Population trapping in two-level models: Spectral and statistical properties

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The interaction of a two–level atom with a single-mode radiation field through multiphoton transitions is studied in terms of the dressed-state formalism. It is shown that when only certain dressed states are initially excited, population trapping occurs. We particularize these results for some well-known models of one- and two-photon transitions. Photon statistics of the cavity mode is analyzed in terms of the quasiprobability distribution $Q(\alpha)$. It is shown that when population trapping occurs some two-time correlation functions become stationary, which allows one to obtain the resonance spectrum as the Fourier transform of the two-time dipole autocorrelation function.

I. INTRODUCTION

The recent experiments on single Rydberg atoms involving one- and two-photon transitions in a high-Q microwave cavity have opened new ways for experimental studies of the interaction of a single two-level atom with a single mode of the quantized radiation field [1]. Since photons are bosons and the single atom always acts as a fermion, the interaction is intrinsically nonlinear, which provides many characteristics absent in linear models.

The problem of a two-level atom interacting with a single-mode field has been studied in considerably detail. The Jaynes-Cummings model (JCM) is the prototype model to describe such a situation [2]. Within the rotating-wave approximation (RWA), this model can be solved exactly, exhibiting many features without classical analogy, perhaps the most important being the collapses and revivals of the atomic inversion [3]. This behavior, that is a clear signature of the discreteness of the quantized field, has been found for several initial states of the field, and has recently been studied using the Q function [4] which provides a method to explain this effect in terms of interference in phase space.

The phenomenon of collapses and revivals depends on the initial field statistics, but not on the phase of the field if the atom is initially in one of its two states. However, if the atom is prepared in a coherent superposition of the excited and ground levels, the excitation probability depends on the relative phase between the atomic coherence and the exciting field. This phase sensitivity in the atom-field interaction provides many useful ways of determining not only nonclassical properties, but also the means of testing the predictions of the quantum theory of radiation against semiclassical and neoclassical theories [5], as well as having applications to noise quenching by correlated spontaneous emission [6], quantum beats [7], and noise-free amplification [8]. Recently, Zaheer and Zubairy [9] have shown that in the JCM with a pure and decoupled initial state, under certain phase-matching condition between the atomic dipole moment and the field, the amplitude of the oscillations of the atomic inversion becomes arbitrarily small when the field is initially in a coherent state and in the strong-field limit. In addition, we have shown [10] that when besides this phase condition the atom and the field are in thermal equilibrium, the oscillations are suppressed and therefore population trapping occurs. The states of the field fulfilling this last condition are eigenstates of the well-known Susskind-Glogower phase operator [11]. Slosser, Meystre, and Braunstein [12] have considered the evolution of a single mode of the field driven by a current of two-level atoms, each interacting with the mode for a time τ, a problem widely studied in the context of micromaser theory [13], showing that under certain trapping conditions it evolves towards a new class of pure states which remain unchanged after each interaction time τ. On the other hand, Gea-Banacloche [14] has pointed out that in the JCM, during the collapse time, the atom almost remains in a trapped pure state, which could be used to prepare atoms in certain linear superpositions of their levels.

The evolution of the atomic inversion in the semiclassical version of the JCM has a simple periodic behavior although Bai *et al.* [15] have shown within this model that when the atom-field system is prepared in a "dressed state" the atom remains stationary. However, in the fully quantum case this evolution is more complicated due to the existence of a photon statistics for the field; the resulting Rabi frequencies are noncommensurate and the atomic inversion must be expressed as a sum which cannot be given in a closed form. Several models involving interactions which give rise to commensurate Rabi frequencies have been introduced these last years in order to obtain exactly summable series for the atomic inversion, perhaps the most interesting being the intensity-dependent coupling [16] and the Raman-coupled model [17], in which two degenerate levels are coupled through a virtual level by a Raman-type transition. These models are exactly summable and the dynamics becomes periodic.

Rydberg atoms are also well suited to study the coupling field atom when the atom makes two or more photon transitions. The multiphoton generalizations of the JCM are sufficently complex to be nontrivial but still they yield nonperturbative results, although the construction of a real multiphoton oscillator results in a formidable experimental problem [18].

In this paper we are concerned with population trapping in two-level systems interacting through m photons with a single mode of the radiation field. We use the dressed-state formalism to explain the features of this trapping, since it gives a simple explanation of this phenomenon independently of the type of interaction. We particularize for the Raman-coupled, the standard two-photon transition [19], the JCM, and the intensity-dependent coupling model. We show that this trapping can be viewed as a consequence of the existence of coherent trapping [20] in three-level atoms after adiabatically eliminating the intermediate level.

The dressed-state formalism allows one to explain as well that under trapping conditions some two-time correlation functions become stationary, which permits one to find the spectrum as the Fourier transform of the two-time dipole autocorrelation function. This spectrum is seen to be a direct consequence of transitions between dressed levels.

In Sec. II we introduce a general Hamiltonian describing the interaction of a two-level atom with an ideal cavity mode. The dressed states and the trapping conditions for this Hamiltonian are given. In Sec. III we study the evolution of the Q function when the trapping condition is fulfilled. In Sec. IV we study some stationary two-time correlation functions and the spectrum emitted by the atom in modes other than the cavity ones.

