Quantum Zeno effect without collapse of the wave packet

Vera Frerichs and Axel Schenzle
Sektion Physik der Universität München, Theresienstrasse 37, 8000 München, Germany
and Max-Planck-Institut für Quantenoptik, Ludwig-Prandtl-Strasse 10, 8046 Garching, Germany
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The change of dynamics in a quantum system under frequent or continuous observation, known as the quantum Zeno effect, is generally derived from the projection or reduction of the wave-packet hypothesis that is the central postulate in the theory of quantum measurements. The only experiment in which the Zeno effect has yet been clearly demonstrated, though, allows no conclusion on the necessity or validity of the projection postulate. This is shown by calculating, in detail, the outcome of the experiment on the basis of the standard three-level Bloch equations. These equations follow from the quantum theory of irreversible processes with no additional assumptions necessary, such as which part of the system serves as measuring apparatus or how efficient the measurement would be.

Since the early days of quantum mechanics, it has been questioned constantly if the quantum description of physical reality can be considered complete. The nonlocal character of quantum mechanics and von Neumann's state reduction postulate [1] were a continuous source of confusion and resulted in vigorous discussions. Especially the collapse of the wave packet in a quantum measurement was considered a preliminary hypothesis and an artificial addition to quantum theory. In the meanwhile there exists a large and steadily growing literature on the interpretation of quantum measurements, its conceptual aspects, and its philosophical implications. The problem must be serious if in quest for a solution one is compelled to consider the possibility that in every measurement several new universes are created, or the equally mind-boggling concept of a measurement only to be completed in the mind of a conscious experimenter. The general uneasiness with the philosophical aspects of quantum mechanics is in sharp contrast to the theory's success when it comes to predicting the outcome of a given experiment. Most textbooks on quantum mechanics provide an unambiguous and clear recipe for this purpose.

We now briefly review the projection postulate and some common features of quantum measurement. The state of a physical system at a given time t is determined by the density operator $\rho(t)$, which can—but need not—be equivalent to a state vector $|\psi(t)\rangle$. A measuring apparatus determines the values of a certain physical observable Ω, with the discrete spectrum ω_n and the eigenfunctions $|n\rangle$. In the Ω representation the state of the system is

$$\rho(t)=\sum_{n,m}|n\rangle\rho_{n,m}\langle m| \ .$$

After performing an Ω measurement, the state of the system is reduced to

$$\rho(t)=\sum_{m}|m\rangle\rho_{m,m}\langle m| \ .$$

Irrespective of the character of the initial state—pure or mixed—after the measurement the system is in a statistical mixture of Ω eigenstates that no longer have the possibility to interfere. The probability $\rho_{n,n}$ for observing a value ω_n is now a classical one, expressing the observer's ignorance about the particular state the system now occupies, before actually reading the meter. The system itself, though, is supposed to be in one $|n\rangle$ state only after measurement. It is generally assumed that such a discontinuous change in the state must be postulated and the reduction is introduced "by hand," whenever a measurement is carried out. This is the von Neumann projection postulate in terms of the density operator formalism. Conceptually, a realistic physical measurement can always be separated into two steps, the interaction of system and meter with the recording of the measured values and reading off a specific pointer position. In our opinion, the first step, where coherence and the possibility of interference is destroyed, can be completely described in the framework of continuous irreversible quantum dynamics. The last step, the jump into one particular eigenstate, nevertheless remains mysterious. Provided the wave function describes nature completely, there should be no room for such an indeterminism. For the present, as well as for many other problems involving quantum measurements, the part that matters is the first one, since the individual meter readings may be entirely irrelevant. In such a case the problem can be treated without explicit state reduction and the Zeno effect is one of them.

In a refreshingly pragmatic and very compact review, van Kampen [2] has summarized the main questions of the quantum-measurement problem in a series of ten postulates. Physics is a branch of science, so in his and our opinion theoretical physics can be nothing but the theory of what is accessible to experimental observation, and additional postulates that can neither be verified nor falsified should be omitted. Also, as Bell [3] recently pointed out, the very word "measurement," unlike the well-defined mathematical operation of projection in Hilbert space, lacks a precise operational meaning.

Should one attempt a definition, the minimum requirement for calling a physical process a measurement would be that the measured microscopic system influences the state of another system with a large number of quantum-mechanical degrees of freedom in order to produce a signal noticeable to the eye. This makes the concept of irreversibility and statistics inevitable, because, from a practical point of view, the wave function of a macroscopic apparatus is a concept with as little meaning as one particular point in the phase space of a large number of classical particles. A physical process leading to a "meter reading," due to the many degrees of freedom involved, requires the use of a density operator ρ and not the wave function Ψ alone. It is the purpose of the paper to give an example of how statistical quantum theory can describe in detail an effect generally thought to arise entirely from state reduction or wave-packet collapse caused by the measurement.

