Second-order Born calculations for electron capture in relativistic U+U collisions

Frank Decker and Jörg Eichler*
Bereich Schwerionenphysik, Hahn-Meitner-Institut Berlin, D-1000 Berlin 39, Germany
(Received 25 March 1991)

Using an exact evaluation of the relativistic second-order Oppenheimer-Brinkman-Kramers approximation, total and differential cross sections for electron capture in $U^{92+}+U^{91+}$ collisions are calculated for projectile energies between 0.5 and 100 GeV/u. It is shown that (a) at 100 GeV/u the asymptotic cross-section behavior is not yet reached, (b) spin-flip transitions and negative-energy intermediate states become important at high energies, and (c) the Thomas peak in the differential cross section is relativistically suppressed.

The second-order Born approximation has often been regarded as the appropriate method to describe electron transfer in ion-atom collisions at high projectile energies. This should be valid in particular for relativistic collisions. Following a first account [1] of the problem, we have in Ref. [2] (henceforth denoted as I) investigated in more detail charge transfer in capture reactions for collision energies ranging between a few hundred MeV/u up to a few GeV/u. It was found in an *exact* evaluation of the second-order Born or Oppenheimer-Brinkman-Kramers (OBK2) approximation that the calculated cross sections are considerably larger than first-order cross sections that already overestimate the experimental results. This shows that the Born expansion is not a suitable perturbation series in the charge and energy regime considered.

The purpose of the present paper [3] is twofold. First, we want to extend exact OBK2 calculations into the extreme relativistic energy regime up to 100 GeV/u. Second, we want to extend calculations to the highest available charges in order to maximize the effect of the relativistic electron motion.

This aim leads us to consider $U^{92+}+U^{91+}$ collisions and to study the asymptotic behavior of the cross section, the effect of spin-flip transitions, and the contribution of negative-energy intermediate states. For U+U collisions, the matching energy E_m is about 240 MeV/u. This means that an electron traveling along with a projectile accelerated to the specific energy E_m has a kinetic energy equal to the 1*s* binding energy in target or projectile [4]. This rather high matching energy renders it feasible to carry out exact numerical calculations to high projectile energies. The details of the formalism and of the calculational procedure have been given in I, so that we here can concentrate on presenting and discussing some further results.

Total cross sections for charge transfer in $U^{92+}+U^{91+}$ collisions have been calculated for projectile energies between 0.5 and 100 GeV/u. Figure 1 shows exact OBK2 cross sections σ_2 and exact first-order OBK1 cross sections σ_1. When considering the dependence on the projectile energy E, it should be remembered [1–4] that, asymptotically, we expect the behavior $\sigma_1 \sim E^{-1}$ and $\sigma_2 \sim (\ln E)^2/E$. In first order, spin-flip transitions show the asymptotic E^{-1} dependence over the whole energy range while non-spin-flip transitions behave asymptotically only for energies $E > 10$ GeV/u. In second order, the asymptotic energy dependence is not yet reached even at $E = 100$ GeV/u. The decrease of both non-spin-flip and spin-flip cross sections is much slower than $(\ln E)^2/E$ so that with increasing energy the cross section is entirely determined by second-order contributions. The ratio between spin-flip and non-spin-flip cross sections is about $\frac{1}{10}$

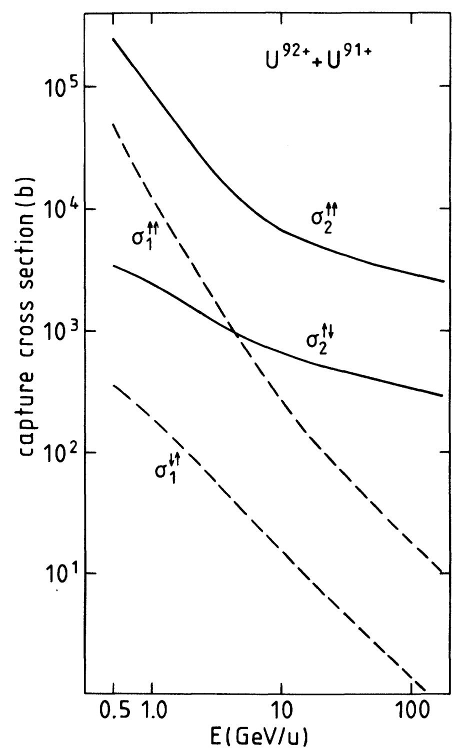

FIG. 1. Total 1*s*-1*s* cross sections for electron capture in $U^{92+}+U^{91+}$ collisions as a function of the projectile energy E. Notation: σ_1, OBK1 cross sections; σ_2, OBK2 cross sections; $\sigma^{\uparrow\uparrow}$, non-spin-flip, $\sigma^{\uparrow\downarrow}$, spin-flip transitions.

for $E > 5$ GeV/u, both for first-order and second-order transitions.

