Two- and three-dimensional behavior of Rayleigh-Taylor and Kelvin-Helmholtz instabilities

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Two- and three-dimensional behavior of the Rayleigh-Taylor (RT) and Kelvin-Helmholtz (KH) instabilities is examined with a hydrodynamic code: cubic interpolated pseudoparticle. The mushroom structure owing to the KH instability in three dimensions is much smaller than that in two dimensions, and hence the nonlinear growth is faster in three dimensions. The simulation without gravity shows a similar behavior and hence this difference between two and three dimensions does not originate from the RT instability. This difference cannot be explained by linear analysis on the KH instability also.

The Rayleigh-Taylor (RT) instability has been a subject of primary interest for many years in many fields of physics. For example, it may be an origin of fuel-pusher mixing during the implosion process in inertial confinement fusion [1,2] (ICF). Recently, the interest in this instability has grown in astrophysics because it may cause a mixing of materials in Super Nova 1987A as suggested from the observation data [3,4]. Hachisu *et al.* [5] made a two-dimensional simulation on this process and concluded that the result is adequate to explain most of observation data that imply mixing. Although there exist three-dimensional simulations [6] on a similar problem, there is no clear evidence of the difference between two and three dimensions. This paper clearly demonstrates the different behavior of the RT instability in two and three dimensions. We will show here that a mushroom structure owing to the Kelvin-Helmholtz (KH) instability in three dimensions is much smaller than that in two dimensions and hence the nonlinear growth is faster in three dimensions. This difference cannot be explained by linear analysis.

We have developed a new general hyperbolic solver CIP (cubic interpolated pseudoparticle) method and applied it to a number of test problems [7–10]. It has been proved that the CIP can give a less-diffusive and quite accurate result [9,10] without any flux-limiting procedure frequently used in most modern schemes. In this paper, we apply the CIP method to the classical RT instability in two and three dimensions.

Let us first describe a configuration used in the simulation. Initially two fluids are placed at rest in contact with each other. The density of those fluids is $\rho=1.0$ for $0\leq x \leq 0.3$ and $\rho=0.3$ for $0.3\leq x$. The gravity g is imposed in the x direction and its magnitude is 1.0. The pressure is obtained from a static force balance $\partial p/\partial x=\rho g$ starting from $p=0.1$ at $x=0$. The boundary is free in the x direction and mirrored in the y and z directions. In order to select the instability mode, the velocity perturbation of the incompressible mode ($\nabla\cdot\mathbf{v}=0$) is imposed around the interface. Its wavelength in the y and z directions is chosen so that the system size in the y and z directions corresponds to one wavelength.

Figure 1 shows the density contours in two- and three-dimensional simulations. In both cases, initial perturbation velocity is set to 0.8. In order to check the accuracy of the code, we have done three simulations for each result by changing the mesh size. In general, the overall structure of the mushroom remains similar even if the mesh size is changed, whereas the detailed structure such as windings inside the mushroom becomes evident with reduced mesh size. It is also shown that the mushroom