Quantum measurement is often discussed by considering a simple closed system that possesses a basis of only two eigenstates of the observed quantity. As long as it is not measured, it evolves according to the Schrödinger equation. Misra and Sudarshan [4], in 1977, proved that the projection postulate, applied to the above system, would produce what they called the quantum Zeno effect: frequent measurement should inhibit unitary temporal evolution from an eigenstate of the measured observable into a coherent superposition, stopping it altogether in the limit of infinitely frequent measurements. The proof given by Misra and Sudarshan is based on an alternating sequence of unitary time evolution and *ad hoc* reduction of the wave function, and therefore implies nothing on spontaneous exponential decay. This obvious restriction is seldom spelled out clearly, and the issue is often confused by the use of the term decay. An illustrating example is the experimental demonstration of quantum jumps in a single trapped ion. Here one concludes from continuously observing no fluorescence that the electron must be in the metastable state. This permanent observation nevertheless does not stop the atom from decaying to the ground state with its natural lifetime.

A trapped ion can also be considered as a nearly ideal experimental realization of a closed two-level system if a transition with frequency in the rf regime is resonantly driven. Spontaneous emission can then safely be neglected. Cook [5] has recently suggested to use such a configuration for an experimental verification of the quantum Zeno effect—see Fig. 1, where ground and excited states are denoted by $|3\rangle$ and $|2\rangle$, respectively. The system, initially prepared in state $|3\rangle$, is subjected to a π pulse driving the 2–3 transition with Rabi frequency β. According to the Schrödinger equation, the two-level system would undergo a complete π rotation into the excited state $|2\rangle$ if there was no additional perturbation.

In order to observe the level populations, the two-level system is now made part of a V-shaped three-level configuration, shown in Fig. 1, where the rf resonance is coupled by a strong optical dipole transition to an additional level $|1\rangle$. During the evolution under the π pulse a number of n equispaced short optical pulses are applied with Rabi frequency α and duration P (see Fig. 1, the

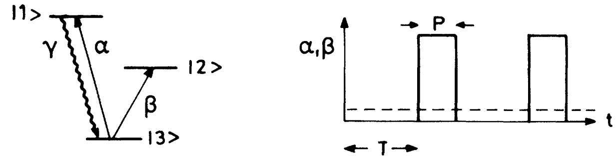

FIG. 1. Left: three-level system. α represents Rabi frequency, γ represents natural linewidth of optical transition, β represents Rabi frequency of weak rf transition. Right: pulse sequence as applied in the experiment (Ref. [6]); P represents pulse length, T represents interval length.

length of an interval between pulses is denoted by T). If the optical pulses are considered as a probe for determining the level population, the conventional measurement concept requires the reduction of the atomic density matrix onto the diagonal elements after each optical pulse. For n such pulses, the projection postulate yields an n dependence of the population of level $|2\rangle$ after the termination of the π pulse [5,6],

$$P_2(T)=\frac{1}{2}\left[1-\cos\left[\frac{\pi}{n}\right]^n\right] . \tag{1a}$$

By expanding $\ln[\cos^n(\pi/n)]$ in a Taylor series for large n one obtains [6]

$$P_2(T)=\frac{1}{2}\left[1-\exp\left[-\frac{\pi^2}{2n}\right]\right]+O(1/n^3) . \tag{1b}$$

The basic reason for the reduction of the level population lies in the fact that the nondiagonal elements which determine the time rate of change of populations are set equal to zero with each projection. Immediately afterwards populations then change quadratically with time.

Itano *et al.* [6] have recently carried out such an experiment by using 5000 laser cooled $^9Be^+$ ions simultaneously stored in a Penning trap. The fluorescence intensity of the optical 1–3 transition was measured at the end of the rf π pulse for different numbers n of interrupting optical pulses. Before the pulse sequence was applied, the ions had been prepared in the ground state $|3\rangle$. The experimentally determined occupation probability for level $|2\rangle$ for different n agreed well with Eq. (1a). However, one could argue that no actual measurement was performed, and no conscious observer meditated on the outcome. The projection postulate was nevertheless assumed to be applicable since in principle the information was present in the scattered light field. As long as a large amount of photons per pulse and ion is scattered, the information is recorded in a detectable signal "more than enough to reduce the wave packet" [6].

