Electron-positron pair production in relativistic heavy-ion collisions

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(Received 26 March 1991)

First-order Born cross sections for electron-positron pair production in relativistic heavy-ion collisions are calculated. Sommerfeld-Maue wave functions are employed to describe the continuum state of the emitted electron and positron in the field of the heavy target nucleus. Double-differential, single-differential, and total cross sections are presented in a wide energy regime from 1 GeV/u up to 20 TeV/u. We compare our results with first-Born calculations using a partial-wave expansion and with lowest-order QED calculations. The target-charge dependence is investigated.

I. INTRODUCTION

The electromagnetic production of electron-positron pairs in collisions of relativistic heavy ions is a field of growing interest [1]. At increasing relativistic energies, cross sections for pair production become very large. Pair production has therefore important practical implications for the planned Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC), where fixed-target energies of 23 TeV/u (Au) and 34 000 TeV/u (Pb) are envisioned [2]. In such extreme relativistic heavy-ion collisions, electron-positron pairs of electromagnetic origin might possibly mask signals from the formation and the decay of the quark-gluon plasma [3]. The pair-production mechanism also contributes to the electromagnetic stopping power for the heavy-particle component of cosmic rays [4] and might allow the measurement of the energy of cosmic rays by counting the pairs produced in emulsions [5]. It has also been proposed to use pair production in relativistic heavy-ion collisions as a real-time nondestructive luminometer [6].

Calculations of cross sections for pair production by fast charged particles have a long history. They have either been based on a lowest-order quantum-electrodynamical (QED) treatment [7,3] or on the equivalent-photon or Weizsäcker-Williams method [8] where the pair-production cross section may be determined by using the already known photon-nucleus pair-production cross section [9,4,10,5]. The equivalent-photon method is best suited for very high collision energies but suffers from an undetermined parameter, which corresponds to the minimum impact parameter assumed. These inadequacies are avoided in the lowest-order (QED) calculations. Here electrons and positrons are treated as free particles interacting with the colliding nuclei via one-photon exchange. Pair production is thus represented by two-photon diagrams, which have recently been evaluated exactly [3].

Lowest-order perturbation theory, however, is believed to give an incomplete description of pair production in heavy-ion collisions [3,5]. The parameter αZ is comparable with unity so that higher-order terms in the target or projectile charge might possibly give important contributions even at very high collision energies.

A more complete description of the pair-production process has been given in the work of Becker, Grün, and Scheid [11]. Here, the lowest-order matrix element in the Furry picture is evaluated, which includes the target interaction to all orders. Only the projectile interaction is assumed to be small and treated in first-order perturbation theory, which can be expected to be a good approximation for very fast collisions. Becker, Grün, and Scheid expand the projectile interaction in multipoles and use exact Dirac continuum states for the description of electron and positron wave functions. This approach, however, is restricted to projectile energies below 10 GeV/u due to numerical difficulties in the evaluation of multipole sums with high angular momentum and has not been extended to energies in the TeV/u region. These problems at high energies can be solved by using Sommerfeld-Maue wave functions [12] for the description of the continuum states of the electron and positron. In this way a multipole expansion is avoided and pair-production cross sections can be obtained for very high collision energies which are of utmost interest.

Calculations employing Sommerfeld-Maue wave functions have been performed by Nikishov and Pichkurov [13] and by Bertulani and Baur [14], who deduced analytical expressions for differential and total cross sections assuming high projectile energies. Although these calculations give important insights into the physical processes of pair production, they rely on specific assumptions regarding the energy of the emitted electron-positron pair. Furthermore, approximations are introduced to make the analytical evaluation of transverse momentum, energy, and angular integrations feasible.

In this paper, we present the results of an exact numerical calculation of double-differential, single-differential, and total cross sections for pair production within the first-order Born approximation. In accordance with Ref. [12] we use Sommerfeld-Maue wave functions for the description of electron and positron continuum states. The integration over the transverse momentum and the integrations over the energy and angular distribution of the emitted electron-positron pair are performed numerically. No approximations are introduced and no restrictions with respect to the collision energy are made. The paper is organized as follows. In Sec. II we establish the

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notation and give a brief outline of the theory together with a description of the numerical procedure. Subsequently, in Sec. III detailed results are presented and discussed. In Sec. IV some concluding remarks are added. Atomic units are used throughout the paper unless explicitly stated otherwise.

