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Sub-shot-noise measurement of modulated absorption using parametric down-conversion

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We use a lithium iodate nonlinear crystal as a source of two quantum correlated beams at about 830 nm. A liquid-crystal cell is placed in one of the beams and is electrically driven to produce a very small modulation of its transmission coefficient. Low-noise high-efficiency photodiodes detect the light in each beam and the spectrum of the photocurrents, and the spectrum of the difference between the photocurrents, are measured. The difference spectrum shows a reduction of the noise below the shot-noise level of 4 dB compared with the classical difference technique, where the shot noise in the two detectors simply adds. However, the classical noise in one beam is low enough to make a single-beam shot-noise-limited measurement. Using an optimized difference technique, we show a reduction of the noise level compared to the single-beam shot noise of 2 dB.

I. INTRODUCTION

Recently much work has been done on the production of nonclassical light, in which the fluctuations of photon number or the amplitude of one quadrature component is reduced below that for a classical coherent state [1–11]. One projected use for such light is in improving the accuracy of transmission (or turbidity) measurements without increasing the amount of light absorbed in the sample [12]. In certain biological or light sensitive samples a maximum absorbed dose limit will exist, limiting the accuracy with which a measurement of the absorption can be performed. In a recent experiment, phase (quadrature) squeezed light was used to reduce the noise to below the shot noise limit in a measurement of absorption in a highly transmissive sample [13]. In this experiment we demonstrate a sub-shot-noise absorption measurement using amplitude correlated or "twin" beams obtained from the nondegenerate parametric down-conversion process [9,11]. These beams show noise correlations at the quantum level and we have previously shown that a sub-shot-noise light source can be created by using negative feedback of the noise on one beam to correct the noise on the other [9]. In this work we have improved both the bandwidth and power of our twin beam source and have used it to demonstrate a signal-to-noise improvement in the measurement of a small fluctuation in the absorption coefficient of a highly transmissive sample. Recent work has demonstrated noise reductions of 2 dB in a similar measurement using an optical parametric oscillator source [14].

II. THEORY

A schematic diagram of the system is shown in Fig. 1. The down-conversion source (crystal) generates pairs of photons directed at the detectors ($D_{1,2}$) at a mean rate r. Each detector actually detects fractions η_1, η_2 of these photons due to its limited quantum efficiency and losses in each channel (for example, reflection losses at the crystal, lenses, and filters). In addition, channel 1 contains the sample (LC) with transmission coefficient α. This transmission coefficient is modulated by a small amount $\Delta\alpha$ at a single frequency f^* and the object of the absorption measurement is to measure $\Delta\alpha$ in the presence of shot noise and, in some situations, classical noise.

The mean photocurrents in each detector are given by

$$i_1 = \alpha\eta_1 re, \quad i_2 = \eta_2 re , \tag{1}$$

where e is the electron charge. If the photocurrents are

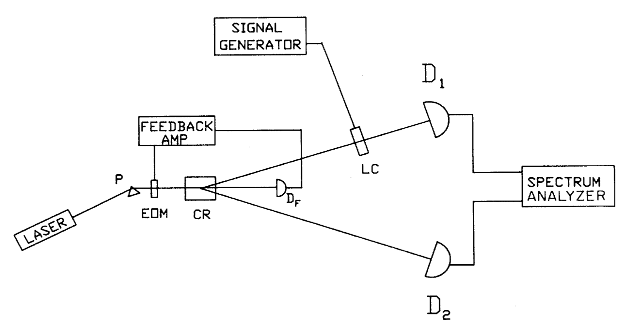

FIG. 1. Schematic diagram of the apparatus. The laser illuminates a lithium iodate crystal (CR) producing twin correlated beams. A liquid-crystal cell LC is placed in one of the beams and its transmission modulated using a signal generator. Detectors D_1 and D_2 view the twin beams and their outputs are analyzed in a Fourier-transform spectrum analyzer. The 413.4-nm laser line is selected by a prism (P). Laser noise is reduced by a feedback loop involving detector D_F, a feedback amplifier, and an electro-optic modulator (EOM).

band limited with a bandwidth Δf and have instantaneous values I_1, I_2, then the current fluctuations $\Delta I_j = I_j - i_j$, $j=1,2$, have a mean-square value given by the formula for shot noise

$$\langle \Delta I_j^2 \rangle = 2 e i_j \Delta f, \quad j=1,2 \tag{2}$$

in the absence of classical noise and away from f^*.