From the quantum optics point of view, however, the obvious way to look at this system would not at all involve any worries whether a measurement is performed or state reduction occurs. Instead, the experiment is a typical example of *optical coherent transients* and one would naturally use optical Bloch equations for the complete three-level V configuration. This level structure is familiar from the demonstration of quantum jumps [7–9] and from its possible application as a frequency standard. Therefore it should be quite straightforward to predict the outcome of such an experiment without resorting to ambiguous assumptions. Since strong spontaneous emission is involved along the optical transition path, the entire system is subject to dissipation and must be described by irreversible dynamic equations. We integrate the corresponding density-matrix equation numerically for a sequence of n optical pulses while the rf transition is driven continuously for a period of a π pulse. Approximate analytical results are presented as well to provide some insight into the physical mechanism. In the experiment of Itano *et al.* [6], the ratio of lifetimes for the rf transition 2–3 and the optical dipole transition 1–3 is at least of the order 10^7, which for our purpose is a very good approximation to infinity.

The optical Bloch equations can be derived systematically from the interaction of a multilevel atom with the quantized electromagnetic field. The enormous number of degrees of freedom of the multimode vacuum leads to an irreversible dynamic process. Whereas the derivation involves the usual approximations necessary for obtaining irreversible dynamics, no need arises anywhere to have measurement or state reduction in mind. The advantage is that one can then study unambiguously the dependence on system parameters as pulse length, optical field intensity, or resonance conditions, on the basis of ordinary, nonmeasurement quantum mechanics. The three-level Bloch equations, in the rotating-wave approximation, are

$$
\begin{aligned}
\dot{\rho}_{11} &= -i\alpha(\rho_{13}-\rho_{13}^{*})-\gamma\rho_{11} , \\
\dot{\rho}_{13} &= i\alpha(\rho_{33}-\rho_{11})-i\beta\rho_{12}-\gamma\rho_{13} , \\
\dot{\rho}_{22} &= -i\beta(\rho_{23}-\rho_{23}^{*}) , \\
\dot{\rho}_{23} &= i\beta(\rho_{33}-\rho_{22})-i\alpha_{12}^{*} , \\
\dot{\rho}_{33} &= -(\dot{\rho}_{11}+\dot{\rho}_{22}) , \\
\dot{\rho}_{12} &= i\alpha\rho_{23}^{*}-i\beta\rho_{13}-\frac{\gamma}{2}\rho_{12} ,
\end{aligned}
\tag{2}
$$

where for convenience exact resonance was assumed. For the Zeno concept to work, the spontaneous decay rate of state 2 must be negligible and can safely be set equal to zero. Three of the resulting eight real equations decouple from the others and are irrelevant for the present purpose. We denote by u_{ij} the real and by v_{ij} the imaginary part of an element ρ_{ij} $(i<j)$. The remaining five equations read

$$
\begin{aligned}
\dot{\rho}_{11} &= 2\alpha v_{13}-\gamma\rho_{11} , \\
\dot{v}_{13} &= \alpha(\rho_{33}-\rho_{11})-\beta u_{12}-\frac{\gamma}{2}v_{13} , \\
\dot{\rho}_{22} &= 2\beta v_{23} , \\
\dot{v}_{23} &= \beta(\rho_{33}-\rho_{22})-\alpha u_{12} , \\
\dot{u}_{12} &= \alpha v_{23}+\beta v_{13}-\frac{\gamma}{2}u_{12} .
\end{aligned}
\tag{3}
$$

For numerical calculation we chose the duration of each rectangular optical pulse, denoted by P, to be 20 lifetimes $(1/\gamma)$ of the optical transition, and the optical field strength to yield a Rabi frequency of $\alpha=\gamma$, i.e., saturation. The ratio of pulse lengths to the total π pulse was chosen as $\frac{3}{5}$ for $n=64$, in accordance with the conditions realized in the experiment. The slight systematic error of finite pulse length occurring in the experiment is thus included in our treatment. If no optical pulses are applied (see Fig. 2), level population and coherence evolve freely as in a closed two-level system. The values obtained for ρ_{22} at the end of the rf π pulse, when starting in the ground state $|3\rangle$, are shown in Fig. 3 for different numbers of interrupting pulses. The histogram compares the experimental results in white with the results calculated from Eq. (3) in black. Examples of the detailed time evolution for 4 and 32 optical pulses are shown in Figs. 4 and 5.

