Simple laser accelerator: Optics and particle dynamics

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When a laser is focused it develops a longitudinal component. This could be used to develop a laser particle accelerator. A lens waveguide array is discussed, and it is shown that such a system could generate high-energy particle beams. The possibility of using a "diffraction-free" Bessel beam is discussed, and a possible configuration is suggested. To accelerate electrons from one to a few MeV seems possible using a well-focused, 1-J, 1-ps laser pulse. This would provide a simple proof-of-principle experiment. In order to accelerate heavier particles, such as protons, the injected particle beam would have to be ultrarelativistic such as that produced by the superconducting supercollider.

I. INTRODUCTION

The possibility of using high-power lasers to accelerate particles is an area of active current research. Among the more extensively studied proposals associated with these devices are the inverse free electron-laser accelerators [1] (IFEL) and the plasma-wave accelerator [2] schemes.

In the case of the IFEL acclerator, the wiggler plus the laser field yield a ponderomotive force that allows exchange of energy from the laser field to the particle beam. While this is a realistic possibility, one notes that the particles are undergoing oscillatory motion in the transverse direction in such a device. Therefore, particle acceleration is limited to lower energies; i.e., extremely high energies will not be achievable by such an IFEL scheme, due to radiation emitted as a result of these transverse oscillations.

The plasma-wave acclerator devices have been intensely studied and are very interesting. They have already been shown to lead to acceleration (in the few-MeV range). There is, however, always the problem of plasma instabilities and subtle features of plasma physics to contend with.

Other possible proposals involve, for example, evanescent waves [3], inverse Cherenkov acceleration [4], transverse injection of particles [5], and ponderomotive potential (radiation pressure) schemes [6]. The present proposal [7] is unique in that it is remarkably simple: we show here that, in principle, the radiation field of a focused laser beam has a longitudinal component large enough to produce substantial acceleration if the particles are injected with a high enough initial energy.

It is interesting to compare the present work with that of Ref. [6], which considers acceleration of thermal particles by a focused laser field having a longitudinal component. In that work the acceleration force is derived from a ponderomotive force and thus the force density acting on the particles goes as the laser *field squared,* whereas in our scheme the particles are driven by the Lorentz force eE_z, which is *linear in the field.*

In Sec. II, we show that a longitudinal electric-field component is associated with a focused "transverse" field, and we derive an equation relating the longitudinal to the transverse component of the electric field. In Sec. III we consider schemes to generate significant longitudinal electric fields appropriate for the laser acceleration of an injected particle beam. In particular, we consider a scheme involving a lens waveguide array and another scheme involving the so-called diffraction-free or Bessel beams [8]. In Sec. IV, we briefly discuss the electron dynamics, investigate the conditions under which we are able to extract net energy from the laser field, and address the question of beam stability. Our main results are summarized in Sec. V, where it is noted that a 1-J, 1-ps pulse focused on an area of λ^2 could provide a proof-of-principle electron acceleration experiment. In order to accelerate heavier particles, e.g., protons, the injected particle beam would most likely have to be ultrarelativistic, such as that produced by the superconducting supercollider (SSC).

II. LONGITUDINAL FIELDS ASSOCIATED WITH THE TRANSVERSE FIELDS

In this section we show that the longitudinal component associated with a nominally transverse field, propagating in the z direction, is given by

$$E_z(x,y,z)=\frac{1}{ik}\nabla_\perp\cdot E_\perp \ , \tag{1}$$

where the field propagates with wave vector k and $\boldsymbol{\nabla}_\perp$ and $\mathbf{E}_\perp$ denote the transverse gradient operator and field, respectively. The existence of a longitudinal component is a direct consequence of Maxwell equations. It was shown by, for example, Lax, Louisell, and McKnight [9] that, in general, a purely transverse field would be inconsistent with the Maxwell equations.

