PHYSICAL REVIEW A VOLUME 44, NUMBER 5 1 SEPTEMBER 1991

Sub-Poissonian laser light by dynamic pump-noise suppression

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(Received 16 January 1991)

We identify a mechanism of dynamical pump-noise suppression in lasers. It is based on the recycling of the active laser electron from the lower to the upper laser level by a sequence of incoherent step processes. Although each of these steps corresponds to a Poisson process, i.e., is stochastic, the combination of many incoherent steps leads to a regular (deterministic) recycling of the laser electron and, correspondingly, a pump-noise suppression in the laser. The mechanism predicts sub-Poissonian laser output and intensity fluctuations beyond the shot-noise limit for incoherently pumped systems.

Recently considerable effort has been made to reduce the intensity fluctuations in the laser output by suppressing pump fluctuations. There have been a number of theoretical models [1–9] which have shown that given the pump fluctuations are reduced, a sub-Poissonian photon output is possible. Sub-Poissonian output in semiconductor lasers has been achieved by Yamamoto and coworkers [3,10] and Richardson and Shelby [11] who attribute the noise reduction to noise suppression in the electron current by space change effects. In all the above models the regularization of the pump is treated as being generated externally. This may be by space-charge suppression of the electron current in semiconductor lasers [3,10,11] or by pumping with amplitude squeezed light [5–9] or a sequence of regularly spaced short laser pulses [1,2].

In this paper we shall introduce a mechanism of dynamic pump suppression where the dynamics of the pumping mechanism are included as an integral part of our laser model. This mechanism relies on having a number of incoherent step processes in recycling of the active laser electron from the lower to the upper laser level. Such a succession of incoherent steps may occur, for example, in recycling an electron in a semiconductor material or in a multilevel atom. The mechanism we identify gives an explanation for the sub-Poissonian statistics of light predicted in calculations with three-level laser systems [12].

The requirements for a laser to exhibit sub-Poissonian behavior are as follows.

(i) The active laser electron is recycled from the lower state of the laser transition to the upper laser state via a multistep process. These processes are assumed to be incoherent and will, in practice, be a sequence of pump and spontaneous transitions. In addition, the atomic system for the recycling of the electron to the upper state has to be a closed m-level system, i.e., there is no population loss in the pump cycle. Furthermore, we require the various pump and spontaneous transition rates to be of the same order of magnitude.

(ii) The number N of active atoms should be constant. This is the case, for example, for a solid-state laser provided the quantum efficiency of the pump is close to unity.

We will show that under the above assumptions the cycling of the active atomic electron becomes regular in time and the pump noise of the laser is suppressed. Such a mechanism has not been considered in standard laser theory [13–16]. In existing laser theories the electron is recycled from the lower to the upper laser level in a single-step Poisson process in two-level models [13], or the electrons are excited by an incoherent weak pump field in three-level models [14]. In both cases the pumping process contributes to the shot noise.

As a first example we consider the four-level system shown in Fig. 1. The laser transition is $|1\rangle - |2\rangle$ with the dipole transition matrix element μ. From $|1\rangle$ electrons decay to $|4\rangle$ where they are incoherently pumped to $|3\rangle$ and decay by spontaneous emission to $|2\rangle$. The atomic transition rates from $|i\rangle \leftarrow |j\rangle$ denoted by w_{ij}; Γ_{ij} are the corresponding transverse damping rates. In the good cavity limit one can adiabatically eliminate the atoms and derive a Fokker-Planck equation for a generalized P representation $P(\alpha,\alpha^\dagger,t)$ for the cavity mode [15,16]. The associated Langevin equation is

$$\dot{\alpha} = -\tfrac{1}{2}\gamma\alpha\left[1-\frac{Cd_0}{1+\alpha^\dagger\alpha/n_s}\right]+\zeta_\alpha(t) \ . \tag{1}$$

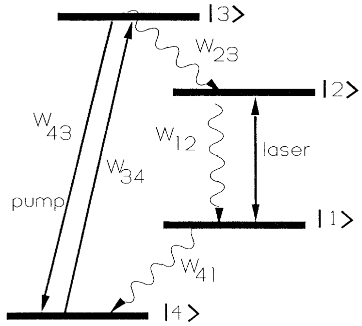

FIG. 1. Four-level laser scheme. $|2\rangle-|1\rangle$ and $|4\rangle-|3\rangle$ are the laser and pump transition, respectively.