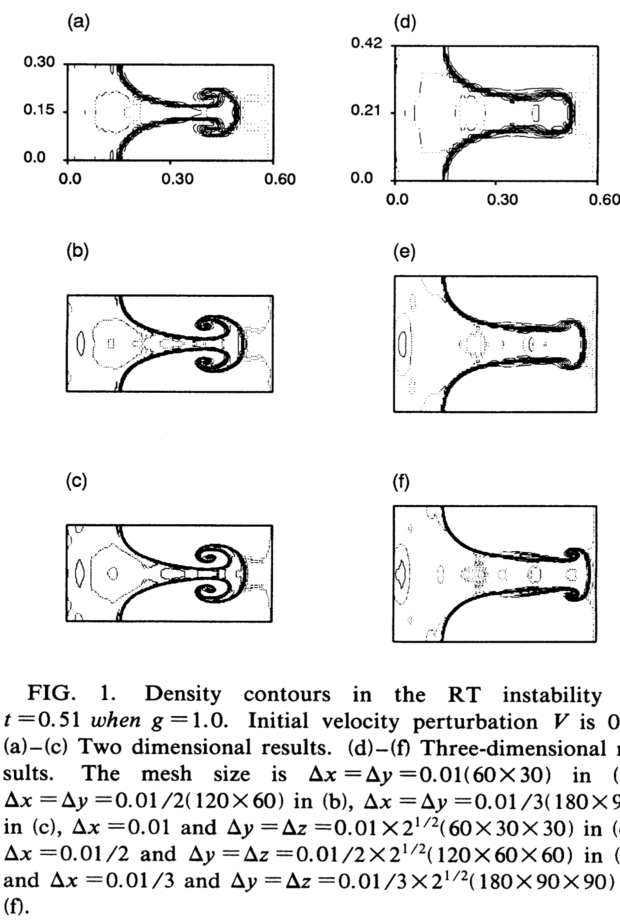

FIG. 1. Density contours in the RT instability at $t=0.51$ *when* $g=1.0$. Initial velocity perturbation V is 0.8. (a)–(c) Two dimensional results. (d)–(f) Three-dimensional results. The mesh size is $\Delta x=\Delta y=0.01(60\times30)$ in (a), $\Delta x=\Delta y=0.01/2(120\times60)$ in (b), $\Delta x=\Delta y=0.01/3(180\times90)$ in (c), $\Delta x=0.01$ and $\Delta y=\Delta z=0.01\times2^{1/2}(60\times30\times30)$ in (d), $\Delta x=0.01/2$ and $\Delta y=\Delta z=0.01/2\times2^{1/2}(120\times60\times60)$ in (e), and $\Delta x=0.01/3$ and $\Delta y=\Delta z=0.01/3\times2^{1/2}(180\times90\times90)$ in (f).

structure in the three-dimensional case is much smaller than that in the two-dimensional case. If the wave number in y and z directions is denoted by k and l, respectively, the growth rate of the RT instability is proportional to $(gk)^{1/2}$ and $[g(k^2+l^2)^{1/2}]^{1/2}$ in two and three dimensions. Thus, in three dimensions we must use $2^{1/2}$ times smaller wave numbers that make the growth rate equal to that in two dimensions. In reality, wave numbers in Figs. 1(d)–1(f) are $2^{1/2}$ times smaller than that in Figs. 1(a)–1(c). This has been done by using, for example, $\Delta y = 0.01$ in two dimensions [Fig. 1(a)] and $\Delta y = \Delta z = 0.01 \times 2^{1/2}$ in three dimensions [Fig. 1(d)]. If we compare the cases having the same wavelength, the difference is even larger because the growth rate of the RT instability is $2^{1/2}$ times larger than that in two dimensions and the KH instability does not have enough time to grow. It is widely recognized that the mushroom structure originates from the KH instability. In order to separate this effect from the RT instability, we set $g = 0$ to eliminate the RT instability. In this case, the interface moves with the speed intially given. The relative motion of two fluids at the interface induces the KH instability. The numerical results are shown in Fig. 2, where 2(a)–2(c) and 2(d)–2(f) are again the two- and three-dimensional results. This result is quite similar to that in Fig. 1. As is easily understood, however, it takes a much longer time to reach the final state at $t = 0.51$ in Fig. 1 because the motion is not accelerated by the RT instability. The result in cylindrically symmetric two dimensions support this conclusion as shown in Figs. 2(g)–2(i). Thus, the three-dimensional behavior is not a consequence of perturbations in three directions but is that of "three-dimensional configuration" as will be shown later.

In order to explain this difference, we derive a linear dispersion relation of the KH instability. In the configuration shown in Figs. 2(a)–2(c), the KH instability occurs on the surface of a plane whose thickness is $2a$, whereas in Figs. 2(d)–2(f) or 2(g)–2(i) the KH instability occurs on the surface of a cylinder whose radius is a. Therefore the dispersion relation is written as [11,12]

$$\frac{\omega}{\kappa V} = \frac{1}{1+F} , \tag{1}$$

where ω is the complex frequency, V is the relative velocity of two fluids at the interface, and κ is the wave number. Here, F is given by

$$F = \left[\frac{\rho_1}{\rho_2} \frac{1+\exp(2\kappa a)}{1-\exp(2\kappa a)} \right]^{1/2} \tag{2}$$

in two dimensions and

$$F = \left[-\frac{\rho_1}{\rho_2} \frac{K_1(\kappa a) I_0(\kappa a)}{K_0(\kappa a) I_1(\kappa a)} \right]^{1/2} , \tag{2'}$$

in three dimensions. In Eq. (2′), K_0, K_1, I_0, I_1 are the zeroth- and first-order modified Bessel functions of the first and the second kind, respectively. In Eqs. (2) and (2′), F is imaginary, and hence this wave propagates on the surface and grows. The growth rate is depicted in Fig. 3, for $\rho_2/\rho_1 = 0.3$.