A simple way of proving Eq. (1) is to first expand the field components, which obey the wave equations $(\nabla^2+k^2)E_j=0$, in the usual Fourier representation [10]

$$E_j(x,y,z)=\int_{-\infty}^{\infty} dk_x dk_y dk_z \tilde{E}_j(k_x,k_y,k_z) \times e^{-i(k_x x+k_y y+k_z z)} , \quad (2)$$

where $j=x,y,z$, and $k=\omega/c$. From the wave equation we have

$$k_x^2+k_y^2+k_z^2-k^2=0 .$$

This imposes a δ function constraint into Eq. (2), i.e.,

$$\tilde{E}_j(k_x,k_y,k_z)=A_j(k_x,k_y)\delta(k^2-k_x^2-k_y^2-k_z^2) .$$

We also make the paraxial approximation wherein $k_x,k_y \ll k$, so that

$$k_z=(k^2-k_x^2-k_y^2)^{1/2}\simeq k\left[1-\frac{1}{2}\frac{k_x^2+k_y^2}{k^2}\right] . \quad (3)$$

Under these conditions Eq. (2) becomes

$$E_j(x,y,z)=\int dk_x dk_y A_j(k_x,k_y) \times \exp\left[-i\left[k_x x+k_y y+kz-\frac{k_x^2+k_y^2}{2k}\right]\right] . \quad (4)$$

It is convenient to introduce the direction cosine notation $k_x=kp$, $k_y=kq$, so that

$$E_x(x,y,z)\simeq \int\int_{-\infty}^{\infty} A_x(p,q)e^{-ik(px+qy)} \times e^{-ikz}e^{(ik/2)(p^2+q^2)z}dp\,dq , \quad (5a)$$

$$E_y(x,y,z)\simeq \int\int_{-\infty}^{\infty} A_y(p,q)e^{-ik(px+qy)} \times e^{-ikz}e^{(ik/2)(p^2+q^2)z}dp\,dq , \quad (5b)$$

$$E_z(x,y,z)\simeq -\int\int_{-\infty}^{\infty} [pA_x(p,q)+qA_y(p,q)] \times e^{-ik(px+qy)}e^{-ikz} \times e^{(ik/2)(p^2+q^2)z}dp\,dq , \quad (5c)$$

where (5c) follows from the divergence condition $\nabla\cdot\mathbf{E}=0$ and we neglected terms in (5c) of order p^2 and q^2. Equation (1) is seen to follow by direct inspection of (5a)–(5c).

In a similar manner the angular spectrum representation for the magnetic-field components in the paraxial approximation is as follows:

$$B_x(x,y,z)=-\int\int_{-\infty}^{\infty}\{[pqA_x(p,q)]+[1+\tfrac{1}{2}(q^2-p^2)]A_y(p,q)\}e^{-ik(px+qy)}e^{-ikx}e^{(ikz/2)(p^2+q^2)}dp\,dq , \quad (6a)$$

$$B_y(x,y,z)=\int\int_{-\infty}^{\infty}\{[1-\tfrac{1}{2}(q^2-p^2)]A_x(p,q)+pqA_y(p,q)\}e^{-ik(px+qy)}e^{-ikz}e^{(ikz/2)(p^2+q^2)}dp\,dq , \quad (6b)$$

$$B_z(x,y,z)=-\int\int_{-\infty}^{\infty}[qA_x(x,y)-pA_y(x,y)]e^{-ik(px+qy)}e^{-ikz}e^{(ikz/2)(p^2+q^2)}dp\,dq . \quad (6c)$$

Another deviation of Eq. (1), this time in the short-wavelength limit $kz \gg 1$, utilizes Bessel-function expansion of the fields [6]. As is shown in Ref. [6], for light that is x polarized, we may write

$$E_x(r,v)=\sum_{n=0}^{\infty}\frac{1}{n!}2^{-n}v^{n+1}h_{n-1}^{(1)}(v)I_n(r) , \quad (7a)$$

$$E_z(r,v)=\frac{1}{k}\frac{\partial}{\partial x}\sum_{n=0}^{\infty}\frac{1}{n!}2^{-n}v^{n+1}h_n^{(1)}(v)I_n(r) , \quad (7b)$$

where $v=-kz$, $h_n^{(1)}$ is the nth-order spherical Bessel function of the third kind and $I_n(r)$ is an integral whose exact form is not important; the only point we care about here is that it appears in the equations for E_x and E_z in the same way.

Now, the asymptotic form for $h_n^{(1)}(v)$ in the limit $v \gg 1$ is

$$\lim_{v\to\infty} h_n^{(1)}(v)=\frac{1}{v}\exp[iv-i\pi(n+1)/2] , \quad (8a)$$

and therefore in this limit

$$h_n^{(1)}(v)=-ih_{n-1}^{(1)}(v) . \quad (8b)$$

Inserting (8b) into (7) we find

$$E_z(r,v)=\frac{1}{ik}\frac{\partial}{\partial x}E_x(r,v) , \quad (9)$$

which is of course Eq. (1); however, we have arrived here at this result in the limit $kz \gg 1$, instead of the paraxial approximation $(k_x^2+k_y^2)/k \ll 1$.

