to grow to the point where spatial dispersion and wave breaking must be considered. These phenomena are closely linked, and both have a similar effect, limiting the maximum amplitude of the resonantly driven plasma wave.

Spatial dispersion arises from the oscillatory motion of the electrons, which generates a nonlocal dielectric function [23,24]. Using Eq. (4), and Maxwell's curl equation for $\nabla\times H$, the resonantly enhanced electric field around the critical density is seen to be

$$E_x(x')=\frac{H_z(x')\sin\theta}{\epsilon(x')}=\frac{H_z(x')\sin\theta}{x'/L+i\nu/\omega_0} . \quad (8)$$

Here, $x'=x_{\rm cr}-x$, and we have assumed that $\nu/\omega_0\ll1$, and that the density profile is locally linear with scale length L. Incorporating the oscillatory motion of the electrons indicates that Eq. (8) should more correctly be written as

$$E_x(x')=\frac{H_z(x')\sin\theta}{(x'+\int_0^T v_z dt)/L+i\nu/\omega_0} . \quad (9)$$

The dispersion term in the denominator ensures that, even with $\nu=0$, the driven field at $x'=0$ remains finite. Harmonics of the laser frequency can also be generated near the resonant point [23].

The second factor which limits the magnitude of the resonantly driven wave is wave breaking [25–30]. If an electron starting from position x_0 oscillates with a displacement given by $\xi(t)$, and the form of ξ changes with x_0 (for example, due to a density variation), then there may come a time during the growth of the oscillation when $|d\xi/dx_0|>1$. If this occurs, particles which were initially adjacent overlap, and the wave breaks. Consider the simple case of a driven, damped harmonic oscillator:

$$\frac{d^2\xi}{dt^2}+\nu\frac{d\xi}{dt}+\omega_p^2\xi=\frac{eE_d}{m}\exp(i\omega_0 t) , \quad (10)$$

where $E_d=H_z\sin\theta$ is the driving field, $\omega_p=(4\pi e^2 n_e/m)^{1/2}$ is the local plasma frequency, and $n_e=n_{cr}(x_0/L)$, that is, the density gradient scale length is locally linear. The steady-state solution to Eq. (10) is

$$\xi(t)=\frac{eE_d\exp(i\omega_0 t)}{m(\omega_0^2-\omega_p^2-i\nu\omega_0)} . \quad (11)$$

Taking the derivative with respect to x_0, we obtain the standard result: the displacement at breaking is given by

$$\xi_{br}=\left[\frac{eE_d L}{m\omega_0^2}\right]^{1/2} , \quad (12)$$

and the breaking field is

$$E_{br}=\left[\frac{m\omega_0^2 E_d L}{e}\right]^{1/2} . \quad (13)$$

So far this does not take account of spatial dispersion (the plasma frequency ω_p depends on x_0, but not on ξ). Including dispersion, Eq. (10) becomes

$$\frac{d^2\xi}{dt^2}+\nu\frac{d\xi}{dt}+\omega_p^2\xi(1+\alpha\xi)=\frac{eE_d}{m}\exp(i\omega_0 t) , \quad (14)$$

where $\alpha=1/x_0$. The solution of Eq. (14) can be obtained in terms of a Fourier series:

$$\xi=\xi_1\exp(i\omega_0 t)+\xi_2\exp(2i\omega_0 t)+\cdots . \quad (15)$$

Taking the first two terms in this expansion, we find that near resonance (for $\nu\to 0$) the displacement does not become infinite, but tends to a limit,

$$\xi_1\to\left[\frac{6eE_d L^2}{m\omega_0^2}\right]^{1/3} . \quad (16)$$

The limiting displacement from Eq. (16) is less than the breaking limit from Eq. (12), in other words, the wave will break before it reaches the maximum amplitude imposed by spatial dispersion. This is a general result which holds for higher-order expansions in Eq. (15), so in the model we need only include a mechanism for limiting the wave amplitude to the breaking value. Particle-in-cell simulations have shown that wave breaking leads to absorption and to the generation of hot electrons [31–35] but it is beyond our dielectric function approach to include multiple distributions of electron energies. Instead we model the wave breaking by including an extra term in the dissipative part of the dielectric function around the resonance region, in order to limit the field to E_{br}. This has been shown to give approximately the same total reflectivity as the full treatment [27]. Effectively, we impose a minimum value for ν, found by substituting Eq. (12) into Eq. (11):

$$|\nu|_{min}=\left[\frac{eE_d}{mL}\right]^{1/2} . \quad (17)$$

With the reduction in collision rate due to the oscillatory motion, and the correction due to wave breaking, the overall effect of laser intensity on plasma reflectivity can be calculated.