A fraction $\eta_1\eta_2\alpha$ of the photon pairs are simultaneously detected in both detectors. This corresponds to a photocurrent $i_3 = \alpha\eta_1\eta_2 r e$, which generates identical shot noise in each detector. Since the remainder of the current in each detector gives rise to uncorrelated shot noise, the correlation between the two currents is given by the shot noise in i_3,

$$\langle \Delta I_1 \Delta I_2 \rangle = 2 e i_3 \Delta f \ . \tag{3}$$

The normalized correlation coefficient is defined as

$$s_{12} = \frac{\langle \Delta I_1 \Delta I_2 \rangle}{(\langle \Delta I_1^2 \rangle \langle \Delta I_2^2 \rangle)^{1/2}} = \sqrt{\alpha\eta_1\eta_2} \tag{4}$$

on combining Eqs. (1) and (3). When channel 1 contains the signal of interest together with the shot noise, we can use the correlated noise in channel 2 to reduce the shot noise in channel 1 by subtraction. A fraction k of the current noise in channel 2 is subtracted to form a new signal I_1',

$$I_1' = I_1 - k \Delta I_2 \ . \tag{5}$$

The mean-square noise in this new signal is given by

$$\langle (\Delta I_1')^2 \rangle = 2 e^2 r \Delta f (\alpha\eta_1 + k^2\eta_2 - 2k\alpha\eta_1\eta_2) \ . \tag{6}$$

Minimizing this noise level with respect to k gives

$$k = \alpha\eta_1 = s_{12}\sqrt{\alpha\eta_1/\eta_2} \ , \tag{7}$$

and using this value of k gives the following noise level in the processed signal:

$$\langle (\Delta I_1')^2 \rangle = 2 e^2 r \Delta f \alpha\eta_1 (1 - \alpha\eta_1\eta_2) \ . \tag{8}$$

The part of this equation before the bracket is identical to the noise level before processing [Eq. (2)], and so it is clear that this approach always improves the signal-to-noise ratio. The greatest reduction in noise occurs when the detector efficiencies and sample transmission are close to unity. It is of interest to compare this case with the case of balanced subtraction of the two signals. The ratio of the two signal currents is clearly independent of any classical intensity fluctuations in the rate of pair production due to intensity changes in the pump laser. If we normalize this ratio with the mean photocurrent in detector 2 then we get a new definition of the processed signal

$$I_1' = i_2 I_1 / I_2 \ . \tag{9}$$

If we assume that the current fluctuations $\Delta I_1, \Delta I_2$ are small compared with the respective mean currents, then we can make a binomial expansion of this equation and find

$$I_1' = I_1 - \Delta I_2 \left[\frac{i_1}{i_2} \right] \ . \tag{10}$$

Comparing this result with Eq. (5) shows that when intensity fluctuations dominate the shot noise, then it is necessary to choose the constant k as

$$k = \frac{i_1}{i_2} = \frac{\alpha\eta_1}{\eta_2} \ . \tag{11}$$

Substituting this into Eq. (6) gives the noise level in this case as

$$\langle (\Delta I_1')^2 \rangle = 2 e^2 r \Delta f \alpha\eta_1 \left[1 - 2\alpha\eta_1 + \frac{\alpha\eta_1}{\eta_2} \right] \ . \tag{12}$$

This noise level is always larger than the optimized noise level given in Eq. (8), and it is also worse than the shot noise in channel 1 before any signal processing unless $\eta_2 > \frac{1}{2}$. However, if balanced subtraction is essential to remove classical intensity fluctuations, then this noise level is always an improvement over the shot noise that would be obtained in a classical two-beam experiment using a 50/50 beam splitter.

III. EXPERIMENT

A large frame krypton ion laser produces two wavelengths in the violet, and the beam is passed through a prism to separate out the 414.3-nm line. The beam passes through an electro-optic modulator, and the remaining 400 mW is weakly focused into a lithium iodate nonlinear crystal. The intensity of the beam leaving the crystal is monitored by a detector that is connected to the electro-optic modulator. The resulting feedback loop is used to reduce the classical fluctuation in the laser output intensity, caused mainly by vibrations of the mirrors in the laser cavity.

