Electric fields from steady currents and unexplained electromagnetic experiments

T. Ivezić
Institute of Physics of the University, P.O. Box 304, 41000 Zagreb, Yugoslavia
(Received 14 August 1990)

It is shown that so-called "relativistic" electric fields [T. Ivezić, Phys. Lett. A **144**, 427 (1990)] from stationary current-carrying conductors are responsible for the existence of longitudinal forces that subject the conductor to tension. The exploding-wire phenomenon [P. Graneau, IEEE Trans. Magn. **MAG-20**, 444 (1984)] is explained in terms of these longitudinal electric forces. Other unexplained electromagnetic experiments are also discussed.

I. INTRODUCTION

Different unexplained electromagnetic experiments have been reported over a period of many years. Some recent experiments are (i) the demonstration of the existence of a radial electric field **E** near a superconducting wire [1], (ii) wire fragmentation by pulse currents [2], (iii) railgun recoil experiments [3], and (iv) electrodynamic explosions in liquids [4].

Since the experiments mentioned above have never been adequately explained within the standard electromagnetic theory (SET), some authors have tried to explain them (with the exception of the radial-field experiments) in terms of the old Ampère action-at-a-distance electromagnetic theory. An important example is the experiment performed in Ref. [2] [see Ref. [2], Fig. 12(a)] in which a straight aluminum wire of diameter 1.19 mm was shattered into many pieces when a particular pulse current level was reached in a circuit containing the wire. But, in an arrangement as was used in Ref. [2], magnetic forces act perpendicularly to the wire axis and cannot produce tensile stress in the direction of the current. Therefore, it is argued [2] that the experiment mentioned gives the most conclusive evidence of the existence of Ampère tension. Thus, according to Ref. [2], the whole relativistic field theory is questioned in this type of experiment.

It will be shown below that all the above-mentioned unexplained experiments can be consistently explained without invoking concepts, e.g., action-at-a-distance, that were abandoned long ago. In our approach the longitudinal ponderomotive forces observed in current-carrying conductors (CCC's) in unexplained electromagnetic experiments have *an electrical origin,* contrary to Ampère's formulation, in which they are of magnetic origin. They represent the longitudinal part of electric forces caused either by the second-order "relativistic" electric fields from *stationary* CCC's, predicted theoretically [5], and/or by the well-known zero-order electric fields arising from surface charges that exist in conductors of finite conductivity carrying steady currents (for surface charges in circuits, see, e.g., Ref. [6], and references therein). Electric forces from "relativistic" electric fields alone give a satisfactory explanation for many pulse current events, which are dominated by longitudinal forces as are, for instance, the explosion of wires and particularly the previously mentioned explosion of straight wires shown in Fig. 12a of Ref. [2]. In the following we shall be mainly concerned with the explanation of that experiment both qualitatively and quantitatively. Both the relativistic and the zero-order electric fields play a part in almost all the other unexplained experiments, e.g., in experiments with railguns in which longitudinal recoil forces produce buckling and distortion of rails.

II. FORCES BETWEEN CURRENT ELEMENTS

Let us start with a short review of the Ampère expression for the magnetic force exerted by the current element $i_1 dl_1$ upon another current element $i_2 dl_2$. This force, acting on $i_2 dl_2$ is

$$d\mathbf{F}_{2,A} = (-\mu_0 i_1 i_2/4\pi)(\mathbf{r}_{12}/r_{12}^3) \times [2(dl_1 dl_2) - (3/r_{12}^2)(dl_1 \mathbf{r}_{12})(dl_2 \mathbf{r}_{12})] . \quad (1)$$

It contains both a part that is perpendicular to the current element $i_2 dl_2$ and a longitudinal part that is directed along the current element. It is argued by proponents of the Ampère force law that the longitudinal part of Eq. (1) is responsible for the longitudinal internal stresses observed in CCC's in many of the unexplained experiments. They point that the relativistic field-theoretical formulation of force on charges, i.e., the Lorentz force, is correct only for charges traveling in vacuum, while for the currents in metallic conductors, and perhaps in dense plasmas, one has to use the Ampère-Neumann formulation of electrodynamics.