II. THEORY

Let us consider a relativistic projectile ion (charge Z_P) colliding with a target ion of nuclear charge Z_T. At relativistic velocities, it is a very good approximation to assume a classical straight-line trajectory $\mathbf{R}=\mathbf{b}+\mathbf{v}t$ for the projectile motion where $\mathbf{v}$ is the projectile velocity and $\mathbf{b}$ is the impact parameter. Associated with the two inertial frames, we have two sets of space-time coordinates. While $\mathbf{r}_T$ and t measure the space-time position of the electron-positron pair from the target nucleus, the coordinates $\mathbf{r}'_P$ and t' refer to the space-time position of the pair as seen from the projectile nucleus. Both sets of coordinates are connected by the Lorentz transformation

$$\begin{aligned}\mathbf{r}'_P&=\mathbf{r}_T+\hat{\mathbf{v}}z_T(\gamma-1)-\mathbf{v}\gamma t-\mathbf{b}\ ,\\ t'&=\gamma(t-vz_T/c^2)\ ,\end{aligned}\tag{1}$$

where $\beta=v/c$, $\gamma=(1-\beta^2)^{-1/2}$, and c = 137.036 a.u. is the velocity of light. The quantity $\hat{\mathbf{v}}$ is a unit vector in the direction of the projectile velocity $\mathbf{v}$. The first-order transition amplitude for electron-positron pair production in the laboratory (target) frame is given by

$$A_{e\text{-}p}=-i\int dt\int d\mathbf{r}_T\psi_e^\dagger(\mathbf{r}_T,t)\gamma(1-\beta\alpha_z)\times\left[-\frac{Z_P}{r'_P}\right]\psi_p(\mathbf{r}_T,t)\ .\tag{2}$$

We have denoted by ψ_e the electron continuum state and by ψ_p the positron continuum state, which satisfy the time-dependent Dirac equation

$$i\frac{\partial}{\partial t}\psi=\left[-ic\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}-\frac{Z_T}{r_T}+c^2\gamma_4\right]\psi\ ,\tag{3}$$

where α_x, α_y, α_z, and γ_4 are the Dirac matrices [15]. A derivation of the transition amplitude (2) using second quantization is given in Ref. [1] and shall not be repeated here. From (2) we derive the differential cross section for electron-positron pair production as

$$\frac{d^6\sigma}{dE_edE_pd\Omega_ed\Omega_p}=\sum_{\mu_e\mu_p}\frac{k_ek_pE_eE_p}{c^4}\int d\mathbf{b}|A_{e\text{-}p}(\mathbf{b})|^2\ .\tag{4}$$

Here E_e, E_p, Ω_e, Ω_p, k_e, and k_p denote the energy, solid angle, and momentum of the emitted electron and positron, respectively. The spin projections are indicated by μ_e and μ_p. In order to evaluate the amplitude $A_{e\text{-}p}$ defined in Eq. (2), we rewrite the electron and positron continuum states as

$$\begin{aligned}\psi_e(\mathbf{r}_T,t)&=\varphi_e(\mathbf{r}_T)e^{-iE_et}\ ,\\ \psi_p(\mathbf{r}_T,t)&=\varphi_p(\mathbf{r}_T)e^{iE_pt}\ .\end{aligned}\tag{5}$$

With the aid of appropriate Fourier transforms, we now can move space-time dependence to the exponent and execute the space-time integrations in Eq. (2). As a result, we obtain

$$A_{e\text{-}p}=i\frac{Z_P}{v\pi}\int d\mathbf{q}_b\frac{H(\mathbf{q}_b,(E_e+E_p)/v)}{q_b^2+[(E_e+E_p)/(v\gamma)]^2}e^{-i\mathbf{q}_b\cdot\mathbf{b}}\ ,\tag{6}$$

where

$$H(\mathbf{p})=\int\varphi_e^\dagger(\mathbf{r}_T)(1-\beta\alpha_z)\varphi_p(\mathbf{r}_T)e^{i\mathbf{p}\cdot\mathbf{r}_T}d\mathbf{r}_T\ .\tag{7}$$

