Berry's phase in rotating systems

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It is shown that, in addition to the Aharonov-Bohm-like phase studied previously [M. V. Berry, Proc. R. Soc. London Ser. A **392**, 45 (1984); Y. Aharakov and J. Anandan, Phys. Rev. Lett. **58**, 1593 (1987); C. H. Tsai and D. Neilson, Phys. Rev. A **37**, 619 (1988)], Berry's topological phase also appears for purely mechanical reasons in systems rotating at slowly-time-varying angular velocity about a fixed center. A possible experiment to probe this manifestation of Berry's phase is discussed.

As first recognized by Berry [1], the wave function of a quantum-mechanical system will acquire a topological phase factor $\exp[i\gamma(C)]$ in addition to the familiar dynamical phase factor $\exp[i\int dt\, E(t)]$ if the evolution of the system is induced through a Hamiltonian which is varied adiabatically around a closed path C in parameter space. This new topological phase has been of interest in a number of recent investigations, including the Aharonov-Bohm effect [2], fractional statistics [3], the quantum Hall effect [4], non-Abelian gauge theories [5], and chiral anomalies [6]. In addition, various experimental verifications of Berry's phase have been reported via techniques such as photointerference [7], neutron-spin rotation [8], polarized-light rotation [9], electron diffraction [10], nuclear magnetic resonance [11], and laser interferometry [12]. Berry's result has also been generalized [13,14] further to cases where the adiabatic assumption and cyclic evolution in parameter space are not necessary.

Reference [15] is a discussion of a quantum interference effect (the mechanical counterpart of the Aharonov-Bohm effect) in general rotating systems. Recently this effect has been experimentally verified [16]. The purpose of this Brief Report is to study further another related phenomenon, i.e., the topological effect, known as Berry's phase, in general rotating systems, and suggest a possible experimental test of this effect.

Consider a transformation from an inertial frame $\mathbf{r}'=(x',y',z')$ to a reference frame $\mathbf{r}=(x,y,z)$ attached to a physical system that is being rotated at slowly varying angular velocity $\boldsymbol{\omega}(t)$ about a fixed center related to the inertial frame. By means of a canonical transformation in the active sense, the dynamical system with a Hamiltonian in the inertial frame

$$\hat{H}'=-\frac{\hbar^2}{2m}\nabla_{r'}^2+U(r')\ , \tag{1}$$

with $U(r')$ a central-field potential (for a free particle, $U\equiv 0$), can be shown [15] to be described equivalently in the rotating frame by a time-dependent Hamiltonian with $\boldsymbol{\omega}(t)$ as a parameter:

$$\hat{H}(\omega)=-\frac{h^2}{2m}\nabla_r^2+U(r)-\boldsymbol{\omega}(t)\cdot\hat{\mathbf{L}}\ , \tag{2}$$

where $\mathbf{L}$ is the angular momentum of the system. The last term in expression (2) might be inferred from the Larmor theorem as discussed by Richardson *et al.* for the case of a neutron spin [8]; however, it is here derived exactly and no approximation is implied. The instantaneous eigenstates of $\hat{H}$, Eq. (2), can be written as

$$\psi_{lm}(\boldsymbol{\omega})=R_l(r)Y_{lm}(\Theta(\boldsymbol{\omega}),\Phi(\boldsymbol{\omega}))\ , \tag{3}$$

where $R_l(r)$ are radial functions depending on $U(r)$, Y_{lm} are spherical harmonics, and $\Theta(\boldsymbol{\omega})$ and $\Phi(\boldsymbol{\omega})$ are the functions of the spherical coordinates θ and ϕ related by an orthogonal matrix $\hat{A}[\boldsymbol{\omega}(t)]$:

$$\begin{bmatrix}\sin\Theta\cos\Phi\\ \sin\Theta\sin\Phi\\ \cos\Theta\end{bmatrix}=\hat{A}[\boldsymbol{\omega}(t)]\begin{bmatrix}\sin\theta\cos\phi\\ \sin\theta\sin\phi\\ \cos\theta\end{bmatrix}\ . \tag{4}$$

The corresponding eigenvalues are

$$E_{lm}(\boldsymbol{\omega})=E_l-m\omega\hbar\ , \tag{5}$$

where E_l is the eigenvalue of $\hat{H}'$. Equation (5) shows that there is a $(2l+1)$-fold degeneracy when $\boldsymbol{\omega}=0$.