However, there is a difficulty with today's acceptance of Ampère's force law, since that law is, as we have already said, an empirical action-at-a-distance law. It obeys Newton's third law and accordingly cannot be based on modern relativistic field theory. Obviously, from the theoretical point of view, one cannot be satisfied with such an approach to the theory of electromagnetism, in which two basically different concepts, the action-at-a-distance and the field-theoretical formulation of forces, exist at the same time.

Let us now see how the forces acting between current elements in CCC's are described within standard electromagnetic theory. First, we consider the case in which CCC's are of infinite conductivity. The current element $i_1 dl_1$ creates a magnetic field $d\mathbf{B}_1$ at the place of the current element $i_2 dl_2$. This field acts only on the moving electrons in dl_2 by the Lorentz transverse force $d\mathbf{F}_2 = \lambda_2 \mathbf{v}_2 \times d\mathbf{B}_1 dl_2$ (λ_2 is the negative charge per unit length in dl_2, $\mathbf{v}_2$ is the average drift velocity of the electrons in dl_2), producing the charge separation inside the second element. Then, $d\mathbf{F}_2$ is transmitted to the material of the conductor through the effect of the Hall field ($\mathbf{E}_H$) on the stationary lattice of positive ions, giving the ponderomotive force $d\mathbf{F}_{2,\mathrm{BS}}$ described by the Biot-Savart force law,

$$d\mathbf{F}_{2,\mathrm{BS}} = (\mu_0 i_1 i_2 / 4\pi)(1/r_{1,2}^3) d\mathbf{l}_2 \times (d\mathbf{l}_1 \times \mathbf{r}_{1,2}) \ . \tag{2}$$

This simple treatment is correct only for free electrons in solid CCC's. Deviations from free-electron behavior due to the electron-lattice interaction can be easily treated, as described in Ref. [7]. This force is always normal to the current density vector at the point of the current element $i_2 dl_2$.

However, in addition to the magnetic field, an electric field is created by the charges in dl_1. It can be shown (see, for example, Ref. [8]) that the moving electrons in a CCC, in a steady state, produce an external electric field that just cancels that from the stationary ions. This happens, in fact, since in SET the mean distances between moving electrons in a CCC are the same as the mean distances between ions at rest.

Thus, using SET, in a CCC of infinite conductivity, only the Biot-Savart magnetic force between current elements $i_1 dl_1$ and $i_2 dl_2$ remains, $d\mathbf{F}_{2,\mathrm{tot}} = d\mathbf{F}_{2,\mathrm{BS}}$. Usually, only this kind of conductor is considered theoretically.

In stationary homogeneous CCC's of finite conductivity, there are surface charges that are sources of an extremely weak axial electric field inside the conductor, a field which drives a conduction current according to Ohm's law. Simultaneously, the surface charges provide a zero-order electric field, $\mathbf{E}_{\mathrm{ext}}^0$, outside the wire, whose tangential component is equal to the electric field inside the conductor, while the normal component is determined by the magnitude of the surface charge density σ, $E_n^0 = \sigma / \epsilon_0$. In contrast to the tangential component, E_n^0 can be very high. It has to be mentioned that it is very difficult, in most practical circuits, to determine the surface charges in a quantitative way, since their distribution depends on the detailed geometry of the circuit itself, and even on its surroundings. Thus, in SET, for stationary CCC's of finite conductivity, the force exerted by current element $i_1 dl_1$ upon another current element $i_2 dl_2$ is given as a sum of the Biot-Savart force, Eq. (2), and an electric force $d\mathbf{F}_{2,E}^0$, caused by $\mathbf{E}_{\mathrm{ext}}^0$, $d\mathbf{F}_{2,\mathrm{total}} = d\mathbf{F}_{2,\mathrm{BS}} + d\mathbf{F}_{2,E}^0$.