We have decomposed the vector $\mathbf{q}$ into a transverse and a longitudinal part as $\mathbf{q}=(\mathbf{q}_b,q_z)$. Using the continuity equation, we now cast the matrix element (8) in the following form [14]:

$$\begin{aligned}H(\mathbf{p})=&\frac{1}{\gamma^2}\int\varphi_e^\dagger(\mathbf{r}_T)\varphi_p(\mathbf{r}_T)e^{i\mathbf{p}\cdot\mathbf{r}_T}d\mathbf{r}_T\\ &+\frac{\beta^2c}{E_e+E_p}\int\varphi_e^\dagger(\mathbf{r}_T)(\boldsymbol{\alpha}_b\cdot\mathbf{p}_b)\varphi_p(\mathbf{r}_T)e^{i\mathbf{p}\cdot\mathbf{r}_T}d\mathbf{r}_T\ ,\end{aligned}\tag{8}$$

where $\boldsymbol{\alpha}_b\cdot\mathbf{p}_b=\alpha_xp_x+\alpha_yp_y$. This transformation, valid for exact solutions of Eq. (3), is essential in order to avoid spurious contributions to the matrix element when approximate wave functions are used [16].

The integration over the impact parameter in (4) can now be carried out and yields the result

$$\frac{d^6\sigma}{dE_edE_pd\Omega_ed\Omega_p}=\sum_{\mu_e\mu_p}\frac{k_ek_pE_eE_p}{c^4}\frac{Z_P^2}{v^2\pi^2}(2\pi)^2\int d\mathbf{q}_b\frac{|H(\mathbf{q}_b,(E_e+E_p)/v)|^2}{\{q_b^2+[(E_e+E_p)/(v\gamma)]^2\}^2}\ .\tag{9}$$

As we have pointed out in the Introduction, the usage of exact Dirac wave functions is restricted to projectile energies below 10 GeV/u due to numerical difficulties in the evaluation of high multipole contributions with $l>10$ [11]. The aim of the present paper is to calculate cross sections for pair production at high projectile energies. Therefore, in the following, we use approximate electron and positron continuum wave functions and, consequently, avoid the partial-wave analysis. These wave functions, denoted as Sommerfeld-Maue or Furry wave functions, have been widely applied in the literature and for the electron can be written as [12]

$$\varphi_e(\mathbf{r}_T)=N_ee^{i\mathbf{k}_e\cdot\mathbf{r}_T}\left[1-\frac{ic}{2E_e}\boldsymbol{\alpha}\cdot\boldsymbol{\nabla}_{r_T}\right]\times{}_1F_1(-i\nu_e,1,-i(k_er_T+\mathbf{k}_e\cdot\mathbf{r}_T))u_e\ ,\tag{10}$$

with $N_e = \exp(\pi\nu_e/2)\Gamma(1+i\nu_e)$, $\nu_e = Z_T E_e/(c^2 k_e)$, and u_e denotes the free electron spinor

$$u_e = (2\pi)^{-3/2}\left[\frac{E_e+c^2}{2E_e}\right]^{1/2} \times \left[\begin{array}{c}\chi_{\mu_e} \\ [c/(E_e+c^2)](\boldsymbol{\sigma}\cdot\mathbf{k}_e)\chi_{\mu_e}\end{array}\right] . \tag{11}$$

The positron wave function is given by [12]

$$\varphi_p(\mathbf{r}_T) = N_p e^{-i\mathbf{k}_p\cdot\mathbf{r}_T}\left[1+\frac{ic}{2E_p}\boldsymbol{\alpha}\cdot\nabla_{r_T}\right] \times {}_1F_1(-i\nu_p, 1, i(k_p r_T + \mathbf{k}_p\cdot\mathbf{r}_T))v_p , \tag{12}$$

with $N_p = \exp(-\pi\nu_p/2)\Gamma(1+i\nu_p)$, $\nu_p = Z_T E_p/(c^2 k_p)$, and v_p denotes the free positron spinor

$$v_p = (2\pi)^{-3/2}\left[\frac{E_p+c^2}{2E_p}\right]^{1/2} \times \left[\begin{array}{c}[c/(E_p+c^2)](\boldsymbol{\sigma}\cdot\mathbf{k}_p)\chi_{\mu_p} \\ \chi_{\mu_p}\end{array}\right] . \tag{13}$$

The Pauli spinors χ_μ are given by $\chi_1^\dagger = (1,0)$ and $\chi_{-1}^\dagger = (0,1)$.