The nonlinear crystal is used as a source of a pair of correlated beams of down-converted light [9]. The bandwidth and spatial extent of each of these beams are limited by apertures placed to select matched pairs of photons. One of the beams passes through the sample consisting of a liquid-crystal cell. A signal generator is used to apply a low-voltage electrical signal at about 480 Hz to the sample which modulates its transmission coefficient both at this frequency and at the second-harmonic frequency of 960 Hz. Both beams are focused using antireflection (AR) coated lenses through AR coated color glass filters (cut on greater than 750 nm) onto *p-i-n* photodiodes. These are connected to low-noise transimpedance amplifiers with gains of $R_1 = 1.13$ GΩ (channel 1) and $R_2 = 1.01$ GΩ. The mean-square current noise of the amplifiers and detectors together was about 1.2×10^{-28} $A^2 Hz^{-1}$ at 1 kHz, which was small compared with the typical shot noise levels of 1.5×10^{-27} $A^2 Hz^{-1}$ (4 nA of detected photocurrent). The theoretical noise floor set by Johnson noise could not be achieved at 1 kHz due to capacitative effects in the detectors.

The outputs of the two detector amplifiers were connected to low pass filters with 2.5 kHz corner frequency. The filter outputs were connected to an analog-to-digital converter and a Hewlett-Packard 300 series computer. The computer was made to perform as a spectrum

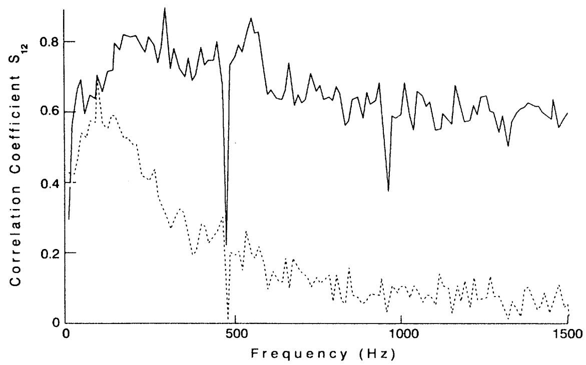

FIG. 2. The spectral correlation coefficient s_{12} as a function of frequency. Real (solid line) and imaginary parts (dashed line) are shown. The imaginary part is nonzero due to mismatch of detector low-frequency response.

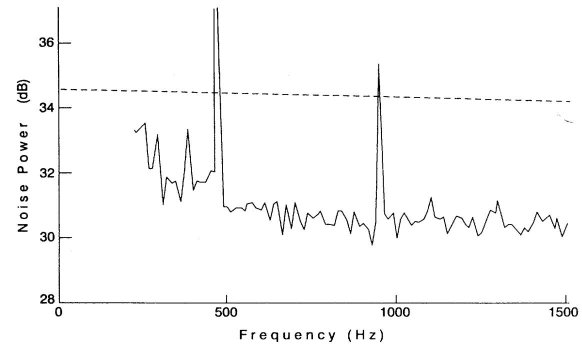

FIG. 4. Measured noise spectra. Solid line, voltage noise in the difference voltage $\Delta V_1 - k'\Delta V_2$ with $k' = 1.12$; dashed line, predicted shot-noise level for channel 1 plus channel 2.

analyzer by Fourier transforming the voltage signals from the two detectors; it also calculated the spectral correlation coefficients between the two signals which was used to check that the detectors and apertures were correctly aligned. The computer also performed the subtraction of one detector output from another in order to generate the reduced noise signal.

IV. RESULTS

The spectral correlation coefficient between the outputs of the two detector amplifiers, after optimization of the alignment, is shown in Fig. 2. The imaginary part is nonzero at low frequencies due to differences in the low-frequency response of the detectors. The average coefficient measured between 800 and 1000 Hz is $s_{12} = 0.60$. Reduction in the correlation at 480 and 960 Hz is due to the modulation of the sample. Unmodulated sample transmission was about 85% and before insertion correlation coefficients close to 0.68 were measured. Figure 3 shows the photovoltage power spectrum $\langle \Delta V_1^2 \rangle$ ($\Delta V_1 = R_1 \Delta I_1$) measured in channel 1 with the sample modulated at 480 Hz. The second harmonic at 960 Hz is clearly seen rising above a noise background, which is near shot-noise limited. The dashed line is the shot-noise level calculated from the measured detector photocurrent of 3.54 nA and amplifier gain 1.13 GΩ. Also plotted on this graph is the difference spectrum $\langle (\Delta V_1 - k'\Delta V_2)^2 \rangle$ with $k' = 0.66$. The photovoltage feedback parameter k' is determined from $k' = kR_1/R_2$ with the optimum value of $k \simeq 0.59$ estimated from Fig. 2 and Eq. (7). Clearly the noise background is lower in this case by 2 dB as expected from the prediction of Eq. (8) using the measured spectral correlation coefficient. Figure 4 shows the difference spectrum $\langle (\Delta V_1 - k'\Delta V_2)^2 \rangle$ with $k' = R_1/R_2$ to minimize macroscopic fluctuations. This is compared to the calculated dual-beam shot-noise level which is roughly twice that of the single beam. Clearly in this situation the noise background reduction is much larger at ~ 4 dB. It must be emphasized that this is only gained when the single-beam measurement is not shot-noise limited.