II. POPULATION TRAPPING

We consider a two-level atom with levels $\mid g\rangle$ and $\mid e\rangle$ coupled to a single mode of the radiation field. A basis for the whole Hilbert space is given by $\{\mid n,g\rangle, \mid n,e\rangle, n=0,1,\ldots\}$, where $\mid n,g\rangle$ ($\mid n,e\rangle$) denotes a state with n photons and the atom in its ground (excited) level. We assume the atomic transitions to be mediated by m photons; i.e., the Hamiltonian in terms of this basis is given, in the RWA, by ($\hbar=1$)

$$H = \sum_{n=0}^{\infty} g_n \mid n,g\rangle\langle n,g \mid + e_n \mid n,e\rangle\langle n,e \mid + \sum_{n=m}^{\infty} R_n \mid n,g\rangle\langle n-m,e \mid + R_n^* \mid n-m,e\rangle\langle n,g \mid , \tag{2.1}$$

where g_n and e_n are the energies of the state $\mid n,g\rangle$ and $\mid n,e\rangle$, respectively, and R_n describes the coupling. This Hamiltonian may be easily diagonalized with the results

$$\begin{aligned} H \mid n,g\rangle &= g_n \mid n,g\rangle, \quad n<m, \\ H \mid \Psi_n^\pm\rangle &= \omega_n^\pm \mid \Psi_n^\pm\rangle, \quad n\geq m, \end{aligned} \tag{2.2}$$

where

$$\omega_n^\pm = \frac{g_n + e_{n-m}}{2} \pm \Omega_{n,\Delta},$$

$$\Omega_{n,\Delta}^2 = (\Delta_n/2)^2 + \mid R_n \mid^2,$$

the detuning being $\Delta_n = e_{n-m} - g_n$. The eigenstates $\mid \Psi_n^\pm\rangle$ are the well-known *dressed states* defined as

$$\mid \Psi_n^\pm\rangle = (\cos\theta_n^\pm) \mid n,g\rangle + (\sin\theta_n^\pm) \mid n-m,e\rangle, \tag{2.3}$$

with

$$\tan\theta_n^\pm = \frac{R_n^*}{-\Delta_n/2 \pm \Omega_{n,\Delta}}. \tag{2.4}$$

In this new basis the evolution for the general initial state,

$$\mid \Psi(0)\rangle = \sum_{n=0}^{m-1} A_n \mid n,g\rangle + \sum_{n=m}^{\infty} (B_n^+ \mid \Psi_n^+\rangle + B_n^- \mid \Psi_n^-\rangle), \tag{2.5}$$

is given by

$$\begin{aligned} \mid \Psi(t)\rangle = &\sum_{n=0}^{m-1} A_n \exp(-ig_n t) \mid n,g\rangle \\ &+ \sum_{n=m}^{\infty} [B_n^+ \exp(-i\omega_n^+ t) \mid \Psi_n^+\rangle \\ &+ B_n^- \exp(-i\omega_n^- t) \mid \Psi_n^-\rangle], \end{aligned} \tag{2.6}$$

which allows us to study all the dynamical features of the evolution of the system. In the following we shall assume that the two-level system and the field are initially decoupled; i.e., the initial state may be written as

$$\mid \Psi(0)\rangle = \sum_{n=0}^{\infty} q_n \, [(\cos\phi) \mid n,g\rangle + (\sin\phi)\, e^{i\psi} \mid n,e\rangle], \tag{2.7}$$

where $q_n = \langle n \mid q\rangle$ and $\mid q\rangle$ represents the initial state of the field. The coefficients appearing in this equation are related to those of Eq. (2.5) through

$$\begin{aligned} A_n &= q_n \cos\phi, \\ B_n^\pm &= \pm \frac{1}{\sin(\theta_n^- - \theta_n^+)} \\ &\quad \times [q_n \cos\phi (\sin\theta_n^\mp) - q_{n-m} (\sin\phi)\, e^{i\psi} (\cos\theta_n^\mp)], \end{aligned} \tag{2.8}$$

and therefore the evolution of the initial state (2.7) is obtained by substituting A_n and $B_n^\pm$ given by (2.8) in Eq. (2.6).

Let us now study population trapping in two-level systems, i.e., a persistent probability of finding the atom in its ground or excited states in spite of the existence of both the radiation field and transitions to the other level [21]. These probabilities are respectively given by the expected values in the state (2.6) of the projectors on the ground and excited levels. On the other hand, these projectors leave invariant the subspaces with n quanta $\mathcal{H}_n = \{\mid \Psi_n^+\rangle, \mid \Psi_n^-\rangle\}$ that are decoupled under the evolution and, in each of them, the dressed states evolve with different frequencies. Hence the trapping condition is that each subspace $\mathcal{H}_n$ *must evolve with only one frequency*, that is, only one of the dressed states in each $\mathcal{H}_n$ must be initially populated. For the initial state (2.7) this condition may be expressed as

$$\frac{q_n}{q_{n-m}} \tan\theta_n^{\pm} = (\tan\phi)\, e^{i\psi}, \tag{2.9}$$

which must be fulfilled for the + or − superscript $\forall n \geq m$.

Before we particularize for some well-known Hamiltonians, note that in order for the state for the field to be normalizable (for $\Delta_n = 0$), $|\tan\phi| < 1$ which indicates that the population inversion is negative. However, for nonzero detuning atoms in states with positive population inversion may be trapped. Besides, for zero detuning trapping conditions do not depend on the interaction between the atom and the field, but only on the number of photons which mediate the atomic transition.