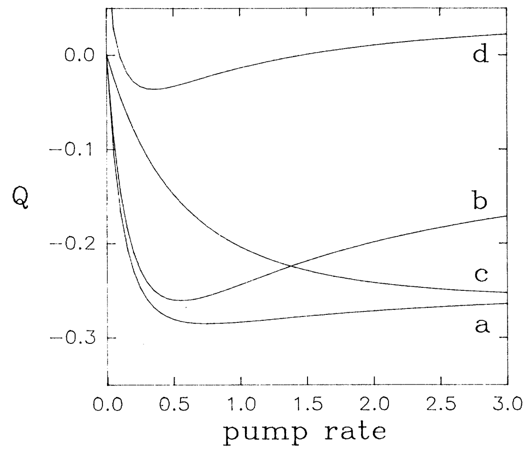

FIG. 2. Mandel Q parameter as a function of the pump rate w_{34}. The parameters for the various curves are $w_{23}=\frac{3}{2}$, $A_{34}=w_{12}=0$ (curve a); $w_{23}=10$, $A_{34}=w_{12}=0$ (curve b); $w_{23}=\frac{3}{2}$, $A_{34}=5$ (curve c); $w_{23}=\frac{3}{2}$, $A_{34}=0$, $w_{12}=\frac{1}{4}$ (curve d). All rates are in units of w_{14}.

$\alpha^\dagger$ obeys a similar equation obtained by the replacement $\alpha \leftrightarrow \alpha^\dagger$. In (1) the first term is the cavity damping while the second term describes the laser gain; γ is the damping constant, $C=N2\mu^2/\Gamma_{12}\gamma$ the cooperativity parameter, d_0 the zero-cavity-field population inversion, and n_s the saturation photon number. $\zeta_\alpha(t)$ is a Langevin force with diffusion coefficients [16]

$$\begin{aligned} D_{\alpha\alpha^\dagger} &= N\mu^2\int_0^\infty d\tau[\langle\sigma^+,\sigma^-(\tau)\rangle+\langle\sigma^+(\tau),\sigma^-\rangle] , \\ D_{\alpha\alpha} &= N\mu^2\int_0^\infty d\tau\langle\sigma^-(\tau),\sigma^-\rangle , \\ D_{\alpha^\dagger\alpha^\dagger} &= N\mu^2\int_0^\infty d\tau\langle\sigma^+,\sigma^+(\tau)\rangle . \end{aligned} \tag{2}$$

Here $\sigma^\pm(\tau)$ refers to the atomic raising (lowering) operator in the Heisenberg picture on the laser transition. The atomic correlation functions in Eq. (2) have to be evaluated with the help of the quantum regression theorem from optical Bloch equations for the multilevel system, where the interaction of the atom with the cavity mode is represented by c-number amplitudes $\alpha,\alpha^\dagger$ [16]. The quantity of interest in our context is the Mandel Q parameter which is defined by $(\langle\hat{n}^2\rangle-\langle\hat{n}\rangle^2)/\langle\hat{n}\rangle=1+Q$ with $\hat{n}$ the photon number operator of the cavity mode. Q measures deviations from Poisson statistics. The intensity fluctuation spectrum of the laser light is related to Q by [1]

$$S(\omega)=1+2Q\frac{\lambda^2}{\omega^2+\lambda^2} , \tag{3}$$

where the first term is the shot-noise contribution. λ is the intensity correlation time of the laser ($\lambda\approx\gamma$ far above threshold $Cd_0 \gg 1$). For $-\frac{1}{2}\le Q<0$ the second term becomes negative and we have noise reduction below the shot-noise level. We have solved Eq. (1) for m-level systems and have derived analytical expressions for the Mandel Q parameter [17]. For the four-level system of Fig. 1 we find

$$Q=-\frac{w_{23}w_{34}w_{41}(2w_{23}+2w_{34}+w_{41}+2w_{43})}{(2w_{23}w_{34}+w_{23}w_{41}+w_{34}w_{41}+w_{41}w_{43})^2} . \tag{4}$$

Note that Q is negative.