The atomic dipole moment corresponding to the coherence v_{23} is destroyed rapidly, but not instantaneously, during each optical pulse, leading to a population ρ_{22} growing quadratically in time again after the pulse [see Eqs. (3)]. The dipole moment, the time evolution of which is shown in greater detail in Fig. 6, is a physical observable and does not switch off suddenly due to the presence of any mysterious observer, but decays naturally with the relaxation rate γ characteristic for the spontaneous decay of the optical transition. No quantum measurement hypothesis of any kind is thus necessary to reproduce the experiment on the Zeno effect. Itano *et al.* [6] also performed the experiment with the ions prepared in the excited state $|2\rangle$. Freezing of the population in the excited state may seem more spectacular, since it seems to prevent the natural decay of a state. However, for the Zeno concept to be applicable the excited state is not allowed to decay spontaneously. Therefore, in our calculation for the "v system," this would only change the values of ρ_{22} into $(1-\rho_{22})$, leaving the rest of the argument unchanged.

During the short optical pulses the action of the rf field is entirely negligible and β can be set equal to zero without error. In a realistic physical situation we can therefore assume that the two external fields alternate,

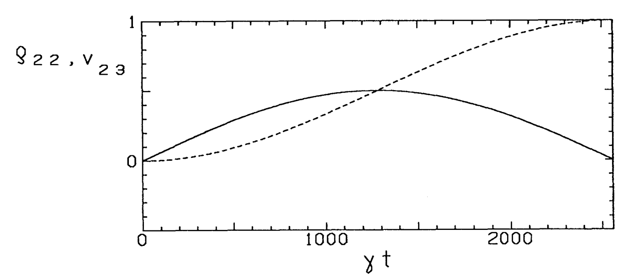

FIG. 2. Free evolution of two-level system 2–3 as calculated from Eqs. (3) with $\alpha=0$ throughout (no pulses are applied). Dashed line: population ρ_{22}; solid line: coherence v_{23}, against time in units of $1/\gamma$.

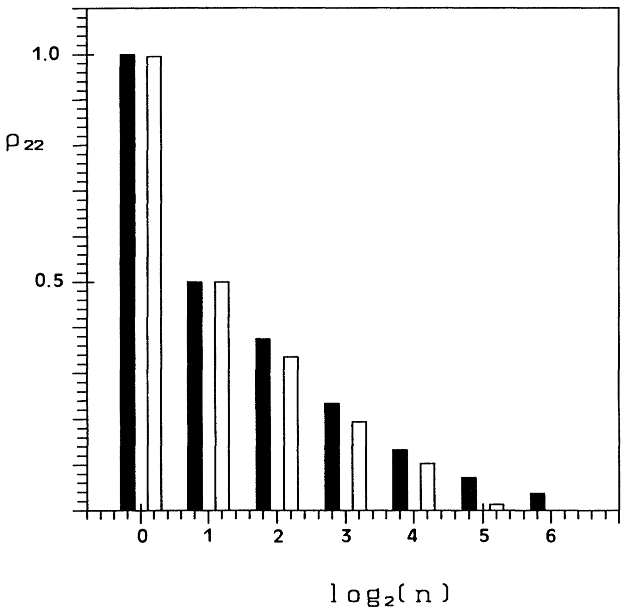

FIG. 3. Final population of state 2 against number n of optical pulses. Solid rectangles: calculation from Eqs. (3); open rectangles: experimental results from Ref. [6].

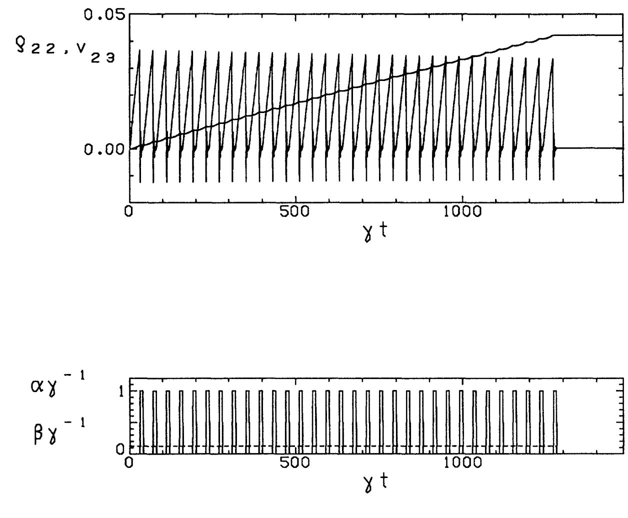

FIG. 5. Temporal evolution of population ρ_{22} (monotonously rising curve) and coherence v_{23}. Number of optical pulses $n=32$. Below: the pulse sequence is shown—optical field α (solid line) and radio field β (dashed line). Time is given in units of γ^{-1}.