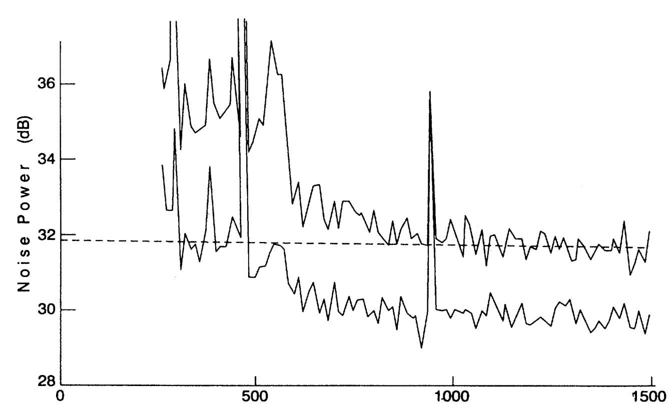

FIG. 3. Measured noise spectra. Upper solid line, voltage noise [referred to $1(\mu V)^2/\mathrm{Hz}$ and amplifier gain 1.13 GΩ] measured in channel 1 as a function of frequency; lower solid line, voltage noise in the difference voltage $\Delta V_1 - k'\Delta V_2$ with $k' = 0.66$; dashed line, predicted shot-noise level for channel 1.

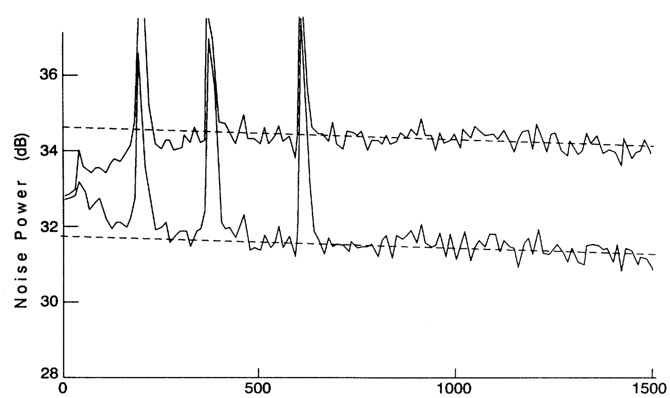

FIG. 5. Calibration spectra taken using a tungsten bulb. Lower solid line, voltage noise [referred to $1(\mu V)^2/\mathrm{Hz}$] measured in channel 1 as a function of frequency; upper solid line, voltage noise in the difference voltage $V_1 - kV_2$ with $k = 1.0$; dashed lines, predicted shot-noise levels in the difference voltage (upper) and in channel 1 voltage (lower).

Figure 5 shows calibration spectra taken with a tungsten lamp source filtered to be of similar bandwidth to our down-conversion source and adjusted to produce similar photocurrents in the detectors. The lower curve shows the photovoltage power spectrum in channel 1, while the upper trace shows the difference spectrum again with $k'=R_1/R_2$. Unexplained peaks seen are due to electrical interference.

V. CONCLUSION

We have demonstrated that fluctuations in turbidity or absorption can be measured with an accuracy 4 dB better than that predicted by the two-beam shot-noise limit at frequencies near 1 kHz. In our experiment we can also arrange that shot-noise-limited single-beam measurements can be made and demonstrate with suitable choice of parameters a reduction in the single-beam noise background of 2 dB. The limitations to the measured noise reductions are detector efficiencies ($\eta \sim 0.85$), sample transmission ($\alpha \sim 0.85$), optical losses in each channel (transmission efficiency $O_t \sim 0.95$) and background amplifier noise ($\langle \Delta I_b^2 \rangle \sim 0.1 \langle \Delta I^2 \rangle$ i.e., approximately 10% of measured noise). These known inefficiencies lead to an effective detector efficiency given by [9]

$$\eta_{\text{eff}} = \frac{\eta O_t}{1 + \langle \Delta I_b^2 \rangle / \langle \Delta I^2 \rangle} . \tag{13}$$

On using η_{eff} in Eq. (4) we obtain a predicted correlation coefficient of 0.68. The measured correlation coefficient is somewhat lower than this, implying that the correlation between the beams selected by the apertures is not perfect possibly due to subtleties of the phase matching. Improved detectors (to 95% efficiency), a higher sample transmission, and lower noise amplifiers could improve the measurement to give near 10-dB noise reduction.

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