Figure 2 shows Q as a function of the pump rate w_{34} for the four-level system (Fig. 1), $Cd_0 \gg 1$ and various combinations of atomic decay rates. All of these curves show a minimum of Q for some intermediate value of w_{34}. If we write $w_{43}=A_{34}+w_{34}$ with A_{34} the Einstein coefficient for the pump transition, we find from (4) that the minimum occurs at $w_{23}=\frac{3}{2}$, $A_{34}=0$, and $w_{34}=\frac{3}{4}$ in units of w_{41} and has a value of $Q=-\frac{2}{7}\approx-0.28$ (Fig. 2, curve a). In the limit that the decay w_{23} becomes very fast all electrons excited to $|3\rangle$ are transferred to $|2\rangle$ and the four-level system reduces to an effective three-level system. In this case we find a minimum $Q=-\frac{1}{4}$ for $w_{34}=\frac{1}{2}$ (curve b), a result which is consistent with Ref. [12]. The occurrence of negative Q values is not particularly sensitive to spontaneous decay on the pump transition ($w_{23}=\frac{3}{2}$, $A_{34}=5$, curve c). The presence of a decay on the laser transition, however, tends to destroy the effect ($w_{23}=\frac{3}{2}$, $A_{34}=0$, $w_{12}=\frac{1}{4}$, curve d). Finally, if one assumes a model with unidirectional rates from $|1\rangle$ to $|2\rangle$ ($w_{43}=0$ in Fig. 1), the minimum value $Q=-\frac{1}{3}$ occurs at $w_{34}=w_{23}=\frac{1}{2}w_{14}$.

The dependence of the Mandel Q parameter on the number of atomic levels is discussed in Figs. 3 and 4. Let us consider an m-level scheme shown in Fig. 3. Again $|2\rangle\rightarrow|1\rangle$ is the laser transition and electrons are recycled from $|1\rangle$ back to the upper state $|2\rangle$ via the levels $|m\rangle,\ldots,|3\rangle$. Each of these steps is modeled by a unidirectional incoherent rate process. Figure 4 shows the Mandel Q parameter as a function of the cooperativity parameter C for $m=3,4,\ldots,10$ assuming optimum conditions of matched rates,

$$w_{23}=\cdots=w_{m-1m}=\tfrac{1}{2}w_{m1} , \tag{5}$$

and no decay on the laser transition, $w_{12}=0$. We see that for large C the Mandel Q parameter decreases with increasing number of atomic levels. We have been able to

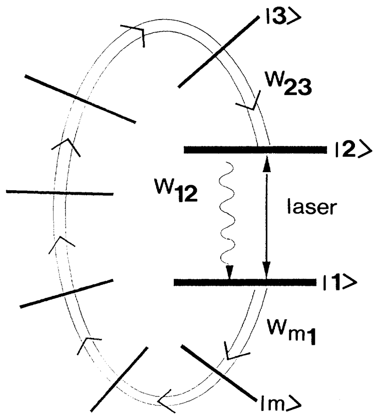

FIG. 3. Laser scheme with m levels.

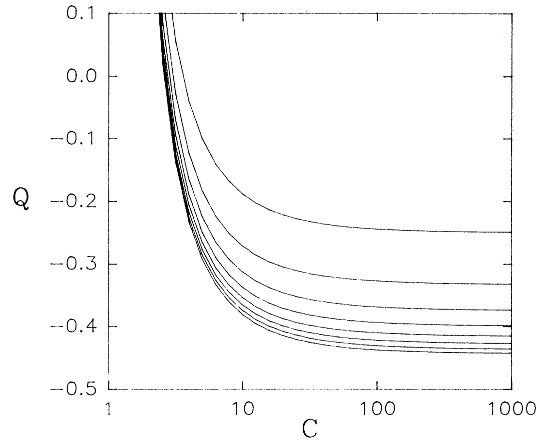

FIG. 4. Mandel Q parameters as a function of the cooperativity parameter C for $m=3,4,\ldots,10$ levels (compare Fig. 3). With increasing m Q approaches $-\frac{1}{2}$ for $C \gg 1$.

show that for operating conditions of the laser where the photon number is much larger than the saturation photon number ($C \gg 1$) the Mandel Q parameter is given by [17]

$$Q=-\frac{1}{2}\frac{m-2}{m-1} \quad (C \gg 1) \tag{6}$$

which predicts $Q=0$ for the two-level system, $Q=-\frac{1}{4}$ for the three-level system, $Q=-\frac{1}{3}$ for four levels, and $Q \to -\frac{1}{2}$ for $m \gg 1$ which corresponds to complete noise suppression.