III. RESULTS

Using the model outlined above, we have calculated the reflectivity of an exponential-profile plasma as a function of the angle of incidence for two different intensities, 10^{14} and 10^{17} W/cm^2. In each case we assumed a constant set of plasma parameters: $T_e=200$ eV, $Z=10$, $n_{solid}=33.5n_{cr}$ (appropriate to the case of solid-density aluminum), and $\lambda_0=249$ nm. Calculations were performed for density gradient scale lengths of $L/\lambda_0=1$, 0.1 and 0.01.

Figure 1 shows the results for s-polarized light. At

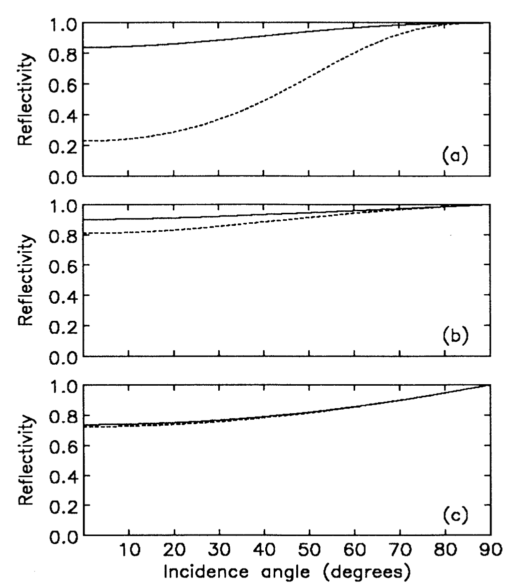

FIG. 1. Angular dependence of reflectivity for s-polarized light and scale lengths (a) $L/\lambda_0=1$, (b) $L/\lambda_0=0.1$, and (c) $L/\lambda_0=0.01$. Curves shown for 10^{14} W/cm^2 (broken line) and 10^{17} W/cm^2 (solid line).

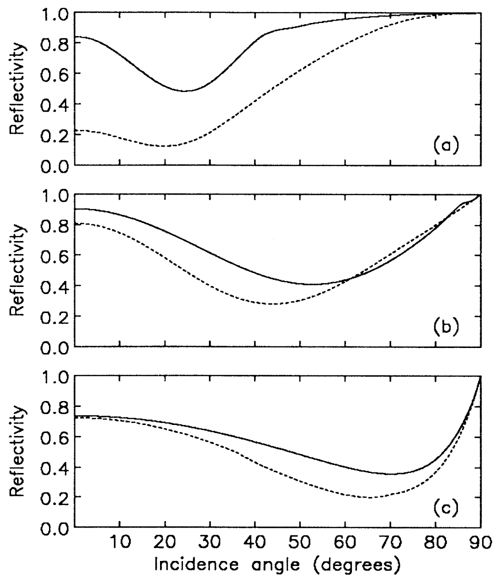

FIG. 2. Angular dependence of reflectivity, as in Fig. 1 but for *p*-polarized light.

each scale length the higher laser intensity shows a higher reflectivity, with the strongest effect for $L/\lambda_0=1$, and an almost insignificant change for $L/\lambda_0=0.01$. The shape of the curves in unaltered by variations in intensity, and in each case the reflectivity increases monotonically from $\theta=0°$ to $90°$.

The angle-dependent reflectivity for *p*-polarized light is shown in Fig. 2. In this case, there is also an overall increase in reflectivity with intensity, by the curves no longer retain their original shape, with the characteristic reflectivity minimum moving to higher angles. For $L/\lambda_0=1$, the resonance is considerably sharper at the higher intensity, and at shorter scale lengths the reflectivity over a certain angular range is still significantly increased. In contrast to the case of *s*-polarized light, an increase in laser intensity makes a considerable difference to the reflectivity at the shortest scale length ($L/\lambda_0=0.01$).

IV. DISCUSSION

Although an intensity-dependent electron-ion collision rate (in other words, nonlinear inverse bremsstrahlung) is a much-studied phenomenon, its effect on the high-intensity reflectivity of a realistic plasma is not immediately obvious. The collision rate is coupled to the local field distribution, and this nonlinearity prevents one from making a simple correction to obtain the high-field reflectivity. If the laser field is strongly attenuated inside the plasma, one might expect there to be little difference, but if the field remains significant, then the nonlinearity will increase the overall reflectivity.