A. Raman-coupled model

This model consists of two degenerate levels coupled through a virtual level by a Raman-type transition [see Fig. 1(a)]. The Hamiltonian is obtained from the Λ-type three-level system coupled to a single mode of frequency ω, far from resonance, after adiabatically eliminating the upper level [21]. It reads

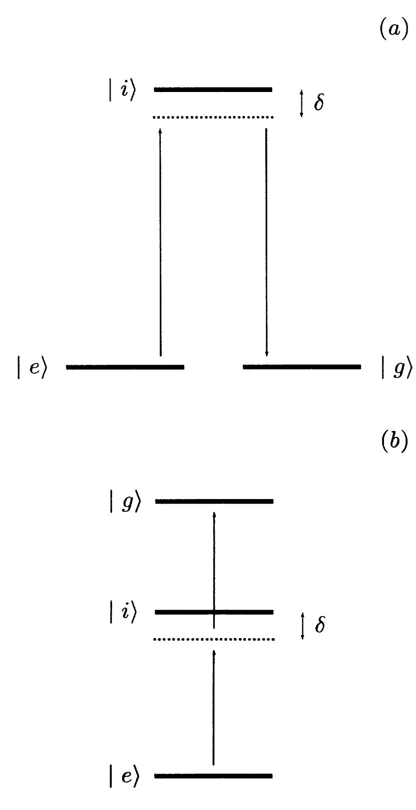

FIG. 1. Atomic level diagrams under two-photon resonance for (a) Λ-type and (b) Ξ-type three-level models.

$$H = \omega a^\dagger a + \lambda a^\dagger a(\sigma^+ + \sigma^-)\ , \tag{2.10}$$

where the σ's are the usual pseudospin operators acting in the space of atomic states, a and $a^\dagger$ are annihilation and creation operators for the mode, and λ is the effective coupling constant.

Note that the interaction does not change the field energy and then the Hamiltonian commutes with arbitrary functions of the photon-number operator. Phoenix and Knight [17] have recently shown that the atomic inversion $\sigma_3 = \sigma^+\sigma^- + \sigma^-\sigma^+$ undergoes periodic collapses and revivals when the atom is initially in its ground or excited state. They have also pointed out that despite the photon statistics remaining constant, the Shannon entropy of the phase is periodic, which indicates that higher moments than the first of the phase distribution are varying quantities. On the other hand, this model has been used successfully in the context of trapping and cooling of two-level atoms with laser fields [22].

In order to apply the trapping condition to this Hamiltonian we identify

$$g_n = e_n = n\omega,$$

$$\Delta_n = 0,$$

$$R_n = R_n^* = \lambda n.$$

Hence, trapping conditions (2.9) impose $(\tan\phi)\, e^{i\psi} = \pm 1$; i.e., the initial state for the atom must be

$$|\,\psi_\pm(0)\rangle_{\rm atom} = |\,g\rangle \pm |\,e\rangle, \tag{2.11}$$

the state for the field being arbitrary. This result is not surprising since, as is well known, in three-level systems with the Λ configuration coherent trapping occurs. In such a case one of the dressed states in every subspace with n quanta depends on the detuning δ between the atomic transition frequency [see Fig. 1(a)] and the mode frequency, but not on the coupling constant; the atom and the field state remain separated under evolution and therefore the population of the levels does not change. With two-level atoms when the initial state for the atom is (2.11) the population inversion remains constant despite the fact that the dressed states depend on the effective coupling constant λ. This dependence is due to the fact that the effective coupling constant λ contains the detuning δ.

B. Two-photon transition model

This model consists of a two-level atom making two-photon transitions at frequency 2ω through a single intermediate level $|\,i\rangle$ [see Fig. 1(b)]. The coupling constants g_1 (for $|\,g\rangle \longrightarrow |\,i\rangle$ at frequency $\omega - \delta$) and g_2 (for $|\,i\rangle \longrightarrow |\,e\rangle$ at frequency $\omega + \delta$) determine the Stark-shift parameters and the coupling between the effective two-level atom and the field mode. After adiabatically eliminating the intermediate level, the Hamiltonian reads [19]

$$H = \omega(a^\dagger a + \sigma_3) + a^\dagger a \frac{g_1^2}{\delta}\,|\,g\rangle\langle g\,| + a a^\dagger \frac{g_2^2}{\delta}\,|\,e\rangle\langle e\,| + \frac{g_1 g_2}{\delta}(a^{\dagger 2}\sigma^- + \sigma^+ a^2). \tag{2.12}$$

In this Hamiltonian the second and third terms are responsible for the Stark shift while $g_1 g_2/\delta$ is the effective coupling constant. By identifying the Hamiltonian (2.12) with (2.1) we find

$$g_n = \omega(n-1) + n\frac{g_1^2}{\delta},$$

$$e_n = \omega(n+1) + (n+1)\frac{g_2^2}{\delta},$$

$$\Delta_n = (n-1)\frac{g_2^2}{\delta} - n\frac{g_1^2}{\delta},$$

$$R_n = \frac{g_1 g_2}{\delta}\sqrt{n(n-1)},$$

and

$$\tan\theta_n^+ = \frac{g_2}{g_1}\sqrt{\frac{n-1}{n}}, \quad \tan\theta_n^- = -\frac{g_2}{g_1}\sqrt{\frac{n}{n-1}}.$$