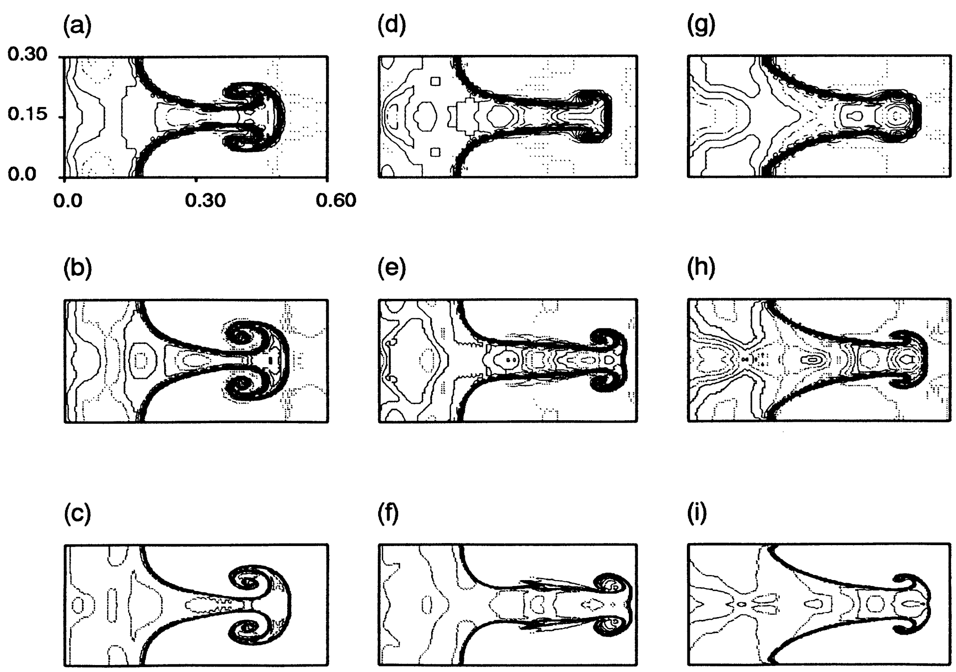

FIG. 2. Density contours at $t = 0.91$ without gravity. (a)–(c) Plane two-dimensional results with $V = 0.8 \times 0.72$. (d)–(f) Three-dimensional results with $V = 0.8$. (g)–(i) Cylindrical two-dimensional results with $V = 0.8 \times 0.72$. The mesh size is $\Delta x = \Delta y (= \Delta z) = 0.01 [60 \times 30 \times (30)]$ in (a), (d), and (g), $\Delta x = \Delta y (= \Delta z) = 0.01/2 [120 \times 60 \times (60)]$ in (b), (e), and (h), and $\Delta x = \Delta y (= \Delta z) = 0.01/3 [180 \times 90 \times (90)]$ in (c), (f), and (i).

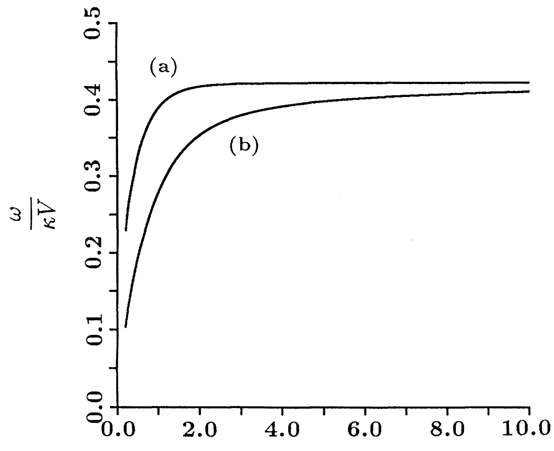

FIG. 3. The growth rate of the KH instability. (a) For a surface wave on a moving plane. (b) For a surface wave on a moving cylinder.

In the configuration shown in Fig. 2, the seed of the KH instability is given at the leading edge of the heavier fluid and hence $\kappa a \sim 1$. At this wave number, the growth rate in three dimensions is about 72% of that in two dimensions. It seems that this difference in the growth rate may explain the difference between two and three dimensions. We can confirm this by reducing the initial velocity of the perturbation by 28% in two dimensions which corresponds to the reduction of V in the dispersion relation Eq. (1). Since ω is proportional to V, this reduction will make the growth rate equal both in two and three dimensions. In Figs. 2(a)–2(c) this reduction has already been done. If we use the velocity $V=0.8$ without reduction in two dimensions, the mushroom structure develops even wider in the y direction. We should note that we have used $\Delta y = \Delta z = 0.01$ in Fig. 2(d) in contrast to Fig. 1(d). By this choice, we can compare the two- and three-dimensional results with the same a in Eqs. (2) and (2′).

From this comparison, we may conclude that the difference in the mushroom structure between two and three dimensions cannot be explained by the linear analysis. It is probably attributed to the nonlinear process. It is natural to imagine that the windings of the fluid inside the mushroom stays within a plane of two dimensions in the two-dimensional case, whereas the windings in the three-dimensional case can escape in the other direction and its effect will decay rapidly with increasing distance from the mushroom, and therefore may not grow so large. Note that the effect may decay in proportion to r^{-1} and r^{-2} in two and three dimensions, respectively, where r is the distance from the surface. This may be a reason why the two-dimensional result in cylindrical geometry corresponds to the three-dimensional result.

More realistic situations similar to SN1987A will be worthwhile to investigate because the growth of the two-dimensional RT is not sufficient to explain the data when a reasonably small perturbation is imposed initially. The present result implies that perturbations can grow large in three dimensions because of less drag resulting from the mushroom structure. In this paper, we will not treat the process in further detail but we will discuss it in a future paper.

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