and are never on simultaneously. This simplifies the equations so far that an analytical treatment becomes possible. The quantity of central interest is the polarization $v_{23}(t)$ which in this approximation evolves according to

$$\dot{v}_{23}(t)=-\alpha u_{12} \ . \tag{4a}$$

This means that the rf transition is driven by the optical field via the coherence between the excited states. Phase coherence among the excited states $u_{12}(t)$ is subject to spontaneous decay,

$$\dot{u}_{12}(t)=+\alpha v_{23}-\gamma/2u_{12} \ . \tag{4b}$$

In the chosen approximation we find a simple closed system of equations which is solved easily. The reversible Hamiltonian interaction leads to a Rabi nutation propor-

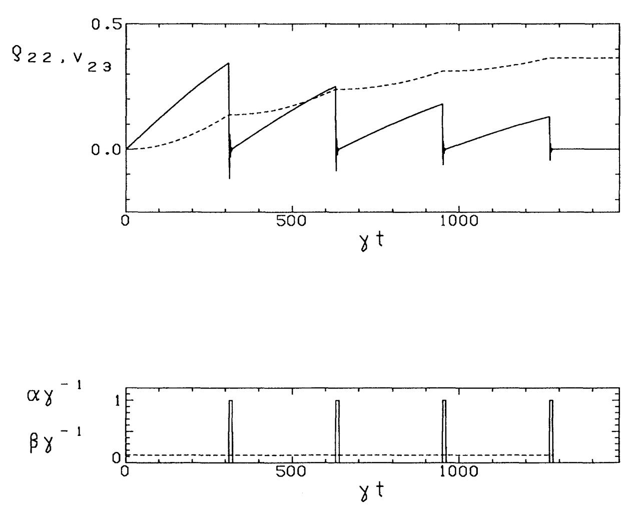

FIG. 4. Temporal evolution of population ρ_{22} (dashed) and coherence v_{23} (solid). Number of optical pulses $n=4$. Below: the pulse sequence is shown—optical field α (solid line) and radio field β (dashed line). Time is given in units of γ^{-1}.

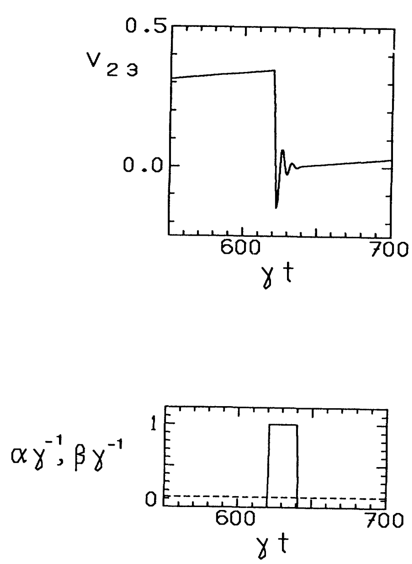

FIG. 6. Enlarged view of Fig. 4 about the optical pulse showing the continuous decay of coherence v_{23}. Below: the pulse sequence is shown—optical field α (solid line) and radio field β (dashed line). Time is given in units of γ^{-1}.

tional to α. The spontaneous decay along the optical transition destroys the coherence u_{12}, which by the optical coupling is transferred to the previously reversible rf transition. This transfer of linewidth in a driven three-level system is well known in spectroscopy and is also relevant for the quantum-jump phenomenon. In the two limiting cases of interest $\alpha \to 0$ and $\alpha \to \infty$, the relaxation of the rf dipole is governed in leading order by

$$v_{23}(t)=v_{23}(t-0)\exp(-2\alpha^2/\gamma t)+O(\alpha) \tag{5a}$$

for $\alpha \to 0$ and

$$v_{23}(t)=v_{23}(t=0)\exp(-\gamma/4t \pm i\alpha t)+O(1/\alpha) \tag{5b}$$

for $\alpha \to \infty$. This simple fact is all that is behind the Zeno effect: the transfer of irreversibility from the measuring apparatus, i.e., the optical transition, to the system under observation.