Hence the trapping condition (2.9) becomes

$$\frac{q_n}{q_{n-2}} = \frac{g_1}{g_2}(\tan\phi)\, e^{i\psi}\sqrt{\frac{n}{n-1}} \tag{2.13}$$

for the +, and

$$\frac{q_n}{q_{n-2}} = -\frac{g_2}{g_1}(\tan\phi)\, e^{i\psi}\sqrt{\frac{n-1}{n}} \tag{2.14}$$

for the − sign in Eq. (2.9). Note that this last condition is fulfilled by the squeezed vacuum state

$$q_{2n+1} = 0, \tag{2.15}$$

$$q_{2n} = \frac{1}{(1-\alpha^2)^{1/4}}\sqrt{\frac{(2n-1)!!}{(2n)!!}}(-1)^n\alpha^n,$$

with $\alpha = (\tan\phi)\exp(i2\psi)g_2/g_1$, and $|\alpha| < 1$. Note also that for $g_1 > g_2$ this condition may be fulfilled with $\tan\phi > 1$, which corresponds to a positive population inversion. As with the Raman-coupled model this can be explained through the coherent trapping appearing in the Ξ model [21]. Finally, we note that if we naively neglect the effect of the Stark shift in the Hamiltonian, we obtain a model that has been worked out by several authors [23], but which predicts other trapping states which is an incomplete result. Hence we conclude that to study population trapping in the two-photon model, the Stark shift must be taken into account.

C. Jaynes-Cummings model

The Hamiltonian for the JCM is[2]

$$H = \tfrac{1}{2}\omega_0\sigma_3 + \omega a^\dagger a + \lambda(a^\dagger\sigma^- + \sigma^+ a), \tag{2.16}$$

where now ω_0 is the atomic transition frequency and λ the coupling constant. Comparing with (2.1) we have

$$g_n = \omega n - \tfrac{1}{2}\omega_0,$$

$$e_n = \omega n - \tfrac{1}{2}\omega_0,$$

$$\Delta_n = \omega_0 - \omega,$$

$$R_n = R_n^* = \lambda\sqrt{n}.$$

Substituting these values in Eq.(2.9) the trapping condition is easily found. Note that, as mentioned above, for $\Delta_n \neq 0$ population trapping with positive population inversion is possible. However, the states for the field in this case are very complicated and it does not seem possible to reach them experimentally. We focus now on the case of zero detuning. Trapped states are then given by

$$|\psi\rangle = (1-\tan^2\phi)^{1/2} \times \sum_{n=0}^{\infty}[(\tan\phi)\, e^{i\psi}]^n \times[(\cos\phi)\,|n,g\rangle + (\sin\phi)\, e^{i\psi}\,|n,e\rangle]. \tag{2.17}$$

With this state population inversion is always negative. The states of the field which fulfill (2.17), the eigenstates of the Susskind-Glogower phase operator, have been extensively studied in Ref. [10] and we refer the reader to this work for a detailed analysis. As has been recently shown, the states appearing in (2.17) are generalized coherent states for the SU(1,1) group [24] corresponding to the representation with Bargmann index $k = \frac{1}{2}$. Using the disentangling theorem for SU(1,1) operators [25] under this representation one can show that [24, 26]

$$|\psi\rangle = \exp(\beta a\sqrt{N} - \beta^*\sqrt{N}a^\dagger)\,|0\rangle, \tag{2.18}$$

where

$$\tan\phi = -\tanh|\beta|, \quad e^{i\psi} = -\beta^*/|\beta|, \tag{2.19}$$

and hence the state (2.17) might be generated with an interaction of the field mode with an external current through the Hamiltonian [26]

$$H = \beta\sqrt{N}a^\dagger - \beta^* a\sqrt{N}. \tag{2.20}$$

On the other hand, the intensity-dependent coupling is represented by the Hamiltonian [16]

$$H = \tfrac{1}{2}\omega_0\sigma_3 + \omega a^\dagger a + \lambda(\sqrt{N}a^\dagger\sigma^- + \sigma^+ a\sqrt{N}). \tag{2.21}$$

Note that this Hamiltonian is constructed in terms of the generators of the SU(1,1) algebra. Comparing with (2.1), this model is identical with the standard JCM except that now $R_n = R_n^* = \lambda n$. So, in this case of zero detuning, trapped states are those given in Eq. (2.17), since now the trapping conditions do not depend on the interaction, as discussed above.

III. PHOTON STATISTICS

As we have shown in the preceding section when trapping conditions are fulfilled the atom has a constant population inversion. In order to derive this condition we have taken into account that the projectors on the atomic levels do not change the subspaces $\mathcal{H}_n$. This property is not only fulfilled by these projectors but also by other operators as the photon-number operator $a^\dagger a$. Hence, when trapping conditions hold, all the diagonal elements of the density operator for the field also remain constant. However, off-diagonal elements vary; their evolution may be characterized by the Q function which, in the JCM-type interaction, has been seen to explain important features such as collapses and revivals of the population inversion. In this context Eiselt and Risken [4] have shown that in the JCM, when the atom is in its upper state and the field in a coherent state, the Q function splits into two peaked functions, counterrotating in the phase space. When these peaks collide a revival of the population inversion is found. In this section we investigate the evolution of the Q function when the atom is initially trapped by the field. This function is defined as

$$Q(\alpha,t) = \mathrm{Tr}\langle \alpha \mid \rho(t) \mid \alpha \rangle, \tag{3.1}$$

where $\mid \alpha\rangle$ is the coherent state and the trace must be taken over the atomic states. For the Hamiltonian (2.1) and the initial state (2.5), it is given by