When $\boldsymbol{\omega}(t)$ changes slowly enough in a periodic manner, a Berry's phase shift will be induced. Following the formula given by Berry [1], the phase shift is readily obtained using Eqs. (3) and (5):

$$\mathbf{V}_{lm}(\boldsymbol{\omega})=m\frac{\boldsymbol{\omega}}{\omega^3} \tag{6}$$

and the Berry's phase factor

$$\exp[i\gamma_{lm}(\boldsymbol{\omega})]=\exp[-im\Omega(C)]\ . \tag{7}$$

Here, m is the eigenvalue of the component of the orbital angular momentum along $\boldsymbol{\omega}$ and $\Omega(C)$ is the solid angle that C subtends at $\boldsymbol{\omega}=\mathbf{0}$. It is interesting to note that this phase exists purely due to mechanical effects, i.e., due to the rotation.

We now conceive an experiment to verify the solid-angle dependence of the Berry's phase, consisting of an apparatus mounted on a table that can be rotated at an arbitarily time-varying angular velocity about a fixed center—for example, given by

$$\boldsymbol{\omega}=(\omega\cos\phi,\omega\sin\phi\cos(\omega_0 t),\omega\sin\phi\sin(\omega_0 t))$$

for $0<t<NT$, $T=2\pi/\omega_0$, with ω, ω_0, and ϕ each independently adjustable. Let us consider an atom beam, initially in a spatial state $l=m=1$ along the $\hat{\mathbf{z}}$ direction, selected by passage through a magnetic filter. The atom beam should be chosen as B, Al, or Ga with ground state $l=1$, because for free particles and atoms such as H, Na and K with ground state $l=0$, it is impossible to observe the effect of Berry's phase in rotating systems. The behavior of the atom beam in general rotating systems is described by the time-dependent Schrödinger equation with the Hamiltonian of Eq. (2), which is readily solved by assuming

$$\psi_l(t)=\exp\left[-\frac{i}{\hbar}E_l t\right]R_l(r)\sum_{m=-l}^{+l}a_m(t)Y_{lm}(\theta,\phi)\ . \qquad (8)$$

Since the initial state is chosen to be $a_{+1}(0)=1$, $a_0(0)=a_{-1}(0)=0$, we obtain

$$a_{+1}(t)=\tfrac{1}{2}\exp(i\beta)\{\sin(\Omega t)\sin(\omega_0 t)+[\cos\beta\cos(\omega_0 t)-i\sin\beta]\cos(\Omega t)+\cos\beta-i\sin\beta\cos(\omega_0 t)\}\ ,$$

$$a_0(t)=\frac{\sqrt{2}}{2}\exp(i\beta)\{i\sin(\Omega t)\cos(\omega_0 t)-i[\cos\beta\cos(\Omega t)-i\sin\beta]\sin(\omega_0 t)\}\ , \qquad (9)$$

$$a_{-1}(t)=\tfrac{1}{2}\exp(i\beta)\{\sin(\Omega t)\sin(\omega_0 t)+[\cos\beta\cos(\omega_0 t)+i\sin\beta]\cos(\Omega t)-\cos\beta-i\sin\beta\cos(\omega_0 t)\}\ ,$$

where

$$\tan\beta=\frac{\omega\sin\phi}{\omega\cos\phi+\omega_0} \qquad (10)$$

and

$$\Omega=\sqrt{(\omega_0^2+\omega^2+2\omega_0\omega\cos\phi)}\ . \qquad (11)$$

The total phase for a cyclic evolution is then

$$\Omega T=2\pi\left[1+\frac{\omega^2}{\omega_0^2}+2\frac{\omega}{\omega_0}\cos\phi\right]^{1/2}-2\pi\ , \qquad (12)$$

where the extra term -2π insures that $\Omega T=0$ when $\omega=0$. In the adiabatic limit $(\omega\gg\omega_0)$

$$\Omega T=\omega T-2\pi(1-\cos\phi)$$
$$=\omega T-\Omega(C)=\phi_+ +\gamma_+(C)-\phi_- -\gamma_-(C)\ , \qquad (13)$$