The general solution of Eqs. (3) with the above approximation takes the form of a 6×6 time evolution matrix $\underline{M}$ if—for convenience—the values of ρ_{ij} are written as a six-component vector $\mathbf{X}$, see the Appendix. This matrix depends on pulse length P, interval length T, and field strengths α and β,

$$\mathbf{X}(t)=\underline{M}\mathbf{X}(0)\ ,$$
$$\underline{M}=[\underline{A}(P,\alpha)\underline{B}(T,\beta)]^n\ ,$$

where $\underline{A}$ stands for the matrix of time evolution during an optical pulse and $\underline{B}$ for the period in between. As the conditions of only optical ($\beta=0$) or only rf field ($\alpha=0$) couple different elements of $\mathbf{X}$ to one another, $\underline{M}$ cannot in general be divided into independent submatrices. However, provided T and P are both considerably larger than $1/\gamma$, quickly decaying transients of order $\exp(-P/\gamma)$, $\exp(-T/\gamma)$ can safely be neglected. In the following we choose $1/\gamma$ as the unit of time $t\gamma \to t$, $T\gamma \to T$, $P\gamma \to P$ and write the Rabi frequencies as multiples of the spontaneous decay rate: $\alpha/\gamma \to \alpha$, $\beta/\gamma \to \beta$. The matrices $\underline{A}$ and $\underline{B}$ for $T,P \gg 1$ are given in the Appendix in all detail. Under the present simplifying conditions, the relevant part of the dynamics can be collected into 2×2 evolution matrices $\underline{a}$ for the optical pulse and b for the rf interval

$$a=\begin{bmatrix} R & 0 \\ 0 & 1 \end{bmatrix}\ ,$$
$$b=\begin{bmatrix} \cos(2\beta 2t) & -\sin(2\beta t) \\ \sin(2\beta t) & \cos(2\beta t) \end{bmatrix}\ , \tag{6}$$

acting on a two-vector which consists of the components

$$\begin{bmatrix} v \\ w \end{bmatrix}=\begin{bmatrix} v_{23} \\ (\rho_{22}-\rho_{11}-\rho_{33})/2 \end{bmatrix}\ ,$$

where

$$R=\exp(-P/4)[\cosh(\Gamma P/4)+\sinh(\Gamma P/4)/\Gamma] \le 1\ ,$$
$$\Gamma=(|1-16\alpha^2|)^{1/2}\ .$$

R is the factor [see Eqs. (5)] which multiplies the coherence when an optical pulse is applied. The hyperbolic functions change into the associated trigonometric ones when α exceeds $\frac{1}{4}$. Dissipation in the rf transition only takes place during an optical pulse and affects coherence while leaving the inversion w unchanged. This result agrees as it should with the interpretation of the process as a nonselective quantum measurement. The essential difference between the measurement approach and the one presented here is that by the Bloch dynamics one can calculate any detail of the "reduction process" involved, whereas in measurement theory, the time scale or the efficiency of measurement are free parameters that cannot be deduced from the information available. We now evaluate the parameter dependence of some quantities associated with the "measurement" aspect of the problem. One should keep in mind, though, that this paper as well as the experiment is only concerned with the evolution of pure states into mixtures under the influence of the measurement, not with the transition of a wave function into a particular eigenstate.

The simplest possible pulse sequency that is of interest here consists of two rf pulses separated by one optical pulse, i.e., $\underline{M}=\underline{B}\ \underline{A}\ \underline{B}$. The reduction efficiency

$$E(\alpha)=\frac{\rho_{22,\underline{B}\,\underline{A}\,\underline{B}}-\rho_{22\underline{B}\,\underline{B}}}{\rho_{22,\underline{B}\,\underline{B}}} \tag{7}$$

is a relative measure of the population changes that occurred after applying either $\underline{B}\ \underline{A}\ \underline{B}$ or $\underline{B}\ \underline{B}$ to an atom initially prepared in the ground state. [Starting from the metastable state $|2\rangle$, the population change could be obtained by replacing ρ_{22} by ρ_{11} in Eq. (6).] We find

$$E(\alpha)=\tfrac{1}{2}(1-R)\ . \tag{8}$$

For a strongly saturating optical field R is essentially zero and therefore $E(\alpha)=\frac{1}{2}$. For a weak pulse, however, sine and cosine change into their hyperbolic counterparts if $\alpha \le \frac{1}{4}$ and coherence is no longer reduced efficiently during the pulse. In this case the simple picture of state reduction fails to describe the process. An example of this is shown in Figs. 7 and 8, where four relatively weak optical pulses are applied during the rf π pulse. In each optical pulse, the coherence v_{23} is reduced, but not completely and as a result, the population reduction is not as dramatic as before.