$$Q(\alpha,t) = e^{-|\alpha|^2}\left(\left| \sum_{n=0}^{m-1} \frac{\alpha^{*n}}{\sqrt{n!}} A_n \exp(-ig_n t) + \sum_{n=m}^{\infty} \frac{\alpha^{*n}}{\sqrt{n!}} [B_n^+ (\cos\theta_n^+) \exp(-i\omega_n^+ t) + B_n^- (\cos\theta_n^-) \exp(-i\omega_n^- t)] \right|^2 \right. \tag{3.2}$$
$$\left. + \left| \sum_{n=m}^{\infty} \frac{\alpha^{*n-m}}{\sqrt{n-m!}} [B_n^+ (\cos\theta_n^+) \exp(-i\omega_n^+ t) + B_n^- (\cos\theta_n^-) \exp(-i\omega_n^- t)] \right|^2 \right).$$

When we substitute $B_n^\pm$ given in Eq. (2.8) we obtain a very complicated function that becomes much simpler when we use one of the trapping conditions [e.g., that of the + superscript in Eq. (2.9)]

$$Q(\alpha,t) = e^{-|\alpha|^2}\left((\cos^2\phi) \left| \sum_{n=0}^{m-1} \frac{\alpha^{*n}}{n!} q_n \exp(-ig_n t) + \sum_{n=m}^{\infty} \frac{\alpha^{*n}}{\sqrt{n!}} q_n \exp(-i\omega_n^+ t) \right|^2 \right.$$
$$\left. + (\sin^2\phi) \left| \sum_{n=0}^{\infty} \frac{\alpha^{*n}}{\sqrt{n!}} q_n \exp(-i\omega_{n+m}^+ t) \right|^2 \right). \tag{3.3}$$

It is easily shown that by defining $\alpha = \mid \alpha \mid e^{i\xi}$ and integrating $Q(\alpha,t)$ over ξ the result depends on $\mid \alpha \mid$ but not on time. Thus, in the phase space, the Q function in every circle centered at $\alpha = 0$ has constant area. This is due to the fact that, as mentioned at the beginning of this section, all the expected values of antinormally ordered functions of the photon-number operator are constant and they may be calculated as the expected values of these functions in $\mid \alpha \mid^2$ with the probability distribution given by $Q(\alpha,t)$.

Let us particularize now for the case of one-photon transition and zero detuning. For the intensity-dependent coupling model ($R_n = \lambda n$) and in a rotating frame at the mode frequency, expression (3.3) simplifies to

$$Q(\alpha,t) = e^{-|\alpha|^2}(1-\tan^2\phi) \times \left| \sum_{n=0}^{\infty} \frac{[(\tan\phi)\, e^{i\psi}\alpha^* e^{(-i\lambda t)}]^n}{\sqrt{n!}} \right|^2, \tag{3.4}$$

and therefore $Q(\alpha,t)$ rotates around $\alpha = 0$ without changing its shape. For the JCM ($R_n = \lambda\sqrt{n}$) we have

$$Q(\alpha,t) = e^{-|\alpha|^2}(1-\tan^2\phi) \times \left((\cos^2\phi) \left| \sum_{n=0}^{\infty} \frac{[(\tan\phi)\, e^{i\psi}\alpha^*]^n}{\sqrt{n!}} e^{-i\lambda\sqrt{n}t} \right|^2 + (\sin^2\phi) \left| \sum_{n=0}^{\infty} \frac{[(\tan\phi)\, e^{i\psi}\alpha^*]^n}{\sqrt{n!}} e^{-i\lambda\sqrt{n+1}t} \right|^2 \right). \tag{3.5}$$

In Fig. 2 we have plotted the Q function for $\langle a^\dagger a\rangle = 4$, and $\psi = 0$ in the complex phase space for several interaction times λt. Initially [Fig. 2(a)] this function has a maximum near $\langle\alpha\rangle$. As time increases it is broadened, its maximum decreases and is displaced from the initial situation [Fig. 2(b)]. For $\lambda t = 3$ [Fig. 2(c)] the Q function has rotated, the height is smaller and another maximum appears. For longer interaction times [Fig. 2(d)] it is clearly deformed, tending to an annular but coarse structure, as reported in Ref. [4]. Hence we can conclude that population inversion remains constant despite the fact that the Q function is deformed and rotates in the phase space.

IV. TWO-TIME CORRELATION FUNCTIONS

Under the same condition as when trapping occurs, it can be easily shown that some two-time correlation functions become stationary. This is also due to the existence of only one frequency in each subspace $\mathcal{H}_n$ and to the invariance of these subspaces under the action of the whole operator. In particular, for the following first- and second-order atomic correlation functions we find

$$\langle \sigma^+(t)\sigma^-(t+\tau)\rangle = \sum_{n=m}^{\infty} \mid B_n^{\pm} \mid^2 \frac{\sin^2\theta_n^{\pm}}{\sin^2(\theta_n^- - \theta_n^+)} \exp(-i\omega_n^{\pm}\tau)[(\sin^2\theta_{n-m}^-)\exp(i\omega_{n-m}^+\tau) + (\sin^2\theta_{n-m}^+)\exp(i\omega_{n-m}^-\tau)], \tag{4.1}$$

$$\langle \sigma^+(t)\sigma^+(t+\tau)\sigma^-(t+\tau)\sigma^-(t)\rangle = \sum_{n=2m}^{\infty} \mid B_n^{\pm} \mid^2 (\sin^2\theta_n^{\pm}) \left(\frac{(\sin\theta_{n-m}^-)\sin\theta_{n-m}^+}{\sin(\theta_n^- - \theta_n^+)} \right)^2 4\sin^2(\Omega_{n\Delta}\tau), \tag{4.2}$$