III. GENERATION OF LONGITUDINAL FIELD

A. Lens waveguide

It is well known that a focused laser beam in a confocal resonator or a lens waveguide array has a transverse field with a Hermite-Gaussian distribution. Here we consider a lens waveguide array as shown in Fig. 1. The field distributions between alternate pairs of lenses are different. We consider only the propagation of the Hermite-Gaussian (0,1) mode, as it is the lowest laser mode with a nonvanishing longitudinal component on axis. The x-polarized (1,0) mode is given by [11]

$$E_x(x,y,z)=\frac{4w_0E_0x}{\sqrt{2\pi}w^2(z)}\exp\left[\frac{-ik}{2\tilde{q}(z)}(x^2+y^2)-ikz+2i\psi(z)\right] , \quad (10)$$

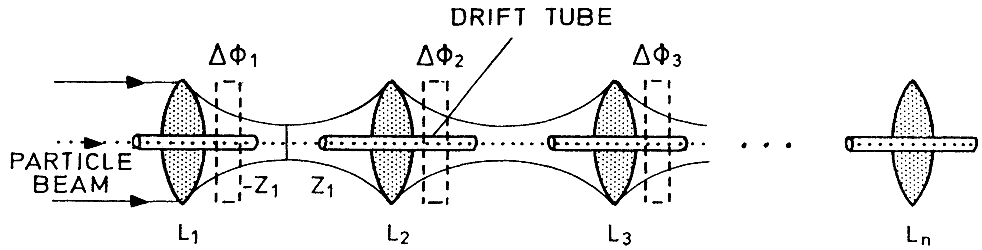

FIG. 1. Laser linac via lens guiding array. Drift tubes and "phase shifters" $\Delta\Phi_1$, $\Delta\Phi_2$, etc., are indicated only as possible ways of dealing with conceivable problems.

where E_0 is the field amplitude, $k=2\pi/\lambda$ is the propagation constant, and

$$w(z)=w_0\left[1+\left[\frac{z}{z_q}\right]^2\right]^{1/2}, \tag{11a}$$

$$\tilde{q}(z)=z+iz_q, \tag{11b}$$

$$\psi(z)=\tan^{-1}\left[\frac{z}{z_q}\right]. \tag{11c}$$

Here w_0 is the beam radius in the center at $z=0$ and $z_q=\pi w_0^2/\lambda$ is the Rayleigh length. The longitudinal field associated with the (1,0) mode, Eq. (10), is then found from Eq. (1) to be

$$E_z(x,y,z)=\frac{-4iE_0w_0}{\sqrt{2\pi}w^2(z)k}\left[1-\frac{ikx^2}{\tilde{q}(z)}\right]\times\exp\left[-\frac{ik}{2\tilde{q}(z)}(x^2+y^2)-ikz+2i\psi(z)\right]. \tag{12}$$

On axis, this reduces to

$$E_z(0,0,z)=\frac{-4iw_0E_0}{\sqrt{2\pi}w^2(z)k}e^{-ikz+2i\psi(z)}. \tag{13}$$

It is clear that, at $z=0$, $E_z\simeq E_0/kw_0$. Since $E_0\simeq(P/\epsilon_0cw_0^2)^{1/2}$, where P is the power of the laser, we have

$$E_z\simeq\frac{1}{kw_0^2}\left[\frac{P}{\epsilon_0c}\right]^{1/2}. \tag{14}$$

For $\lambda\simeq10$ μm radiation, beam radius $w_0\simeq0.5$ mm, the field $E_z\simeq40\sqrt{P}$ V/m, where P is the power in watts.

The phase velocity for the field is given by

$$v_0=\frac{\omega}{k-2\psi'(z)}=\frac{c}{1-(1/k)[2z_q/(z_q^2+z^2)]}\simeq c\left\{1+\left[\left[\frac{kw_0}{2}\right]^2+\left[\frac{z}{w_0}\right]^2\right]^{-1}\right\}. \tag{15}$$

Typically $w_0\simeq10^{-3}$ m and for 10-μm radiation, $k\simeq10^6$ m^{-1}. We then have $v_0\simeq c(1+10^{-6})$. The phase velocity of the field is therefore greater than the speed of light in vacuum c by a small amount. Implications of this "larger-than-c" phase velocity will be discussed later. For now we may ignore this small effect and take $v_0\cong c$.