The Sommerfeld-Maue wave functions are a very good approximation to the exact Dirac continuum wave functions for angular momenta $l \gg Z_T/137$ [12] and can be used for the calculation of processes where angular momenta $l \simeq 1$ are not important. A detailed analysis for bremsstrahlung and photon-nucleus pair production shows that high l values become important when the electron-positron energy is large, which holds for all charges Z_T [12]. The validity of the Sommerfeld-Maue approximation for pair production in relativistic heavy-ion collisions is discussed in Sec. III.

After inserting the wave functions (10) and (12) into the matrix element (8) and neglecting in the product of φ_p with φ_e the term of the order $1/(E_e E_p)$ we obtain

$$H(\mathbf{p}) = N_e^* N_p\left[\frac{1}{\gamma^2}I_a + \frac{\beta^2 c}{E_e+E_p}I_b\right] , \tag{14}$$

where

$$I_a = u_e^\dagger v_p I_1 + \frac{ic}{2E_p}u_e^\dagger(\boldsymbol{\alpha}\cdot\mathbf{I}_2)v_p + \frac{ic}{2E_e}u_e^\dagger(\boldsymbol{\alpha}\cdot\mathbf{I}_3)v_p , \tag{15}$$

$$I_b = u_e^\dagger(\boldsymbol{\alpha}_b\cdot\mathbf{p}_b)v_p I_1 + \frac{ic}{2E_p}u_e^\dagger(\boldsymbol{\alpha}_b\cdot\mathbf{p}_b)(\boldsymbol{\alpha}\cdot\mathbf{I}_2)v_p + \frac{ic}{2E_e}u_e^\dagger(\boldsymbol{\alpha}\cdot\mathbf{I}_3)(\boldsymbol{\alpha}_b\cdot\mathbf{p}_b)v_p , \tag{16}$$

and

$$I_1 = \int F'F e^{i\mathbf{q}\cdot\mathbf{r}_T}d\mathbf{r}_T , \tag{17}$$

$$\mathbf{I}_2 = \int F'(\nabla F)e^{i\mathbf{q}\cdot\mathbf{r}_T}d\mathbf{r}_T , \tag{18}$$

$$\mathbf{I}_3 = \int (\nabla F')F e^{i\mathbf{q}\cdot\mathbf{r}_T}d\mathbf{r}_T , \tag{19}$$

where $\mathbf{q} = \mathbf{p} - \mathbf{k}_e - \mathbf{k}_p$ and

$$F' = {}_1F_1(i\nu_e, 1, i(k_e r_T + \mathbf{k}_e\cdot\mathbf{r}_T)) , \tag{20}$$

$$F = {}_1F_1(-i\nu_p, 1, i(k_p r_T + \mathbf{k}_p\cdot\mathbf{r}_T)) . \tag{21}$$

The integrals (17)–(19) are given in Ref. [12] and shall not be repeated here. The further evaluation of the transition amplitude is straightforward but the resulting expressions are too lengthy to be presented here. We merely note that the final expression for the differential cross section can be expressed as a one-dimensional integral over the transverse momentum q_b. The computation of total cross sections, therefore, requires the numerical evaluation of a seven-dimensional integral. The integrations were evaluated by the Monte Carlo technique [17]. By comparing various steps in the iteration procedure we estimate the accuracy of the total cross section to be better than 10%. The accuracy of double- and single-differential cross sections presented in this paper is better than 1%.