In certain cases, particularly for $L/\lambda_0>0.1$ and *s*-polarized light, the increase in reflectivity due to the nonlinear collision rate is dramatic. As seen in Fig. 1, the reflectivity at normal incidence increases from 23% at 10^{14} W/cm^2 to 84% at 10^{17} W/cm^2. As the density gradient steepens, the nonlinear effect becomes weaker, until for $L/\lambda_0=0.01$ the reflectivity (for *s*-polarized light at least) is virtually independent of intensity. This change with scale length is directly related to the rate of attenuation of the field inside the plasma. For $L/\lambda_0=1$ there is a large region of underdense plasma where the field is significant, and thus the nonlinearity acts over an extended length of plasma, whereas for shorter scale lengths the field is attenuated more rapidly as the density profile approaches a step interface. For *p*-polarized light, in comparison, the increase in reflectivity with laser intensity is not as marked at long scale lengths. In all cases there is a small shift in the characteristic position for minimum reflectivity towards higher angles at higher intensity—the reduction in collision rate throughout the underdense region makes the plasma gradient appear steeper, and this has an effect on the resonance angle. In fact, resonance effects seem to remain important even for a very short scale length. For $L/\lambda_0=0.01$, in contrast to the case of *s*-polarized light, an increase in intensity still has a moderate effect on the reflectivity. For $L/\lambda_0=1$, the absorption due to the growth and breaking of the resonant plasma wave at critical density reduces the intensity-related effect significantly. The shape of the reflectivity curve changes at higher intensities because the underlying component of collisional absorption is reduced, and only the resonance component remains. In fact, for very high intensities the situation is quite similar to collisionless resonance absorption.

Analytical solutions to the problem of plasma reflectivity are available in a number of specific cases. For a vacuum-to-plasma step interface and a constant collision rate, the Fresnel equations describe reflectivity for *s*-polarized and *p*-polarized light in terms of the angle of incidence and the (complex) angle of refraction [18]. For a dielectric function which varies slowly compared with a laser wavelength, a WKB solution can be obtained in the case of *s*-polarized light [6]. We have used the Fresnel equations and the WKB solution as a check for the accuracy of our calculation method. However, none of these approximate or exact analytic solutions are valid in the case where the dielectric function varies with the local field.

Our model is inherently a static one, and assumes a fixed density profile for the plasma. In this way, it is difficult to include time-dependent phenomena, both related to the electrons (growth and breaking of plasma waves), and the ions (expansion of the plasma, and ponderomotive density profile steepening). Density profile steepening in particular is expected to be quite significant. Particle-in-cell simulations [30,31] have shown that an initially linear density profile can develop a sharp step around the critical density, as a result of pressure generated both by the reflecting laser beam and by the resonantly driven electrostatic wave. The time taken for this feature to develop is on the order of 60 laser cycles, in simulations with an electron-ion mass ratio of 1:100. In the present case, with an aluminum plasma, the mass ratio is almost 500 times smaller, so we would expect the characteristic time to be appropriately longer. However, in the case of *p*-polarized light, movement across the narrow resonance region can occur on a much shorter time scale, given the enhanced electric field and the small size of the region itself. Our model predicts that the local field can be more than an order of magnitude larger than the incident field before wave breaking occurs and, in addition to causing profile modification, this extremely large field in a buried layer of the plasma could lead to enhanced ionization due to direct field-induced processes (multiphoton ionization, tunneling, or over-the-barrier ionization). The detailed investigation of this time-dependent behavior goes far beyond the scope of our simple model.

A further high-field effect which we have been forced to neglect is a modification to the electron distribution function. Langdon [15] has shown that even for intensities where $v_{osc} \simeq v_{th}$, the resultant non-Maxwellian distribution can reduce the inverse bremsstrahlung absorption in certain cases by more than half. At very high fields, where $v_{osc} \gg v_{th}$, the absorption is actually independent of the exact form of the distribution (as long as it remains isotropic) [16], but in between these two limits one expects the modification to be important. If a self-consistent distribution function were used in place of the present Maxwellian, the reflectivity would increase more than predicted by our model, but in the cases of very short scale length (where the field decays rapidly inside the plasma) or *p*-polarized light (where the response is dominated by resonance effects) we would not expect the difference to be large. Wave breaking will result in hot electron generation, but given the extremely high effective temperature their effect on the averaged collision rate is likely to be small. In the results presented, we have also assumed that the electron temperature is the same at the higher intensity; if allowance were made for an increase in temperature at higher intensities, then the reflectivity would be expected to increase still further.

V. CONCLUSIONS

Reflectivity experiments performed in the high-intensity regime ($>10^{15}$ W/cm^2) can no longer be modeled by a linear wave equation. Once the incident intensity is such that $v_{osc} \gtrsim v_{th}$, the nonlinearity in the collision rate will act to increase the reflectivity above that predicted by a linear model. The effect is most severe for *s*-polarized light, and for density gradient scale lengths $L/\lambda_0 > 0.1$. A high fractional absorption can still be achieved, but only in the cases where the laser pulse is so short that expansion is restricted to $<0.1\lambda_0$, or *p*-polarized light is used near the optimum angle of incidence.

The reduction in collision frequency at high intensities suggests that for *p*-polarized light and even for very short scale lengths a large-amplitude resonance wave can be driven at the critical density. The intense electric field in this wave could result in rapid density profile modification and enhanced ionization, and wave breaking would lead to the production of very hot electrons. The full investigation of these phenomena would require a time-dependent treatment which includes a dynamical treatment of the electron and ion populations.

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