The components of the magnetic field can be obtained from Maxwell's equations. For the present x-polarized (1,0) mode, we obtain

$$B_y\simeq\left[1+\frac{2}{(kw_0)^2}\right]E_x\simeq\left[\frac{v_0}{c}\right]^{1/2}E_x. \tag{16}$$

The field distribution, after passage through a lens, is given by [12]

$$E_x'(x,y,z)=\frac{ik}{2\pi(z+f)}\int\int_{-\infty}^{\infty}E_x(x_1,y_1,f)\exp\left[\frac{ik}{2f}(x_1^2+y_1^2)\right]\exp\left[\frac{-ik}{2(z+f)}[(x-x_1)^2+(y-y_1)^2]\right]dx_1dy_1, \tag{17}$$

where f is the focal length of the lens and, for simplicity, we have chosen the origin $z=0$ at the forward focal point. After carrying out the necessary integrations, we obtain

$$E_x'(x,y,z)=\frac{4xE_0w_0}{\sqrt{2\pi}w'^2(z)}\exp\left[-\frac{ik}{2\tilde{q}'(z)}(x^2+y^2)-ikz+2i\psi'(z)\right], \tag{18}$$

where

$$w'(z)=w_0\frac{f}{z_q}\left[1+\left[\frac{z_qz}{f^2}\right]^2\right]^{1/2}, \tag{19a}$$

$$\tilde{q}'(z)=z+\frac{if^2}{z_q}, \tag{19b}$$

$$\psi'(z)=\frac{\pi}{2}+\tan^{-1}\left[\frac{zz_q}{f^2}\right]. \tag{19c}$$

The longitudinal component along the axis is

$$E_z(0,0,z)=\left[\frac{-4iE_0}{\sqrt{2}\pi w'^2(z)}e^{-ikz+2i\psi'(z)}\right](w_0/k) \quad (20)$$

and the phase velocity is

$$v_0'=\frac{\omega}{k}\left[1-\frac{2z_q/k}{f^2+(z_0/f)^2z^2}\right]^{-1}$$
$$\simeq c\left\{1+\frac{2z_q}{k}\left[f^2+\left[\frac{z_q}{f}\right]^2z^2\right]^{-1}\right\} . \quad (21)$$

It is clear that when the focal length f is chosen to be equal to the Rayleigh length z_q, the field distribution between any pair of lenses is identical. However, for $f>z_q$, the field is alternately well focused and weakly focused.

B. "Diffraction-free" Bessel beams

In order to avoid the problems of beam spreading and the z dependence of the phase velocity, an alternate "lens guide" uses the "diffraction-free" Bessel beams discussed by Durnin and co-workers [8]. It has been shown that a beam whose field distribution is given by

$$E(x,y,z)=E_0e^{-i\beta z}J_0(\alpha\rho) , \quad (22a)$$

where $\alpha^2+\beta^2=k^2$, $\rho^2=x^2+y^2$, J_0 is the zeroth-order Bessel function of the first kind, and E_0 is the field on axis, represents a nondiffracting beam for $0<\alpha<k$. It would be impossible to generate such a beam over the entire z plane as that would require an infinite amount of energy. However, an experimental realization of such beams in a finite domain has been realized and their diffraction-free nature has been demonstrated. It is shown, for example, that if laser light passes through a circular slit of diameter d and width Δd then a lens of radius R at a distance equal to the focal length f (as shown in Fig. 2) will generate a Bessel beam that will remain diffraction free over a region [8]

$$z_B\simeq kRw . \quad (22b)$$

Here the beamwidth $w\simeq\alpha^{-1}\simeq(4f^2+d^2)^{1/2}/kd$, which, for $f\gg d$, becomes

$$w\simeq\alpha^{-1}\simeq\frac{2f}{kd} .$$

This description of beamwidth assumes that the energy of the beam is mostly concentrated between the axis and the first minimum associated with the first zero of the Bessel function. The slit width Δd is chosen such that

$$\Delta d\ll f/kR .$$

It is clear that if the laser beam producing the field distribution (22) is x polarized, then the z component of the field E_z is given by

$$E_z=\frac{1}{i\beta}\frac{\partial E_x}{\partial x} . \quad (23)$$

The field component E_z is therefore zero on the axis.