III. RESULTS AND DISCUSSION

Using the formulation of Sec. II, we have calculated double-differential cross sections $d^2\sigma/(dE_e dE_p)$, single-differential cross sections $d\sigma/dE_p$, and total cross sections σ for pair production in relativistic heavy-ion collisions. The cross sections have been obtained from Eq. (9) by exact numerical integration over the transverse momentum q_b, the solid angles Ω_e and Ω_p, and the electron and positron energies E_e and E_p. We note that first-order perturbation theory has a projectile charge dependence like Z_P^2 [cf. Eq. (9)]. All results presented in this section can therefore be scaled to get cross sections for arbitrary projectile charges.

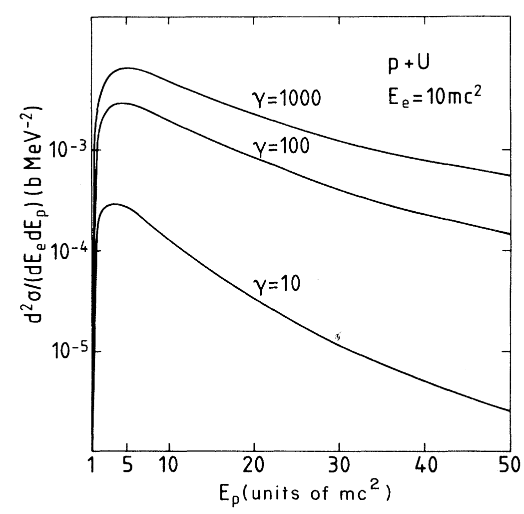

FIG. 1. Double-differential cross sections for the creation of free electron-positron pairs in proton-uranium collisions as a function of the positron energy E_p for various Lorentz factors $\gamma = 10$, 100, and 1000, respectively. The electron energy is $E_e = 10mc^2$.

A. Double-differential cross sections

In Fig. 1 we show double-differential cross sections $d^2\sigma/(dE_e dE_p)$ for a collision of a proton and a uranium nucleus as a function of the positron energy E_p. The electron energy is fixed at $E_e = 10mc^2$. The projectile energy is 8.4, 92.2, and 930.6 GeV/u, which corresponds to Lorentz factors γ of 10, 100, and 1000, respectively. It is seen from Fig. 1 that the positrons are preferably emitted with energies $E_p \simeq 3-8mc^2$. The cross section decreases slowly with increasing positron energy, which shows that high values of E_p are important for the total cross section. The steep decrease for small positron energies is due to the repulsion of the positron by the Coulomb field of the target nucleus. For high values of γ we observe a weaker decay of the cross section and the maximum value is shifted to larger values of E_p.

Figure 2 presents double-differential cross sections for proton-uranium collisions as a function of the electron energy E_e. The positron energy is fixed at $E_p = 10mc^2$. The projectile energies are the same as in Fig. 1. We observe that electrons are preferably emitted with energies $E_p \simeq 2-6mc^2$. Electrons and positrons, as a consequence, are emitted with approximately the same maximum energies. In contrast to the positron spectrum shown in Fig. 1, we obtain a nonzero value for the cross section at $E_e = mc^2$, which reflects the fact that the produced electron is attracted by the target nucleus.

Finally, we shall discuss the asymptotic behavior of the double-differential cross section at asymptotically high projectile energies. Becker, Grün, and Scheid [11] show that the double-differential cross section increases in proportion to $\ln\gamma$. This behavior can readily be obtained from Eq. (9) by noting that the denominator in Eq. (9) has a singularity for $\gamma \to \infty$ at $q_b = 0$. The matrix element $H(\mathbf{q}_b)$ is proportional to q_b so that an elementary integration over q_b yields a γ dependence like $\ln\gamma$. We note that the asymptotic behavior of the total cross section is given by $\sigma \sim (\ln\gamma)^3$ [13,14]. In Fig. 3 double-differential cross sections for proton-uranium collisions, which are divided by $\ln\gamma$, are displayed as a function of the Lorentz factor γ. The electron energy is $10mc^2$, the positron energy $2mc^2$. The curve tends to a constant value for $\gamma > 10^9$ thus demonstrating the correct asymptotic behavior of the differential cross sections calculated from Eq. (9). In this way, it is ensured that the influence of spurious terms [16] has been eliminated.