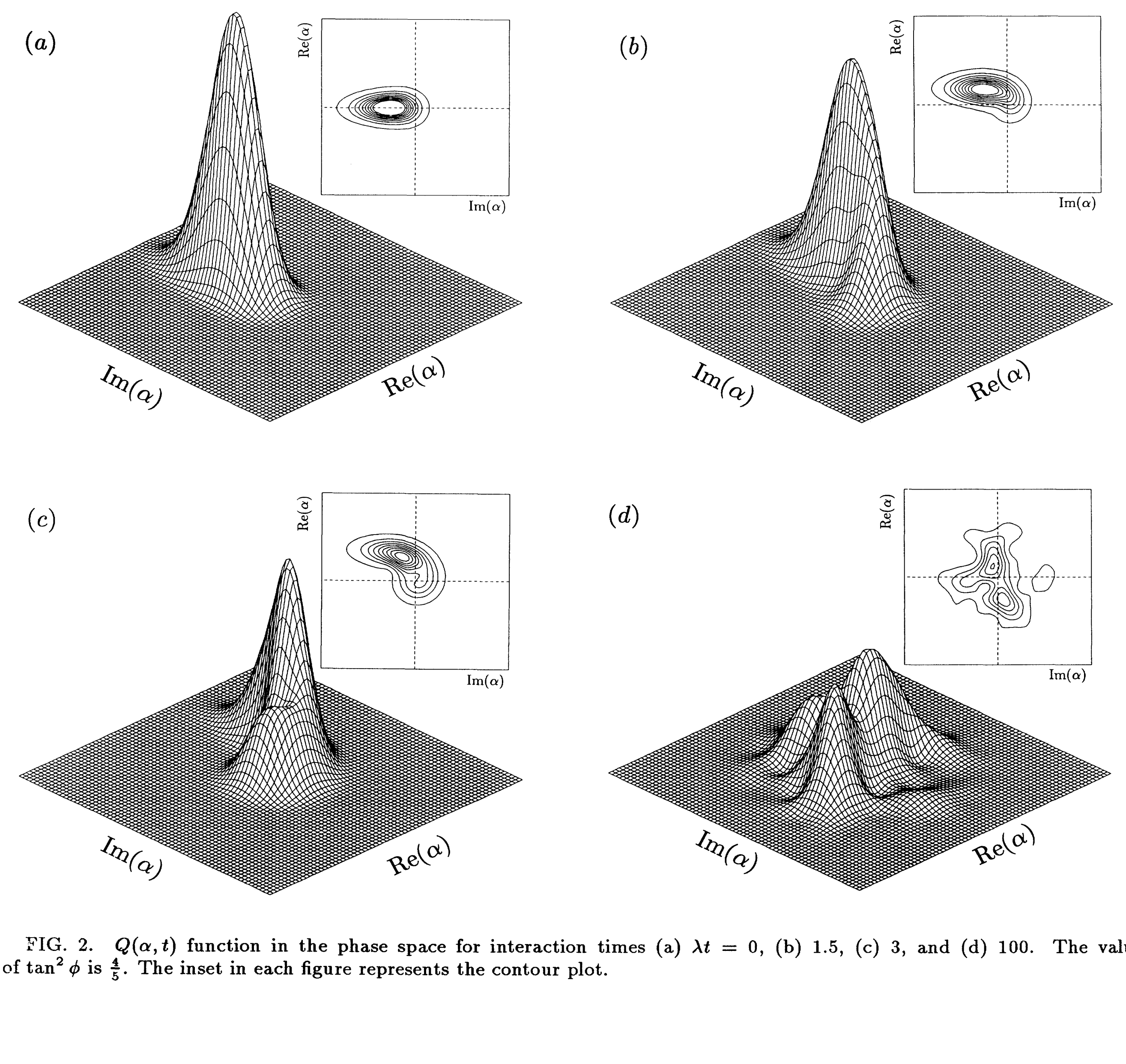

FIG. 2. $Q(\alpha,t)$ function in the phase space for interaction times (a) $\lambda t = 0$, (b) 1.5, (c) 3, and (d) 100. The value of $\tan^2\phi$ is $\frac{4}{5}$. The inset in each figure represents the contour plot.

where the + or − sign depend on which of the B's coefficients is not zero. The second function describes the joint probability of detecting one photon at time t and another photon at time $t+\tau$ [27], and the first is used to obtain the spectrum of the light emitted by the atom [28]. In nondissipative models this correlation function is not stationary in general and therefore the *physical spectrum* of Eberly and Wódkiewicz [29] must be used. However, in the case under study, we can find it as the Fourier transform of the two-time correlation function (4.1). The result is

$$S(\omega) = \sum_{n=m}^{\infty} \mid B_n^{\pm} \mid^2 \frac{\sin^2 \theta_n^{\pm}}{\sin^2(\theta_n^- - \theta_n^+)}[(\sin^2 \theta_{n-m}^-)\delta(\omega - \omega_n^{\pm} + \omega_{n-m}^+) + (\sin^2 \theta_{n-m}^+)\delta(\omega - \omega_n^{\pm} + \omega_{n-m}^-)]. \quad (4.3)$$

It is composed by a summation of δ functions each one corresponding to the transition of an initially populated dressed level with n quanta to others with $n-m$ quanta. The intensity of each line is proportional to the initial (and constant) population of the dressed level. This is due to the fact that, contrary to the general case, here there are no interferences between transitions from different dressed levels with the same amount of quanta since only one is populated.