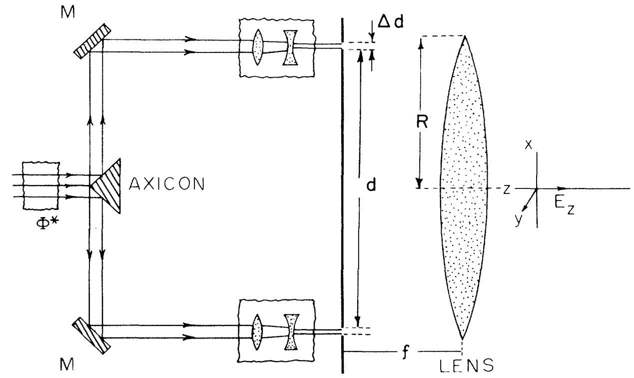

FIG. 2. An "in principle" scheme for producing a "Bessel beam" without losing most of the laser energy. Laser light from left passes through a phase conjugate element, reflects off the axicon, is collected by a cylindrical 45° mirror, passes through a beam condenser, and is then passed through a "rim" aperture. The resulting light is collected by the lens and focused on the axis to generate a high-power Bessel beam. It is emphasized that this is very schematic and is only intended to show that it is not in principle necessary to waste most of the light in order to generate a $J_m(\alpha\rho)$ beam.

However, it is maximum at a distance $\rho=2.4/\alpha$, where $E_x=0$. The field E_z is then

$$E_z\simeq\frac{\alpha}{\beta}E_0\simeq\frac{1}{kw}E_0 ,$$

where we have assumed $\alpha/k\ll1$.

We note, however, that there is no reason we cannot produce a Bessel beam having the characteristic asymmetric form of Eq. (1). That is, the general form of the nondiffracting Bessel beam is given, for $\hat{\mathbf{x}}$ polarized light, by

$$\mathbf{E}_\perp(r,\phi,z,t)=\hat{\mathbf{x}}E_0e^{im\phi}J_m(\alpha\rho)e^{-i(\beta z-\omega t-\Theta)} , \quad (24)$$

where ϕ is the azimuthal phase angle, J_m is the Bessel function of mth order, and Θ is a constant.

The Bessel beam, which goes as J_1, then has the desired asymmetry. That is, we may arrange (by, e.g., appropriate phase-retarding plates) and the x-polarized field has the form

$$E_x(r,\phi,z,t)=E_0e^{i\phi}J_1(\alpha\rho)e^{-i(\beta z-\omega t-\Theta)} . \quad (25)$$

Inserting Eq. (25) into Eq. (23), using the elementary fact that

$$\frac{\partial}{\partial x}=\cos\phi\frac{\partial}{\partial\rho}-\frac{\sin\phi}{\rho}\frac{\partial}{\partial\phi} ,$$

and the Bessel function relation

$$J_1'(\xi)=J_0(\xi)-\frac{1}{\xi}J_1(\xi) ,$$

we find

$$E_z(r,\phi,z,t)=\frac{-E_0}{\beta}\left[\frac{\sin\phi}{\rho}e^{i\phi}J_1(\alpha\rho)e^{-i(\beta z-\omega t+\Theta)}+i\alpha\cos\phi e^{i\phi}\left[J_0(\alpha\rho)-\frac{1}{\alpha\rho}J_1(\alpha\rho)\right]e^{-i(\beta z-\omega t+\Theta)}\right] . \tag{26a}$$

On or near the axis such that $\alpha\rho \ll 1$ we may use the Bessel function expansions,

$$J_0(\xi)\cong 1-\frac{\xi^2}{4}+\cdots ,$$

$$J_1(\xi)\cong \frac{\xi}{2}-\frac{\xi^3}{16}+\cdots ,$$

and find that we may write the real part of Eq. (26a) on axis as

$$\mathrm{Re}E_z(r,\phi,z,t)\cong -\frac{\alpha E_0}{2\beta}\sin(\beta z-\omega t-\Theta) . \tag{26b}$$

Thus the "nondiffracting" Bessel beams are, in principle, able to provide a simple on-axis E_z field component.

Let us return now to the question of application of such beams to the present problem. It follows from a comparison of Eq. (22b) with the corresponding expression of the Rayleigh length $z_R \simeq kw_0^2$ in the usual case, e.g., confocal laser mode that the diffraction-free beam propagates a distance R/w_0 larger without spreading.