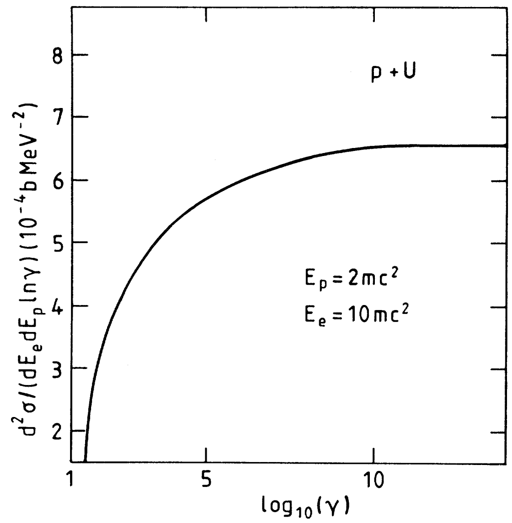

FIG. 3. Double-differential cross sections for the creation of free electron-positron pairs in proton-uranium collisions as a function of the Lorentz factor γ. The cross sections are divided by $\ln\gamma$. The electron energy is $10mc^2$, the positron energy $2mc^2$.

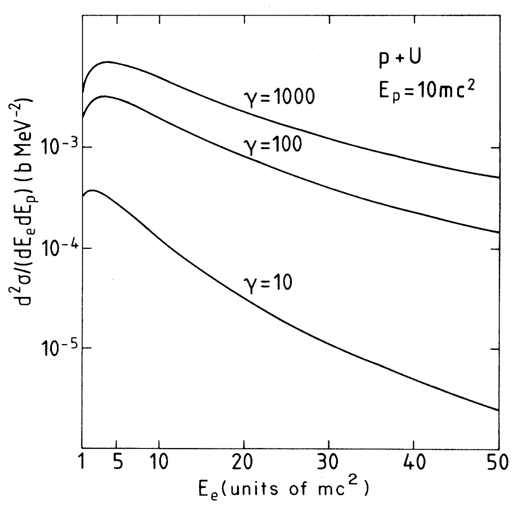

FIG. 2. Double-differential cross sections for the creation of free electron-positron pairs in proton-uranium collisions as a function of the electron energy E_e for various Lorentz factors $\gamma = 10$, 100, and 1000, respectively. The positron energy is $E_p = 10mc^2$.

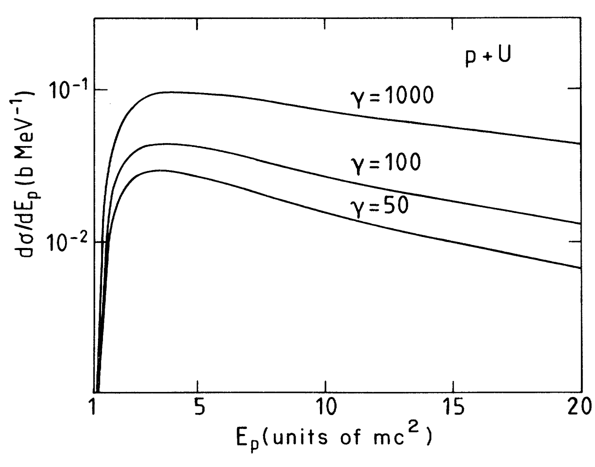

FIG. 4. Single-differential cross sections for the creation of free electron-positron pairs in proton-uranium collisions as a function of the positron energy E_p for various Lorentz factors $\gamma = 50$, 100, and 1000, respectively.

B. Single-differential cross sections

Single-differential cross sections $d\sigma/dE_p$ are obtained by integrating $d^2\sigma/(dE_e dE_p)$ over the electron energy E_e. Figure 4 displays single-differential cross sections in proton-uranium collisions as a function of the positron energy E_p. The Lorentz factor is choosen as 50, 100, and 1000, respectively. The curves show a behavior already discussed in connection with the double-differential cross sections presented in Fig. 1. Small positron energies are strongly suppressed. With increasing γ more and more positrons with high energies are produced. The curves also reveal that single-differential cross sections decrease slowly with increasing positron energy, which implies that high positron energies give important contributions to the total cross section.