Since it is difficult to calculate, or to measure, the surface charge distributions and the corresponding external fields, they are simply ignored in many experiments (for example, in all experiments with recoil forces in railguns), or even misunderstood. Thus, in the experiment described in Ref. [9], it is argued that the highest value of the electric field outside CCC's induced by surface charges is the value of the axial electric field inside the conductor, which is obviously wrong. Neither $d\mathbf{F}_{2,\mathrm{BS}}$ nor $d\mathbf{F}_{2,E}^0$ can explain the tensile fracture of long straight wires that carry high currents shown in Fig. 12a of Ref. [2].

However, there is another source of electric field outside a stationary CCC of infinite conductivity, which has been discovered [5] quite recently. The starting points in work described in Ref. [5] were (i) the requirement of relativistic invariance of macroscopic charge in all cases, i.e., for a CCC too; and (ii) the requirement that a CCC has to be globally and locally (section by section) charge neutral in every inertial frame of reference (IFR), as is the corresponding conductor without current. None of these requirements is satisfied in SET. In order to satisfy the mentioned physical requirements, which are based on the principle of relativity, the mean distances Δ_e between moving electrons in a stationary CCC have to be Lorentz contracted with respect to the mean distances Δ between ions at rest; $\Delta_e = \Delta / \gamma$. As a consequence, on an element of length dl of the "wire," or more precisely, of the lattice of the ions, there is a negative charge $\delta Q = (1-\gamma)\lambda dl$, Eq. (4) [5], which creates a real second-order ($\propto v^2/c^2$) electric field, $\mathbf{E}_{\mathrm{ext}} = [(1-\gamma)\lambda / 2\pi\epsilon_0 r]\hat{\mathbf{r}}$ [Eq. (5) of Ref. [5]], outside an infinite stationary wire with current; λ is the positive charge per unit length, $\gamma = (1-\beta^2)^{-1/2}$, $\beta = v/c$, v is the average drift velocity of the electrons, $\hat{\mathbf{r}}$ is the unit vector in the $\mathbf{r}$ direction.

It can be easily shown that this field agrees in a qualitative way (an I^2 dependence) and also quantitatively with the electric field observed in the unexplained electromagnetic experiment [1]. We remark that the experiment [1] is performed with a superconductor, in which case there is no zero-order electric field from surface charges. The quantitative comparison with experiment [1] was performed by generalizing the theory developed for an infinite stationary CCC [5] to a closed CCC. It is given elsewhere, but this generalization will be also briefly described here.

When an infinite wire with a current in different inertial frames of reference is considered (as in Ref. [5]) the Lorentz contraction is relative. However, when a stationary ideal or superconducting loop is considered, the electrons are accelerated and therefore the Lorentz contraction of the mean spacing between moving electrons becomes absolute. This causes the moving-electron subsystem to shrink to a smaller length in the laboratory frame. The total negative and positive charge is of equal magnitude and *the loop as a whole is charge neutral.* It can be easily shown that again no radiation is emitted by a closed CCC. The Lienard-Wiechert fields of N moving electrons spaced by the distance Δ/γ will be again a static, time-independent field. However, the electric field of N stationary positive ions situated on a ring with length $L_+ = L = 2R\pi$ (and the total charge $\Delta Q_+ = \lambda L = Ne$), will no longer cancel the electric field caused by N moving electrons situated on a smaller, contracted, ring with the length $L_- = L/\gamma$ [and the total charge $\Delta Q_- = (-\gamma\lambda)(L/\gamma) = -Ne = -\Delta Q_+$]. The electric field outside the closed CCC can be calculated as the vector sum of the fields produced by positive and negative charges situated on corresponding rings; this field is found to be in quantitative agreement with experiments [1] (actually, the potentials have been calculated). A very long stationary straight wire with current can be considered now as the limiting case, for $R \to \infty$, of a closed CCC. The electric field outside an element of length dl of such a long straight wire can be then calculated taking into account the presence of a negative charge δQ [Eq. (4) in Ref. [5]] on the element of length dl of the "wire."