In Fig. 2, we consider a possible experimental setup. A Gaussian laser beam is incident on a conical axicon. After reflection from the spherical mirror it is transformed into an annular mode that is incident on an appropriate circular aperture that fills the lens. For purposes of discussion consider the case in which the particle beam is sent in at a distance $x=2.4/\alpha$ away from this axis where the field component E_z is maximum. It follows that if $k\simeq 10^6$ m^{-1}, $f\simeq 4$ m, $d\simeq 8$ mm, and $R\simeq 1$ cm; then $\Delta d\simeq 0.1$ mm, $w\simeq 1$ mm, and $z_{\max}\simeq 10$ m. We thus obtain an order-of-magnitude longer interaction region as compared to the lens array discussed earlier.

Finally we note that the diffraction and/or optical flaws may prevent the attaining of a neat "diverging ring of light." Should this be the case one would hope that a holographic element (or phase conjugator) such as indicated in Fig. 2 may be used to "clean up" the beam.

IV. ELECTRON DYNAMICS

A. Particle acceleration

Next we consider the dynamics of particles in the electromagnetic field. For simplicity, we assume the field to be a plane wave

$$E_z(0,0,z)=iE_{z_0}e^{i(\omega t-kz)} , \tag{27}$$

where, according to Eq. (11), $E_{z_0}\simeq E_0/kw_0$. In this approximation we have ignored the diffraction effects leading to the spreading of beamwidth and the phase variations. This approximation is valid for $z \lesssim z_R$. The phase velocity of the wave is then equal to the speed of light.

The relativistic particle will generally be slipping behind the laser field and, in the process, will be alternately accelerated and decelerated. Hence, no net energy will be transferred to the particle. It is therefore required that the particles be removed from the field when they pass from the accelerating part of the field to the decelerating part. Thus a particle entering the field at a certain phase will be allowed to travel in the field only for a certain interaction length to realize a net extraction of energy.

An estimate of the interaction length can be obtained as follows. The "slip" distance traveled by the particle with respect to the wave should be, at most, equal to $\lambda/2$ so that the maximum time of interaction

$$t_i \simeq \frac{\lambda}{2(c-v)} . \tag{28}$$

The distance traveled by the wave during this period is

$$l=ct_i\sim\left[\frac{mc^2}{m_0c^2}\right]^2\lambda , \tag{29}$$

where m_0 is the rest mass of the particle and we have used the fact that

$$\frac{v}{c}=\left[1-\left[\frac{m_0c^2}{mc^2}\right]^2\right]^{1/2}\simeq 1-\frac{1}{2}\left[\frac{m_0c^2}{mc^2}\right]^2 , \tag{30}$$

for relativistic particles. The energy transferred to the particle during the interaction length [Eq. (29)] is then

$$\mathcal{E}_f\simeq eE_zl=eE_z\lambda\left[\frac{mc^2}{m_0c^2}\right]^2 . \tag{31}$$

It follows that for $\lambda\simeq 10$ μm, $w_0\simeq 0.5$ mm, $P\simeq 10^{14}$ W, we have $\mathcal{E}_f\simeq 1$ MeV for 10-MeV injected particles.

The simple calculation for the energy transfer underestimates the interaction length and the energy transferred, as it assumes the same velocity for the particle during its interaction with the field. In fact, as the particle picks up energy it will "speed up" and the interaction length will correspondingly increase. We can derive this result rigorously as follows.

The equations of motion for the energy $\mathcal{E}=\gamma m_0c^2$ and the phase $z'=z-ct$ of the particle, which is moving with a velocity v along the z axis, are found from the differential expression $d\mathcal{E}=m_0c^2d\gamma=eE_{z_0}\sin kz'dz$ and $dz'=dz-cdt$, so that [7]

$$m_0c^2\frac{d\gamma}{dz}=-eE_{z_0}\sin kz' , \tag{32a}$$

$$\frac{dz'}{dz}=1-\frac{1}{\beta} , \tag{32b}$$

where $\beta=v/c$ and $\gamma=1/\sqrt{1-\beta^2}$.

Equations (32a) and (32b) can be combined to yield

$$-\frac{eE_{z_0}}{m_0c^2}\sin kz'dz'=\left[1-\frac{1}{\beta}\right]d\gamma , \tag{33}$$

and upon carrying out the implied integration we obtain [13–15]

$$H=(p^2c^2+m_0^2c^4)^{1/2}-pc-\frac{eE_{z_0}}{k}\cos kz' , \quad (34)$$

where p is the momentum of the particle and the Hamiltonian H appears as a constant of integration. In deriving Eq. (34), we have used $\beta=(\gamma^2-1)^{1/2}/\gamma$.