C. Total cross sections

In Fig. 5 total cross sections for pair production in uranium-uranium collisions are presented as a function of the Lorentz factor γ. Figure 5 shows four calculations: (a) The results of our exact numerical integration of Eq. (9) employing Sommerfeld-Maue wave functions; (b) first-Born cross sections given by Becker, Grün, and Scheid [11] based on a partial-wave expansion; exact Dirac continuum states are employed but only multipoles up to $l=10$ are included; (c) exact lowest-order QED calculations by Bottcher and Strayer where electron and positron are treated as free particles [3]; and (d) results derived from an approximate integration of Eq. (9) [14], where (i) the first term on the right-hand side of Eq. (8) is neglected, (ii) peaking approximations are used in order to simplify the integration over q_b, (iii) it is assumed that only electrons and positrons with energies $mc^2 \ll E_{e,p} \ll \gamma mc^2$ contribute to the total cross section, and (iv) cutoff parameters for the impact parameter integration are introduced.

It is seen from Fig. 5 that the agreement of our results with the results of Becker, Grün, and Scheid [11] is satisfactory at low projectile energies ($\gamma \sim 2$) but the curves tend to disagree at larger projectile energies. It has been pointed out by Becker, Grün, and Scheid that their cross sections might be inaccurate at projectile energies larger than 10 GeV/u due to the neglect of multipole contributions with $l > 10$, which leads to an underestimation of the total cross section. In order to assess the accuracy of the Sommerfeld-Maue approximation for the continuum states employed in our calculation, we have computed total cross sections using "ultrarelativistic" continuum wave functions [8], i.e., we have neglected the terms containing α in Eqs. (10) and (12) or (15) and (16), respectively. The cross section for $\gamma=100$ obtained in this way is only by about 15% larger than the value shown in Fig. 5. This behavior is expected when large electron-positron energies give the main contribution to the total cross section and indicates that the Sommerfeld-Maue approximation is accurate.

For Lorentz factors $\gamma > 17$ our results show unexpectedly good agreement with the values derived from QED calculations [3] in which the electron and the positron are described by plane waves. Accordingly, higher-order terms in the target charge give only a small contribution

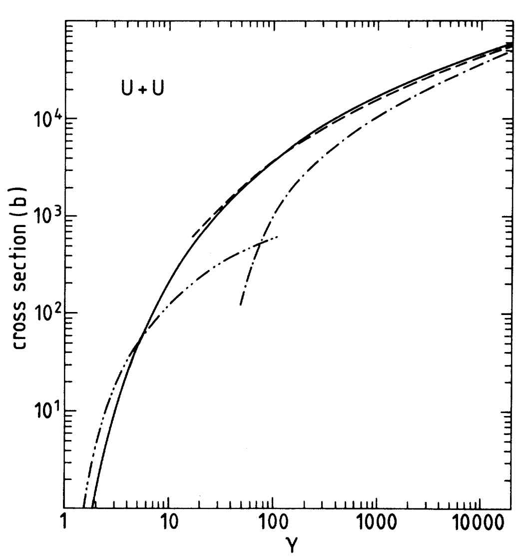

FIG. 5. Total cross sections for the creation of free electron-positron pairs in uranium-uranium collisions as a function of the Lorentz factor γ. Solid line, present work (first-Born calculations using Sommerfeld-Maue continuum states); dashed line, Ref. [3] (lowest-order QED calculations); dashed-dotted line, Ref. [14] (approximate first-Born calculations); dashed-double-dotted line, Ref. [11] (first-Born calculations employing a partial-wave expansion with $l \leq 10$).