For numerical reasons, it is difficult to pursue the energy dependence for $E > 100$ GeV/u. However, the on-shell matrix element in which the intermediate states have negative energies [cf. Eq. (56) in I] can still be evaluated at much higher energies. The matrix element has a maximum value at about 10^3 GeV/u and beyond that point decreases more slowly than the asymptotic dependence $(\ln E)/\sqrt{E}$. It can be inferred [3] that for U+U collisions the asymptotic energy dependence is not yet reached at 10^6 GeV/u.

In Table I we present calculated OBK2 cross sections for non-spin-flip and spin-flip transitions. We also include cross sections obtained by disregarding intermediate states of negative energy. In this way, one may study the influence of virtual electron-positron pair production and subsequent annihilation. It is seen that for non-spin-flip transitions at 0.5 GeV/u negative-energy intermediate states contribute with 4%. As the projectile energy increases, these states become more important, and at 100 GeV/u are responsible for about 50% of the cross section. For spin-flip transitions the negative-energy contributions increase from 20% to 50%.

When studying charge transfer in second-order perturbation theory, it is important to examine the differential cross section. At high projectile energy, one expects to find a peak in the angular distribution, caused by the classical Thomas double-scattering mechanism [5]. From relativistic classical mechanics, the Thomas angle associated with this peak is derived [6] as

$$\theta^T_{\rm lab} = \frac{m}{M_P}\frac{\sqrt{2\gamma+1}}{\gamma+1} , \qquad (1)$$

where m and M_P are electron and projectile masses, respectively, and γ is the Lorentz factor. The energy dependence [6] of the classical cross section is proportional to E^{-3} and, in contrast to the nonrelativistic case [5], does not agrees with the asymptotic OBK2 energy

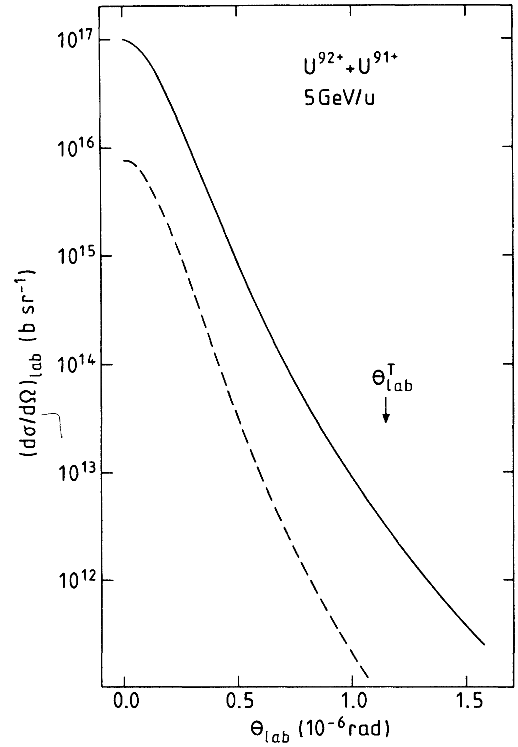

FIG. 2. Differential 1s-1s cross sections for electron capture in $U^{92+}+U^{91+}$ collisions at 5 GeV/u. Only transitions without spin-flip are included. Solid curve, OBK2; dashed curve, OBK1.

TABLE I. Total 1s-1s cross sections (in b) for electron capture in $U^{92+}+U^{91+}$ collisions. The cross sections have been calculated by exact evaluation of the OBK2 approximation, separately for non-spin-flip and spin-flip transitions. Complete cross sections including positive- and negative-energy intermediate states are denoted by OBK2 while OBK2* refers to calculations with positive-energy intermediate states only. The number in square brackets gives the power of 10 multiplying the preceding number.