Now the correct condition for the net energy transfer is [compare Eq. (2.8)]

$$\int_0^t [c-v(t')]dt'=\frac{\lambda}{2} . \quad (35)$$

Here the velocity of particle v is a function of time and it is related to the particle momentum p via the energy relation

$$\frac{m_0c^2}{(1-v^2/c^2)^{1/2}}=(p^2c^2+m_0^2c^4)^{1/2} , \quad (36a)$$

which implies

$$\frac{v^2}{c^2}=\frac{pc^2}{p^2c^2+m_0^2c^4} . \quad (36b)$$

The constant (time-independent) Hamiltonian function (34) yields the expressions for p as a function of z'

$$2pc=-\left[H+\frac{eE}{k}\cos kz'\right]+m_0^2c^4\left[H+\frac{eE}{k}\cos kz'\right]^{-1} . \quad (37)$$

Inserting Eq. (36b) into Eq. (37) we find $v(z')=v(z-ct)$, and Eq. (35) may then be written in terms of $x=ct$ as

$$l-\int_0^l\left[\frac{m_0^2c^4-\{H+(eE_{z_0}/k)\cos[k(z-x)]\}^2}{m_0^2c^4+\{H+(eE_{z_0}/k)\cos[k(z-x)]\}^2}\right]dx =\frac{\lambda}{2} . \quad (38)$$

Recall that the parameter H is a constant determined by the initial momentum p_0 and phase z_0 of the particle.

Now from Eq. (37) we see that if $H=eE_{z_0}/k$, then when $kz'=\pi$ the momentum $p\to\infty$. This is manifest in Eq. (38) in that, in this same limit, l must go to ∞.

In order to find the particle injection energy that yields this infinite interaction length, we return to Eq. (37) and further consider the condition

$$H(p_0,z_0)-\frac{eE_{z_0}}{k}\cos kz'=0 , \quad (39)$$

or

$$(p_0^2c^2+m_0^2c^4)^{1/2}-p_0c-\frac{eE_{z_0}}{k}(\cos kz_0-\cos kz')=0 . \quad (40)$$

The maximum contribution from the field term comes when $kz_0=0$ and $kz'=\pi$; in such a case we find the initial momentum p_0 to be determined by the expression

$$(p_0^2c^2+m_0^2c^4)^{1/2}-p_0c=\frac{2eE_{z_0}}{k} . \quad (41)$$

We easily solve for p_0 by taking the p_0c to the right-hand side (RHS) and squaring; we find

$$p_0c=\frac{1}{2}\left[\frac{m_0^2c^4}{2eE_{z_0}/k}-\frac{2eE_{z_0}}{k}\right] . \quad (42)$$

Inserting Eq. (42) into Eq. (36a) we then obtain the important expression for the injection energy necessary to ensure $l\to\infty$:

$$\mathcal{E}_{\rm inject}=m_0c^2\left[\frac{m_0c^2}{4eE_{z_0}/k}\right]+\frac{eE_{z_0}}{k} . \quad (43)$$

For 10 μm light with $P\simeq10^{14}$ W focused to 0.5 mm, $(mc^2)_{\rm critical}\simeq10$ MeV and the final energy is now determined by the Rayleigh length $z_R=\pi w_0^2/\lambda$, which taken together with Eq. (14) yields

$$\mathcal{E}_{\rm final}\simeq\frac{e}{kw_0^2}\left[\frac{P}{\epsilon_0c}\right]^{1/2}z_R\simeq\left[\frac{P}{\epsilon_0c}\right]^{1/2}\simeq100\text{ MeV} .$$

For comparison if we had taken a "Bessel beam" for the laser field then the effective range

$$z_B\simeq2\pi w_0\frac{R}{\lambda}=2z_R\frac{R}{w_0} .$$

Hence, if we assume $R\simeq10$ cm and $w_0\simeq0.5$ mm, $z_B\simeq400z_R$ and the final energy, in the above example, becomes many GeV. This comparison should not be taken too seriously until further studied. There are many open questions considering the use of Bessel beams, e.g., do we lose too much power in using this approach [16]? It indicates, however, the potential advantage of such a configuration.