To identify the mechanism of noise suppression we note that for a single step $|i\rangle \to |i-1\rangle$ with rate r the conditional probability for the electron jumping to $|i-1\rangle$ in the time interval $[t, t+dt)$, provided it was prepared in $|i\rangle$ at time $t=0$, is given by an exponential decay law $\tilde{c}(t)=re^{-rt}$. On the other hand, $m-1$ consecutive rate steps lead to [6]

$$\tilde{c}(t)=\frac{r(rt)^{m-2}}{(m-2)!}e^{-rt} . \tag{7}$$

Thus the recycling of the electron through these steps is anticorrelated in the sense that $\tilde{c}(t=0)=0$ for $m>2$. For fixed mean jump time $\tilde{c}(t)$ approaches a δ function when m tends to infinity. In this limit there are no more fluctuations and the stochastic process becomes deterministic. In Ref. [13] we have considered a laser model where atoms are injected in state $|2\rangle$ according to the stochastic process (7) by some *external* mechanism, and found a Mandel Q identical to Eq. (6). The physical picture of the noise suppression, therefore, is that the multistep process in Fig. 3 leads to a *regular recycling* of the laser electrons. The important difference in comparison with pump-noise suppression by pumping with squeezed light or a sequence of regularly spaced short laser pulses is that the origin of the regularization is part of the laser dynamics and not generated externally. Note that the laser is pumped incoherently. We emphasize that fluctuations in the number of atoms and decay of the atoms out of the closed excitation cycle leads to noise and degrades the effect. For a fixed number of atomic levels m and the model of Fig. 3 optimum noise suppression is obtained for the condition stated in Eq. (5); if one of the rates is much slower than the other ones a bottleneck occurs which leads again to Poissonian fluctuations and pump shot noise. We have also shown that Q tends to increase in the bad cavity limit [17].

Note added. After this work was completed we received a paper by T. C. Ralph and C. M. Savage prior to publication on amplitude squeezing spectra in three- and four-level laser systems.

We thank M. Marte, R. Graham, and C. Savage for discussions. P.Z. thanks the University of Auckland and the University of Waikato for hospitality where part of this work was done. H.R. and P.Z. were supported by the Fonds zur Förderung der Wissenschaftlichen Forschung under Grant No. P7295.

[1] Y. Golubev and I. V. Sokolov, Zh. Eksp. Teor. Fiz. **87**, 408 (1984) [Sov. Phys.—JETP **60**, 234 (1984)].

[2] F. Haake, S. M. Tan, and D. F. Walls, Phys. Rev. A **40**, 7121 (1989); **41**, 2808 (1990).

[3] Y. Yamamoto, S. Machida, and O. Nilsson, Phys. Rev. A **34**, 4025 (1986); Y. Yamamoto and S. Machida, *ibid.* **35**, 5114 (1987).

[4] M. A. M. Marte, H. Ritsch, and D. F. Walls, Phys. Rev. Lett. **61**, 1093 (1988).

[5] A. W. Troshin, A. I. Trubilko, and R. Ibarra, Opt. Spektrosk. **65**, 1195 (1988) [Opt. Spectrosc. (USSR) **65**, 674 (1988)].

[6] M. A. M. Marte and P. Zoller, Phys. Rev. A **40**, 5774 (1989).

[7] T. A. B. Kennedy and D. F. Walls, Phys. Rev. A **40**, 6366 (1989).
[8] M. A. M. Marte and P. Zoller, Quantum Opt. **2**, 229 (1990).
[9] C. Benkert, M. O. Scully, J. Bergou, L. Davidovich, M. Hillery, and M. Orszag, Phys. Rev. A **41**, 2756 (1990).
[10] S. Machida, Y. Yamamoto, and Y. Itaya, Phys. Rev. Lett. **58**, 1000 (1987); S. Machida and Y. Yamamoto, *ibid.* **60**, 792 (1988).
[11] W. H. Richardson and R. M. Shelby, Phys. Rev. Lett. **64**, 400 (1990).
[12] A. M. Khazanov and G. A. Koganov, Phys. Rev. A **42**, 3065 (1990).
[13] M. Lax and W. H. Louisell, Phys. Rev. A **185**, 568 (1969).
[14] H. Haken, *Laser Theory* (Springer-Verlag, Berlin, 1984), Vol. 2.
[15] F. Haake and M. Lewenstein, Phys. Rev. A **27**, 1013 (1983).
[16] C. W. Gardiner, *Quantum Noise* (Springer-Verlag, Berlin, 1991), Sec. 9.3.2.
[17] H. Ritsch and P. Zoller (unpublished).