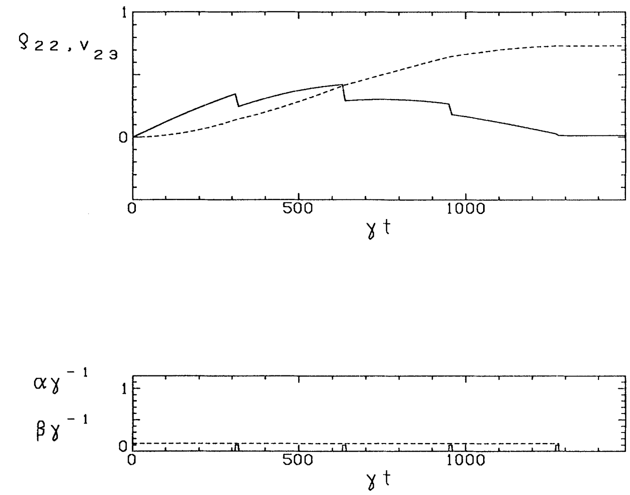

FIG. 7. As Fig. 4, but at low intensity of optical field ($\alpha=0.1$). The amount of photons emitted is 0.4 per pulse on average, resulting in less efficient inhibition of free evolution than in Fig. 4.

For $\alpha \rightarrow 0$, one obtains

$$E(\alpha)=\alpha^2 P \; . \tag{9}$$

The average number of photons scattered by the $3\rightarrow 1$ transition is

$$\langle N \rangle = \int_0^P \rho_{11} dt = \frac{2\alpha^2}{4\alpha^2+1}[1-\rho_{22}(T)]P \; , \tag{10}$$

where again small corrections resulting from rapid transients have been neglected. For small α this reduces to

$$N \simeq 2\alpha^2 P \; . \tag{11}$$

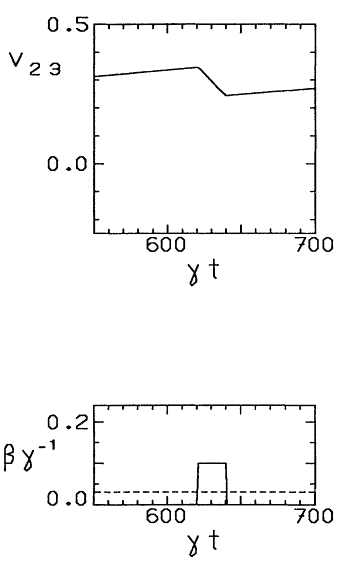

FIG. 8. Enlarged view of Fig. 7 about the optical pulse showing the continuous but not complete decay of coherence v_{23}. Below: the pulse sequence is shown—optical field α (solid line) and radio field β (dashed line). Time is given in units of γ^{-1}.

For a weak optical pulse, the ratio of the average number of optical photons scattered to the inversion $(1-\rho_{22})/2$ of the transition, which the scattering process is supposed to probe, is about $4E(\alpha)$.

In case the optical field is sufficiently strong to make the process look almost like an idealized quantum measurement, there are always much more than one fluorescence photon scattered on average per pulse and per atom. For the purpose of measuring, though, a reasonably reliable occurrence of a single photon would, in principle, be enough. Here the variance rather than the mean number of fluorescence photons would be associated with the quality of the measurement process. However, if, *due to a weak optical field,* it is very rare anyway that a photon might be absorbed, the provided field intensity is expected to limit the reliability of a measurement, in agreement with Eqs. (9) and (10).

In discussing the Zeno effect theoretically, Misra and Sudarshan [4] used the limit of infinitely many, arbitrarily dense quantum measurements. However, they still assumed that they were independent of each other. The atomic three-level system does not allow for this limit: If the spacing T of measurement pulses is of order $1/\gamma$ or shorter, population remains present in level 1 for all times, i.e., outside the system to be measured, leading to rather more efficient population reduction than expected from extrapolating results obtained for the case of well-separated pulses. As can be seen from the Appendix, the dynamics become altogether more complicated if terms falling off as $\exp(-t/4)$ are not negligible, and it is no longer possible to divide the system into a "measured" and a "measuring" part.

Quantum mechanics has retained a grain of mystery in the formulation of the measurement. In the Kopenhagen interpretation two dynamic processes exist side by side: the unitary time evolution of the Schrödinger equation and the mysterious and abrupt collapse of the wave packet when a measurement is performed. If the dynamics is reversible, the state vector rotates continuously through Hilbert space, and introducing the collapse seems to be an undue tampering with theory. Recently, experiments have become possible on single or few atoms, which should exhibit this idealized closed-system dynamics. Observations, however, are made in the macroscopic laboratory world, and the device measuring the observable under question is a part of it. In order to perform a measurement, a coupling has to be established between this device and the observed system, which thereby becomes an open system, connected to the many degrees of freedom of the apparatus. Even the simplest possible such detector, consisting of a single atom only, follows irreversible dynamics, as the coupling to the multitude of vacuum modes makes it part of a large system.