E	Non-spin-flip		Spin-flip	
(GeV/u)	OBK2	OBK2*	OBK2	OBK2*
0.5	2.40[5]	2.30[5]	3.49[3]	2.84[3]
1.0	9.03[4]	8.38[4]	2.48[3]	1.99[3]
2.0	3.40[4]	3.02[4]	1.58[3]	1.24[3]
5.0	1.16[4]	9.50[3]	8.79[2]	6.62[2]
10.0	6.67[3]	5.16[3]	6.47[2]	4.47[2]
50.0	3.54[3]	2.12[3]	4.06[2]	2.19[2]
100.0	2.99[3]	1.60[3]	3.41[2]	1.64[2]

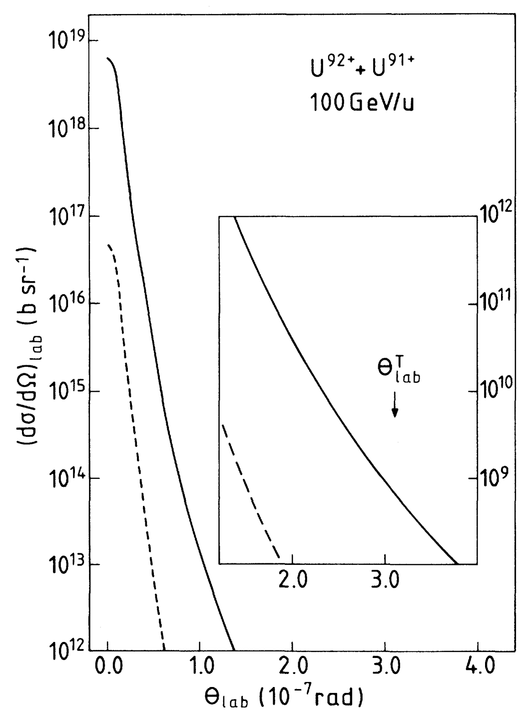

FIG. 3. Same as Fig. 2 but for a collision energy of 100 GeV/u.

dependence. Therefore, one will not expect an asymptotic dominance of the Thomas mechanism.

In Figs. 2 and 3 we show the differential cross section for charge transfer in U+U collisions at 5 and 100 GeV/u, respectively. We only include the dominant non-spin-flip transition. In neither case can one observe an indication of a Thomas peak at the Thomas angle θ_{lab}^{T}. The same is true for $E=0.5$ GeV/u (not shown here). With increasing energy, the differential cross section is more and more peaked at forward angles, so that the neighborhood of the Thomas angle provides a negligible contribution to the total cross section. For 100 GeV/u, the cross section at θ_{lab}^{T} is smaller than the peak value by ten orders of magnitude. The nonexistence of a Thomas peak is in accordance with the observation that the classical cross section decreases much more rapidly as the energy increases than the actual OBK2 cross section and therefore is strongly suppressed.

If, however, a Thomas peak exists at nonrelativistic collision energies, this peak survives in the relativistic energy regime. In I we have considered p+H collisions, where a well-developed peak exists at 50 MeV/u and becomes even more pronounced at 1 GeV/u. For high projectile and target charges the matching energy E_m is so high ($E_m \sim 240$ MeV/u for U) that a Thomas process for nonrelativistic energies is excluded and hence the relativistic suppression of the Thomas peak becomes effective.

From our exact calculation of OBK2 cross sections for U+U collisions we draw the following conclusions: (a) Up to 100 GeV/u and presumably at much higher energies the OBK2 cross section decreases much more slowly with energy than implied by the asymptotic behavior. Therefore capture reactions play a much larger role than assumed so far. (b) The importance of spin-flip transitions and of negative-energy intermediate states is qualitatively assessed. (c) For high projectile and target charges the Thomas peak in the differential cross section is suppressed by relativistic effects.

---

*Also at: Fachbereich Physik, Freie Universität Berlin, D-1000 Berlin 39, Germany.

[1] F. Decker, Phys. Rev. A **41**, 6552 (1990).
[2] F. Decker and J. Eichler, Phys. Rev. A **44**, 377 (1991).
[3] F. Decker, Ph.D. thesis, Freie Universität Berlin, 1990.
[4] See, e.g., J. Eichler, Phys. Rep. **193**, 165 (1990).
[5] L. H. Thomas, Proc. R. Soc. London Ser. A **114**, 561 (1927); R. Shakeshaft and L. Spruch, Rev. Mod. Phys. **51**, 371 (1979).
[6] R. Shakeshaft, Phys. Rev. A **20**, 729 (1979); B. L. Moiseiwitsch and S. G. Stockman, J. Phys. B **12**, L695 (1979).