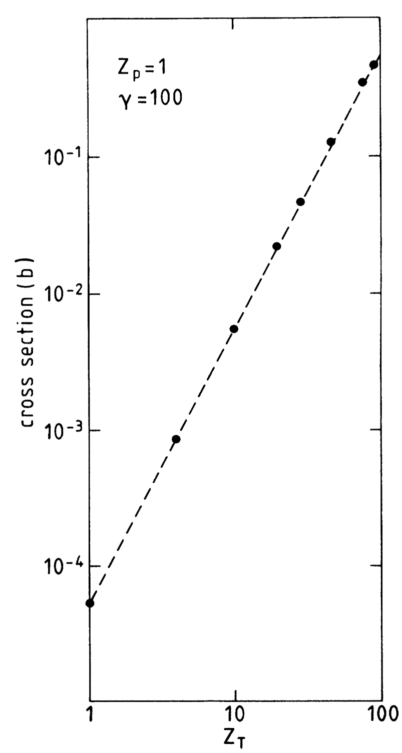

FIG. 6. Total cross sections for the creation of free electron-positron pairs as a function of the target charge Z_T for Lorentz factor $\gamma=100$ and $Z_P=1$. Dots, present work; dashed line, Ref [3].

to the total pair-production cross section. This is in agreement with the findings for photon-nucleus pair production where corrections by about 10% for heavy elements like Pb were found [18]. We thus conclude that pair production is reasonably well described by the lowest-order QED diagrams for Lorentz factors as low as $\gamma \sim 17$, which holds even in the presence of very strong fields. It should be noted that the cross sections derived by Racah as early as 1937 (see Racah [7], Eq. (61) and Ref. [19]) are in close agreement with exact lowest-order QED cross sections [3] and show a good agreement with our results for $\gamma > 10$. The cross sections given by Bertulani and Baur [14], which are obtained from an approximate integration of Eq. (9), underestimate our exact results for Lorentz factors $100 < \gamma < 10\,000$ considerably. The approximations employed seem to be valid only for Lorentz factors as large as $\gamma = 20\,000$.

To further examine higher-order corrections in αZ_T, it is instructive to consider the dependence of the pair-production cross section on the target charge Z_T. In Fig. 6, total cross sections for pair production are displayed as a function of the target charge for a Lorentz factor $\gamma = 100$ and $Z_P = 1$. The dashed curve represents the QED calculation [3], which has a target charge dependence as Z_T^2. The dots are the results of our first-Born calculation, which includes the target interaction to all orders. The Born cross sections show an unexpectedly clear Z_T^2 dependence and agree with the QED cross sections on the percent level. Accordingly, cross sections for all projectile and target combinations can be obtained from Fig. 5 by assuming a scaling law of the pair-production cross section like $\sigma \sim Z_T^2 Z_P^2$.

IV. CONCLUDING REMARKS

In this paper, we present double-differential, single-differential, and total cross sections for electron-positron pair production in relativistic heavy-ion collisions. Electron and positron continuum states are described by Sommerfeld-Maue wave functions and the resulting first-Born matrix elements are evaluated exactly. Our formulation provides a treatment of pair production, which covers a wide energy regime from 1 GeV/u up to 20 TeV/u. Good agreement is found with first-Born calculations employing exact continuum states [11] at low projectile energies and with lowest-order QED calculations using plane waves [3] at projectile energies 15 GeV/u $< E <$ 20 TeV/u. It turns out that higher-order terms in αZ_T give only a small contribution to the total pair-production cross section. Our results indicate that perturbation theory gives an accurate description of electron-positron pair production even in the presence of strong fields.

We finally remark that for projectile energies of several TeV/u not only single but also multiple electron-positron pairs are expected to be produced in a relativistic heavy-ion collision [1,20]. Since the exact numerical evaluation of higher-order diagrams will become extremely difficult, new approaches are needed to give reliable estimates of the corresponding cross sections. Here the theoretical development is just at its beginning.

ACKNOWLEDGMENTS

The author is very grateful to Professor T. Ishihara and Professor N. Toshima for their support and hospitality at the Institute of Applied Physics, Tsukuba University. He gratefully acknowledges support from the Japan Society for the Promotion of Science (JSPS) and a Grant-in-aid of the Japanese Ministry of Education, Science and Culture (Monbusho). He is also indebted to Professor J. Eichler for valuable discussions and critical reading of the manuscript. Dr. P. J. Fons contributed with stimulating discussions on supercolliders and helped to check several formulas with the MATHEMATICA program.

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