B. Beam stability

Assuming $\hat{\mathbf{x}}$ polarized laser radiation, we may write

$$E_x\cong E_0\frac{x}{w_0}e^{-(x^2+y^2)/w_0^2}e^{-i(kz-\omega t)} \quad (44)$$

and, near the axis, the force in the x and z directions, as calculated via the Lorentz force using Eqs. (5) and (6), then goes as

$$F_x\simeq2e\left[1-\frac{v}{c}\left[\frac{v_0}{c}\right]^{1/2}\right]E_0\frac{x}{w_0}e^{-(x^2+y^2)/w_0^2}\cos(kz-\omega t) , \quad (45)$$

$$F_z\cong2e\frac{1}{kw_0}E_0e^{-(x^2+y^2)/w_0^2}\sin(kz-\omega t) . \quad (46)$$

For maximum force, $kz-\omega t\cong\pi/2$ and, as shown in Ref. 7, particles reaching this point will evolve to phase $kz-\omega t=\pi/2+\epsilon$. Hence the force (45) goes as

$$F_x=-\left\{2e\left[1-\frac{v}{c}\left[\frac{v_0}{c}\right]^{1/2}\right]\frac{E_0}{c^2}e^{-(x^2+y^2)/w_0^2}\sin\epsilon\right\}x ,$$

and the beam is stable as long as $v^2 v_0 < c^3$.

We further note that the force F_x vanishes for $vv_0 = c^2$ and is first order in x and ϵ. Thus there is reason to believe that the "defocusing" effects of E_x and B_y can be mitigated. This problem will be further discussed elsewhere [17].

V. SUMMARY AND POSSIBLE PROOF OF PRINCIPLE EXPERIMENT

Collecting together our main results, we have the following.

(i) Longitudinal electric field,

$$E_{z_0} \cong \frac{1}{kw_0^2} 10\sqrt{P} \ \text{V/m} .$$

(ii) Interaction lengths z for (a) ordinary lens

$$z_{\text{Rayleigh}} = \frac{\pi w_0^2}{\lambda} ,$$

(b) Bessel beam

$$z_{\text{Bessel}} = 2 z_{\text{Rayleigh}} \frac{r_{\text{lens}}}{w_0} ,$$

where r_{lens} is the radius of the lens.

(iii) Injection energy for "infinite" acceleration range,

$$\mathcal{E}_{\text{inject}} \cong m_0 c^2 \left[\frac{m_0 c^2}{e E_{z_0} \lambda} \right] + \frac{e E_0 \lambda}{2\pi} .$$

(iv) Final energy of particle,

$$\mathcal{E}_{\text{final}} \cong \frac{e}{kw_0^2} 10\sqrt{P} z .$$

These results suggest a possible proof of principle electron acceleration experiment. In order to minimize the injection energy we focus our laser beam to $w_0 \simeq \lambda$, then

$$E_{z_0} \simeq \frac{1}{2\pi\lambda} 10\sqrt{P} .$$

Once a pulsed laser having 1 J energy in 1 ps at $\lambda = 1\ \mu\text{m}$ is well within the state of the art [17], then the field $E_{z_0} \simeq 10^{12}$ V/m. For a field of this magnitude, the injection energy is of order 1 MeV, and the particle energy gained in one Rayleigh length is then given by

$$\mathcal{E}_{\text{final}} \simeq 5e\sqrt{P} \simeq 5 \ \text{MeV} .$$

Thus a 1-MeV electron will be accelerated to 5 MeV in roughly one wavelength. Note, however, that $\mathcal{E}_{\text{final}}$ is independent of the focal area since the "gain" of a tightly focused beam is offset by the "loss" due to a shorter Rayleigh length.

Finally we note that a focused laser, in perhaps a Bessel-beam configuration, could be used to accelerate protons to very high energies. However, this would require high-energy protons to begin with. Consider the injection energy necessary to secure an extended interaction length for protons,

$$\epsilon_{\text{inject}} \cong m_p c^2 \left[\frac{m_p c^2}{eE\lambda} \right] ,$$

where m_p is now the proton rest mass, i.e., around 2000 times that of the electron $m_0 c^2$. If we take, as discussed earlier, $E \simeq 10^{12}$ V/m and $\lambda \simeq 1\ \mu\text{m}$ then $\epsilon_{\text{inject}} \simeq 1$ TeV, which is in the superconducting-supercollider (SSC) ball park; and the possibility of using a laser linac as an SSC booster is interesting.

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[17] For example, W. Mori has reported experiments using a 6-J, 0.75-ps laser at 1 μm (unpublished).