Using the language of measurement, the three-level system studied in the present paper contains the observed system and the detector in one atom. The quantum Zeno effect is traditionally considered as a paradigm for the concept of wave-packet collapse. In the present paper, however, we demonstrated that the experimentally observed freezing of the unitary motion, which was attributed to the collapse, can be understood naturally in terms of continuous quantum dynamics without ever making use of the projection postulate. The microscopic system attains irreversibility due to the interaction with the measuring device. Thereby the off-diagonal elements of the density operator are not quenched in a discontinuous jump, as it is assumed by the collapse hypothesis, but decay steadily with the correlation time of the detector. Only in the limit of an intense probing pulse and rapid spontaneous emission of the detector transition is the collapse a useful approximation to the fast decay of atomic coherence. The optical Bloch equations provide a complete description of all measurable details in full agreement with experiment. Obviously, one does not have to decide which part of the system can be considered a detector, at what time a measurement takes place, how many photons it takes to reduce the wave packet, or on other ambiguous issues of measurement. The Bloch-equation approach also allows one to predict the dependence on all system parameters, whereas the projection hypothesis only mimics the observed dynamics in one extreme parameter limit.

Any scientific theory is based on observation. Theories that differ only in predictions, which in principle are inaccessible to experimental observation, are equivalent, and preferring one over the other is just a matter of taste or convenience. For the present example, the formulation in terms of irreversible quantum dynamics, which never leaves the conventional framework of quantum mechanics, is superior to the *ad hoc* collapse hypothesis. Since all measurable results of the Zeno experiment are predicted without difficulty as long as we restrict ourselves to physical and not metaphysical questions, one might ask whether a separate theory of measurement is really needed in this case. There is no logical requirement that, for a theory to be complete, it should assign one particular real number to every measurable quantity. From a positivistic point of view, a scientific theory is complete, when it is capable of predicting the outcome of every possible physical experiment.

APPENDIX

It is convenient to write the six density-matrix elements of interest in the form of a vector,

$$\mathbf{X}=(v_{13},\rho_{11},\rho_{22},v_{23},u_{12}) \, .$$

The time evolution during an optical pulse takes $\mathbf{X}$ at time into $\underline{A}X$ at time $t+P$ with

$$\underline{A}(P,\alpha)=\begin{pmatrix} t(P) & \frac{\alpha}{4\alpha^2+\frac{1}{2}} & \frac{\alpha}{4\alpha^2+\frac{1}{2}} & 0 & 0 & 0 \\ t(P) & \frac{2\alpha^2}{4\alpha^2+\frac{1}{2}} & \frac{2\alpha^2}{4\alpha^2+\frac{1}{2}} & 0 & 0 & 0 \\ t(P) & \frac{2\alpha^2+\frac{1}{2}}{4\alpha^2+\frac{1}{2}} & \frac{2\alpha^2+\frac{1}{2}}{4\alpha^2+\frac{1}{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & (c+s) & -4\alpha s \\ 0 & 0 & 0 & 0 & 4\alpha s & (c-s) \end{pmatrix},$$

where

$$c:=\cos(\Gamma/4)\ ,\quad s:=\sin(\Gamma/4)/\Gamma \quad \text{for } \alpha>\tfrac{1}{4}$$

$$c:=\cosh(\Gamma/4)\ ,\quad s:=\sinh(\Gamma/4)/\Gamma \quad \text{otherwise}\ ,$$

and

$$\Gamma=(|1-16\alpha^2|)^{1/2}\ .$$

Elements falling off rapidly on the time scale of the spontaneous lifetime $\exp(-t/4)$ are denoted by $t(P)-t$ for transients.

Applying a rf pulse results in $\mathbf{X}\to\underline{B}\mathbf{X}$ with

$$c:=\cos(2\beta T)\ ,\quad s:=\sin(2\beta T)\ ,$$

$$\underline{B}(T,\beta)=\begin{pmatrix} t(T) & 0 & 0 & 0 & 0 & t(T) \\ 0 & t(T) & 0 & 0 & 0 & 0 \\ 0 & t(T)+\frac{1+c}{2} & \frac{1+c}{2} & \frac{1-c}{2} & -s & 0 \\ 0 & \frac{1-c}{2} & \frac{1-c}{2} & \frac{1+c}{2} & s & 0 \\ 0 & \frac{s}{2} & \frac{s}{2} & -\frac{s}{2} & c & 0 \\ t(T) & 0 & 0 & 0 & 0 & t(T) \end{pmatrix}.$$

If T and P are sufficiently long all the elements $t(P),t(T)$ can be neglected. In particular, once α has been off for some time (matrix $\underline{B}$ for $T\gg1$), no elements of ρ involving state 1 are left. It is then easily seen that the time evolution takes the form (9).

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