In a real experiment, in order to see the resonance spectrum, it is necessary that part of the light emitted by the atom be outside the cavity, and therefore, there must be some spontaneous decay at rate γ into modes other than the cavity mode. In such a case a full master equation accounting for this effect should be used, but now the trapping condition does not hold. However, if we consider that the coupling between the atom and the cavity mode is much stronger than the coupling between the atom and the external modes, the model considered here holds, at least for times $t \ll 1/\gamma$. The spectrum measured then is given by Eq. (4.3) but instead of δ's there will be Lorentzian-type functions of γ width. In Fig. 3 we have plotted the spectrum obtained for the JCM with the initial state (2.17), with $\tan^2 \phi = \frac{1}{2}$ and $\psi = 0$, assuming that the decay due to spontaneous emission in the external modes is larger than the atom-field coupling ($\lambda/\gamma = 0.05$). It is composed of Lorentzian functions of width γ centered at $\omega - \omega_0 + \lambda(\sqrt{n+1} \pm \sqrt{n})$ (n integer). Note that the Lorentzian functions at the left of $\omega - \omega_0 + \lambda$ overlap, and therefore the peaked structure is lost, giving rise to a new function of width larger than γ. The recent successful experiments involving high excited Rydberg states of certain atoms in a microcavity seem to be the adequate framework to see this effect since in them the condition for the couplings is fulfilled and it is possible to observe several Rabi oscillations before the atom decays to its ground state due to spontaneous emission [1].

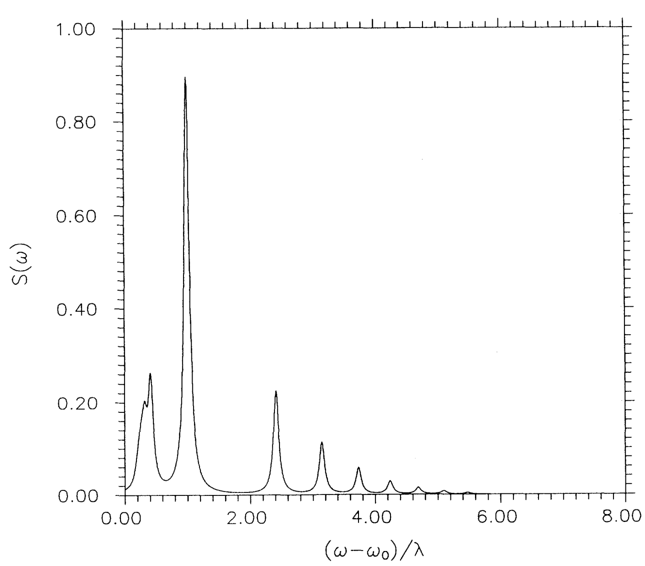

FIG. 3. Resonance spectrum for the JCM with $\tan^2 \phi = \frac{1}{2}$, $\psi = 0$, and $\gamma/\lambda = 0.05$.

V. CONCLUSIONS

Trapping states for a two-level atom interaction with a single cavity mode through an m-photon transition have been found. When the system is prepared in one of these states the atomic population inversion remains constant and therefore there is a persistent probability of finding the atom in its ground and excited levels. The existence of these trapped states may be explained within the dressed-state formalism: the whole Hilbert space splits into one- and two-dimensional subspaces with n quanta which are decoupled under the action of the Hamiltonian and the atomic population inversion operator. When only one of the dressed levels in each subspace is initially populated, there is only one evolution frequency in each subspace and therefore the population inversion becomes constant. We have particularized for some well-known models: for the Raman-coupled model and the two-photon transition model. We have shown that trapping is related to the existence of coherent trapping in the Λ- and Ξ-type three-level atom model, if the intermediate level is eliminated adiabatically. In the last model some care must be taken since one cannot neglect the resulting Stark shift in order to obtain the right trapping condition. For the JCM we have analyzed the trapping condition and proposed a theoretical method to obtain the trapped states.

We have also analyzed the behavior of the Q function; as has recently been shown this function splits into two peaked functions counterrotating in the phase space. The revivals of the population inversion appear when both functions collide. We have pointed out that when the trapping condition holds, in the case of the intensity coupling model, the Q function rotates in the phase space without deforming its shape. In the JCM, the Q function is deformed and also rotates in the phase space.

Finally we have shown that when the trapping condition is fulfilled some two-time correlation functions become stationary. This fact allows one to calculate the emission spectrum as the Fourier-transformed two-time dipole autocorrelation function. It is composed by a summation of δ's centered at the transition frequencies between the dressed levels of $n+1$ and n quanta, and the intensity is proportional to the population of the dressed level from which the transition is performed. In a real experiment it is necessary for the atom to be coupled to other modes than the cavity in order to see the light emitted by the atom. In this case the δ functions become Lorentzian with a width of the order of the spontaneous decay rate in these other modes.