In this case, the total force exerted by current element $i_1 dl_1$ upon the another current element $i_2 dl_2$, for conductors of finite conductivity, is given as a sum of a magnetic and two electric forces of different origin,

$$d\mathbf{F}_{2,\text{tot}} = d\mathbf{F}_{2,\text{BS}} + d\mathbf{F}^0_{2,E} + d\mathbf{F}^r_{2,E} \ . \tag{3}$$

The first two terms represent the total force on $i_2 dl_2$ in SET, while the third term represents the electric force caused by the second-order "relativistic" electric field. Both the usual zero-order and the "relativistic" electric fields are time independent and the forces they induce obey Newton's laws. The normal components (to dl_2) of the electric forces acting on free electrons in dl_2 cause a charge separation. Thereby a transverse electric field $\mathbf{E}_i$ is created in the interior of the wire. Since in a steady state there are no transverse currents, the sum of the electric forces, transverse to the motion of the electrons, must be zero. This condition determines $\mathbf{E}_i$. We mention that, due to the validity of the principle of superposition, the effects of these electric forces on free electrons can be treated independently of the effects of the previously mentioned magnetic force and the electric force induced by $\mathbf{E}_H$. Then, as can be easily shown, the positive ions of the lattice also experience zero net transverse electric force. Thus, only the effects of longitudinal components (along dl_2) of the external electric fields have to be considered. Their influence on free electrons is of no concern here. However, the longitudinal components of $d\mathbf{F}^0_{2,E}$ and $d\mathbf{F}^r_{2,E}$, attached to the metal lattice of the current element $i_2 dl_2$, are responsible for the internal stresses observed in "troubling" experiments. For instance, they can account in a natural way for the appearance and the characteristics of longitudinal recoil forces, which cause deformations of conductors used as rails in railgun experiments. To make a quantitative comparison with such experiments one would need to perform a computer-aided finite-current-element analysis, similar to the one carried out in Ref. [3], but which takes into account all three of the forces. However, this is out of the scope of this letter.

III. EXPLOSION OF STRAIGHT WIRES

In real-world circuits the second-order effects of relative motion found in Ref. [5] are completely masked by the mentioned zero-order effects in almost all cases. The important exceptions are (i) a superconductor, (ii) the case of very high dc currents in conductors of finite conductivity, and particularly, (iii) the explosion of straight wires. In the first case $\mathbf{E}^0_{\text{ext}}$ is zero since there are no surface charges. In the third case the magnetic force $d\mathbf{F}_{2,\text{BS}}$ is ineffective and in this case the same happens with $d\mathbf{F}^0_{2,E}$. Thus, the only remaining force exerted by $i_1 dl_1$ on charges in $i_2 dl_2$ is $d\mathbf{F}^r_{2,E}$, $d\mathbf{F}^r_{2,E} = (1-\gamma_2)\lambda_2 dl_2 \mathbf{E}_{1,\text{ext}}$, where $\mathbf{E}_{1,\text{ext}} = (1/4\pi\epsilon_0)(1-\gamma_1)\lambda_1 dl_1 \mathbf{r}_{12}/r^3_{12}$. $d\mathbf{F}^r_{2,E}$ is directed along the separation vector $\mathbf{r}_{12}$.

Let the two current elements lie in the same straight line, point in the same direction, and carry the same current i. The only effective force in this case is the longitudinal component of $d\mathbf{F}^r_{2,E}$, given for $\beta \ll 1$, by $dF^r_{\text{long}} \simeq \frac{1}{2}(1/4\pi\epsilon_0)(\beta^2\lambda^2 dl_1 dl_2/r^2_{12})$ or

$$dF^r_{\text{long}} \simeq \tfrac{1}{2}(\mu_0/4\pi)(i^2/r^2_{12}) dl_1 dl_2 \ . \tag{4}$$

This force is always repulsive. In this way the remarkable result is obtained that the long straight current-carrying wires find themselves in tension. Comparing this expression with Eq. (4) of Ref. [2] we see that they differ only by a factor $\frac{1}{2}$.

All the theoretical results in Ref. [2] have been obtained using Eq. (4) and the computer-aided finite-current-element analysis. This means that if one applies the same analysis to our Eq. (4), then the results will be the same (up to a factor $\frac{1}{2}$). It also refers to the quantitative comparison of theoretical results and experimental findings performed in Ref. [2]. Thus we reach a conclusion that, not the longitudinal magnetic forces (Ampère's forces), but the longitudinal electric forces, induced by "relativistic" electric fields, explain the brittle tension breaks in the solid straight wire, which has been weakened by Joule heating.