with ϕ_+ and ϕ_- the dynamical phase and $\gamma_+(C)$ and $\gamma_-(C)$ the Berry's phase corresponding to $m=+1$ and $m=-1$, respectively. In this case, from Eq. (9), we immediately have

$$|a_{+1}(T)|^2=\cos^4\{[\phi_+ +\gamma_+(C)]/2\}\ ,$$
$$|a_0(T)|^2=\tfrac{1}{2}\sin^2\{[\phi_+ +\gamma_+(C)]/2 -(\phi_- +\gamma_-(C))/2\}\ , \qquad (14)$$
$$|a_{-1}(T)|^2=\sin^4\{[\phi_- +\gamma_-(C)]/2\}\ .$$

We note from Eq. (14) [or Eq. (9)] that the change of the atomic state is related to Berry's phase and therefore reflects the additional topological effects.

Therefore, after N cycles of evolution of the system at time $t=NT$, we can measure the states of atoms by again using a magnetic filter, and then extract the Berry's phase from the data.

In conclusion, we have shown in this paper that Berry's phase appears in general rotating systems purely for mechanical reasons. A possible experiment is suggested, which could easily be constructed to probe Berry's topological phase.

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[1] M. V. Berry, Proc. R. Soc. London Ser. A **392**, 45 (1984).
[2] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485 (1959).
[3] D. Arovas, J. R. Schrieffer, and F. Wilczek, Phys. Rev. Lett. **53**, 722 (1984); F. D. M. Haldane and Y. S. Wu, *ibid.* **55**, 2887 (1985).
[4] D. Thouless, M. Kohmoto, M. Nightingale, and M. den Nijs, Phys. Rev. Lett. **49**, 405 (1982); G. W. Semenoff and P. Sodano, *ibid.* **57**, 1195 (1986).
[5] F. Wilczek and A. Zee, Phys. Rev. Lett. **52**, 2111 (1984); J. Moody, A. Shapere, and F. Wilczek, *ibid.* **56**, 893 (1986); E. Gozzi and W. Thacker, Phys. Rev. D **35**, 398 (1987).
[6] P. Nelson and L. Alvarez-Gaume, Commun. Math. Phys. **99**, 103 (1985); H. Sonoda, Nucl. Phys. B **266**, 410 (1986); A. J. Niemi, G. W. Semenoff, and Y. S. Wu, Nucl. Phys. B **276**, 173 (1986).
[7] R. Y. Chiao and Y. S. Wu, Phys. Rev. Lett. **57**, 933 (1986).
[8] T. Bitter and D. Dubbers, Phys. Rev. Lett. **59**, 251 (1987); D. J. Richardson, A. L. Kilvington, K. Green, and S. K. Lamoreaux, *ibid.* **61**, 2030 (1988); H. Weinfurter and G. Badurek, *ibid.* **64**, 1318 (1990).
[9] A. Tomita and R.Y. Chiao, Phys. Rev. Lett. **57**, 937

(1986).
[10] D. M. Bird and A. R. Preston, Phys. Rev. Lett. **61**, 2863 (1988).
[11] R. Tycko, Phys. Rev. Lett. **58**, 2281 (1987); D. Suter, K. T. Mueller, and A. Pines, *ibid.* **60**, 1218 (1988).
[12] R. Simon, H. J. Kimble, and E. G. C. Sudarshan, Phys. Rev. Lett. **61**, 19 (1988).
[13] Y. Aharonov and J. Anandan, Phys. Rev. Lett. **58**, 1593 (1987); M. V. Berry, Proc. R. Soc. London Ser. A **414**, 31 (1987).
[14] T. F. Jordan, Phys. Rev. A **38**, 1590 (1988); J. Samuel and R. Bhandari, Phys. Rev. Lett. **60**, 2339 (1988); Y. S. Wu and H. Z. Li, Phys. Rev. B **38**, 11 907 (1988).
[15] C. H. Tsai and D. Neilson, Phys. Rev. A **37**, 619 (1988); H. H. Xu and C. H. Tsai, *ibid.* **41**, 4046 (1990).
[16] F. Hasselbach and M. Nicklaus (unpublished).