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[1] S. Haroche and J. M. Raimond, in *Advances in Atomic and Molecular Physics*, edited by D. Bates and B. Benderson (Academic, New York, 1985), Vol.20, pp.347–411; G. Rempe, H. Walther, and N. Klein, Phys. Rev. Lett. **58**, 353 (1987); P. Filipowicz, P. Meystre, G. Rempe, and H. Walther, Opt. Acta **32**, 1105 (1985); D. Meschede, H. Walther, and G. Muller, Phys. Rev. Lett. **54**, 551 (1985).

[2] E. T. Jaynes and F. W. Cummings, Proc. IEEE **51**, 89 (1963); S. M. Barnett, P. Filipowicz, J. Javanainen, P. L. Knight, and P. Meystre, in *Frontiers in Quantum Optics*, edited by E. R. Pike and S. Sarkar (Hilger, Bristol, UK, 1986), pp.485–520.

[3] J. H. Eberly, N. B. Narozhny, and J. J. Sánchez-Mondragón, Phys. Rev. Lett. **44**, 1323 (1980); N. B. Narozhny, J. J. Sánchez-Mondragón, and J. H. Eberly, Phys. Rev. A **23**, 236 (1981).

[4] J. Eiselt and H. Risken, Opt. Commun. **72**, 351 (1989); J. Eiselt and H. Risken, Phys. Rev. A **43**, 346 (1991).

[5] A. Joshi and R. R. Puri, J. Mod. Opt. **36**, 557 (1989).

[6] K. Zaheer and M. S. Zubairy, Phys. Rev. A **38**, 5227 (1988); W. Schleich and M. O. Scully, *ibid.* **37**, 1261 (1988).

[7] W. W. Chow, M. O. Scully, and J. O. Stoner, Phys. Rev. A **11**, 1380 (1975).

[8] M. O. Scully and M. S. Zubairy, Opt. Commun. **66**, 303 (1988).

[9] K. Zaheer and M. S. Zubairy, Phys. Rev. A **39**, 2000 (1989).

[10] J. I. Cirac and L. L. Sánchez-Soto, Phys. Rev. A **42**, 2851 (1990).

[11] L. Susskind and J. Glogower, Physics **1**, 49 (1964); P. Carruthers and M. M. Nieto, Rev. Mod. Phys. **40**, 411 (1968).

[12] J. J. Slosser, P. Meystre, and S. L. Braunstein, Phys. Rev. Lett. **63**, 934 (1989).

[13] P. Filipowicz, J. Javanainen, and P. Meystre, J. Opt. Soc. Am. B **3**, 906 (1986).

[14] J. Gea-Banacloche, Phys. Rev. Lett. **65**, 3385 (1990).

[15] Y. S. Bai, T. W. Mossberg, N. Lu, and P. R. Berman, Phys. Rev. Lett. **57**, 1692 (1986); N. Lu, P. R. Berman, A. G. Yodh, Y. S. Bai, and T. W. Mossberg, Phys. Rev. A **33**, 3956 (1986).

[16] B. Buck and C. V. Sukumar, Phys. Lett. **81A**, 132 (1981); C. V. Sukumar and B. Buck, J. Phys. A **17**, 885 (1984).

[17] S. J. D. Phoenix and P. L. Knight, J. Opt. Soc. Am. B **7**, 116 (1990); S. J. D. Phoenix and P. L. Knight, Ann. Phys. (N.Y.) **182**, 381 (1988).

[18] M. Brune, J. M. Raimond, P. Goy, L. Davidovich, and S. Haroche, Phys. Rev. Lett. **59**, 1899 (1987); M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. A **35**, 154 (1987); L. Davidovich, J. M. Raimond, M. Brune, and S. Haroche, *ibid.* **36**, 3771 (1987).

[19] R. R. Puri and R. K. Bullough, J. Opt. Soc. Am. B **5**, 2021 (1988).

[20] G. Alzeta, A. Gozzini, L. Moi, and G. Orriols, Nuovo Cimento. B **36**, 5 (1976); G. Orriols, *ibid.* **53**, 1 (1979); H. R. Gray, R. M. Whitley, and C. R. Stroud, Opt. Lett. **3**, 218 (1978); P. M. Radmore and P. L. Knight, J. Phys. B **15**, 561 (1982); B. J. Dalton and P. L. Knight, Opt. Commun. **42**, 411 (1982).

[21] H. I. Yoo and J. H. Eberly, Phys. Rep. **118**, 239 (1985).

[22] A. Aspect, E. Arimondo, R. Kaiser, N. Vansteenkiste, and C. Cohen-Tannoudji, J. Opt. Soc. Am. B **6**, 2112 (1989); A. Aspect, E. Arimondo, R. Kaiser, N. Vansteenkiste, and C. Cohen-Tannoudji, Phys. Rev. Lett. **61**, 826 (1988).

[23] S. J. D. Phoenix and P. L. Knight, J. Opt. Soc. Am. B **7**, 116 (1990) and references therein.

[24] V. Bužek, Phys. Rev. A **39**, 3196 (1989).

[25] K. Wódkiewicz and J. H. Eberly, J. Opt. Soc. Am. **2**, 458 (1985).

[26] C. V. Sukumar, J. Phys. A **21**, L1065 (1988); J. Mod. Opt. **36**, 1591 (1989).

[27] G. S. Agarwal, J. Opt. Soc. Am. B **2**, 480 (1985).

[28] J. I. Cirac and L. L. Sánchez-Soto, Opt. Commun. **80**, 67 (1990).

[29] J. H. Eberly and K. Wódkiewicz, J. Opt. Soc. Am. **67**, 1252 (1977).