Further, the approach of Ref. [5] explains in a natural way the conclusion obtained in Ref. [2] that a current element used in the computer-aided analysis must be of finite size and that the analysis must involve not only electrons but also lattice ions. This question has been recently discussed in Refs. [10] and [11], which appeared during the reviewing process for this paper. There [2,10,11], it is shown that current elements cannot be allowed to coincide, i.e., they must retain finite nonzero volumes in order to avoid the possibility of self-interaction. Therefore, the smallest distance between two current elements in Eq. (4) from Ref. [2] has to be of the order of the interatomic lattice spacing.

According to Ref. [5] a net negative charge δQ for an infinite wire appears due to Lorentz contraction of the mean spacing between moving electrons, $\Delta_e = \Delta/\gamma$. For the same reason a nonuniform charge distribution, different from the one caused by the pinch effect, appears for a closed loop. The electrons concentrate close to the inside surface of the circuit. The resulting charge distribution can be modeled with the previously mentioned positively and negatively charged rings. In both cases, (an infinite wire and a closed loop), the charges of one sort have to be in motion relative to the charges of another sort in a given IFR, so that the Lorentz contraction of the mean spacing between moving charges can be effective. It means that a current element must have finite, physical size, at least of the order Δ, and must be comprised of ions and electrons.

IV. CONCLUSION

It can be concluded from the theory presented that longitudinal ponderomotive forces appearing in CCC's in unexplained electromagnetic experiments are not of magnetic origin but of an electric origin. The proposed theory is in complete agreement with the relativistic field theory, which is not the case with the Ampère formulation of the longitudinal ponderomotive forces. In the explanation of a remarkable phenomenon, the fragmentation of long straight wires by pulse currents, the "relativistic" electric fields arising from stationary wires with steady currents are used. There are no such fields in the usual formulation of electromagnetic theory, and therefore SET is unable to explain these electromagnetic experiments.

Recently [11], Rambaut and Vigier derived in a completely different way the similar result that a steady current around a loop generates a small electric field. They have also discussed longitudinal forces and the unexplained electromagnetic experiments, and pointed out the importance of the existence of such electric fields and longitudinal forces for tokamak devices. In their approach all the results are the consequences of the assumed form for the four-vector potential of *one charged particle* [Eqs. (6), (7), and (12) in Ref. [11]], which is different from the standard one. To justify this change in the standard form of the 4-potential they evoke a model of an extended charged particle and a model of non-zero-mass photons.

Obviously, our approach is conceptually much simpler and uses standard notions of pointlike charges and zero-mass photons. We believe that the explanation for longitudinal forces given in this paper will have consequences in many branches of physics, especially in plasma physics and in hot fusion.

[1] W. F. Edwards, C. S. Kenyon, and D. K. Lemon, Phys. Rev. D **14**, 922 (1976).
[2] P. Graneau, IEEE Trans. Magn. **MAG-20**, 444 (1985).
[3] P. Graneau, J. Phys. D **20**, 391 (1987); J. Appl. Phys. **62**, 3006 (1987).
[4] R. Azevedo, P. Graneau, C. Millet, and N. Graneau, Phys. Lett. A **117**, 101 (1986).
[5] T. Ivezić, Phys. Lett. A **144**, 427 (1990).
[6] M. A. Heald, Am. J. Phys. **52**, 522 (1984).
[7] W. R. McKinnon, S. P. McAlister, and C. M. Hurd, Am. J. Phys. **49**, 493 (1981).
[8] J. D. Jackson, *Classical Electrodynamics* 2nd ed., (Wiley, New York, 1975), Prob. 14.13.
[9] R. Sansbury, Rev. Sci. Instrum. **56**, 415 (1985).
[10] N. Graneau, Phys. Lett. A **147**, 92 (1990).
[11] M. Rambaut and J. P. Vigier, Phys. Lett. A **148**, 229 (1990).