Suppression of spontaneous emission by squeezed light in a cavity

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The behavior of a two-level atom confined within a cavity, pumped by an intense external field, and damped by a squeezed vacuum reservoir is studied. When the lowest squeezed quadrature of the vacuum is in phase with the pumping field, the relaxation time of the population inversion and of the polarization quadratures of the atom are appreciably enlarged, and hence the atom is decoupled from the reservoir. The effects of finite bandwidth of the squeezing are analyzed and it is shown that they can be used to improve the decoupling.

I. INTRODUCTION

The radiative decay of atoms in the free space is one of the central problems in quantum optics. Recently, much attention has been focused on the changes of the atomic population and phase decays appearing when an atom is damped by a squeezed vacuum environment. First, Gardiner [1] showed that in a infinite-band squeezed vacuum, the two polarization components of the atom are damped at different rates, leading to a longer relaxation time for one of these components when compared to normal vacuum radiative decay. The introduction of a driving field in the model leads to a broadening of the sidebands of the fluorescence spectrum while the central band is narrowed or broadened depending on the relative phases of the squeezed vacuum and the driving field [2]. However, the squeezed light generated in the actual experimental situations has finite bandwidth [3], which has lead to the investigation of new methods to study the behavior of a two-level atom in such an environment. It has been shown, using a cumulant expansion and simulation methods, that the effects of the squeezing on the central band of the spectrum is reduced in this case, while the sidebands can be narrowed in comparison with the infinite-band case [4–7].

On the other hand, the suppression and enhancement of spontaneous emission when an atom is placed inside a cavity has been long studied [8–10] and observed [11]. The existence of a finite response time of the reservoir produces a frequency-dependent coupling between the atom and the electromagnetic field which leads to important changes in the atomic decay. As it is well known, the rate of spontaneous emission is proportional to the density of photon modes at the atomic transition frequency. If the density of photon modes close to the spectral region where atomic transitions take place is modified, the corresponding decay rate is also changed. One can therefore tune or detune the cavity to the atomic transition frequency in order to enhance [8] or inhibit [9] the spontaneous emission. Alternatively, as has been shown recently by Lewenstein, Mossberg, and Glauber [10], by driving the atom with an intense laser field one can dynamically change the transition between dressed levels frequencies (i.e., produce a Stark shift), which causes an inhibition in one of the polarization components of the atom decay.

Recently Parkins [5] has shown that when an atom is damped by a reservoir in a infinite-band squeezed vacuum state and driven by a laser field, spontaneous emission from the atom can be switched off, i.e., the atom and the cavity remain decoupled. The main goal of this work is to study the effects of the finite bandwidth of the squeezing in this behavior both in a cavity and in the free space. This paper is organized as follows. In Sec. II we qualitatively analyze the effects produced by the shape of reservoir spectrum in the atomic decay. In Sec. III, following the work of Lewenstein, Mossberg, and Glauber, we derive the general modified Bloch equations describing this model. In Sec. IV we particularize these equations for the case in which the atom is in the free space by assuming a constant cavity mode function and, in Sec. V, for the case in which the atom is in a Lorentzian–type cavity, confirming the results predicted in Sec. II. Finally, in the Appendix, we compare the solutions given by this approach for the case of no external driving field with the numerical simulations of Ref. [6], showing an excellent agreement. This ensures that our solutions may be used for a large range of parameters.

II. MODIFICATIONS ON THE ATOMIC DECAY

We consider here an atom placed in a cavity driven by an intense laser field on resonance with the atomic transition frequency and coupled to an electromagnetic reservoir in a squeezed vacuum state. As it is well known, the atom may be represented by the Bloch vector $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$, where σ_i are the usual pseudospin operators representing the atom. In the absence of damping, the Bloch equations are given, in the rotating frame, by

$$\frac{d}{dt}\boldsymbol{\sigma} = \mathbf{B} \times \boldsymbol{\sigma}, \tag{2.1}$$

where $\mathbf{B}= (\Omega, 0, 0)$, Ω being the Rabi frequency which characterizes the atom–laser-field interaction strength. Equation (2.1) shows that the first component of the Bloch vector $\pi_x = \sigma_x$ (also known as the dressed population inversion [12]) remains constant, while the dressed polarizations $\pi_\pm = (\sigma_y \pm i\sigma_z)/2$ oscillate at frequencies $\pm\Omega$, respectively. However, this behavior breaks down when damping is taken into account. The electromagnetic reservoir that triggers spontaneous emission can be represented in the Bloch space by fluctuating fields in the XY plane [13] (see Fig. 1). Hence $\mathbf{B}= (\Omega + E_x, E_y, 0)$, where E_x and E_y are the slowly varying quadratures of the electromagnetic reservoir along the X and Y directions, respectively. The shape of the fluctuation distribution is elliptical when the reservoir is in a squeezed state, as shown in Fig. 1, where we have assumed for the principal directions of the elliptical distribution to coincide with the X and Y axes. The length of the semiaxes (which is a measure of the fluctuating field strength) are proportional to $N - M + \frac{1}{2}$ and $N + M + \frac{1}{2}$, respectively, where N and M are two parameters describing the squeezing which, for an ideal squeezed vacuum, fulfill $\mid M \mid = \sqrt{N(N+1)}$. Thus for M positive (negative) it is the E_x (E_y) quadrature that is initially squeezed. When these fluctuations are much faster than the typical atomic lifetime, white-noise methods can be used. However, for fluctuations that do not fulfill this condition, other methods must be used. For weakly interacting systems one can describe the fluctuations of E_x and E_y in the Fourier space by its spectrum.

In order to analyze the role played by the reservoir, let us decompose the field operator in the usual way [14]

$$E_{x,y} = E^f_{x,y} + E^s_{x,y}. \tag{2.2}$$

The first term E^f gives the free evolution of the field and the second E^s is the source term, which gives the contribution of the photons emitted by the atom into the cavity modes. Note that with this definition, despite the commutator $[\sigma(t), E_{x,y}(t)] = 0$ for all times, $E^f_{x,y}(t)$ and $E^s_{x,y}(t)$ may not commute separately with the atomic observables. So we chose normally ordered mean values

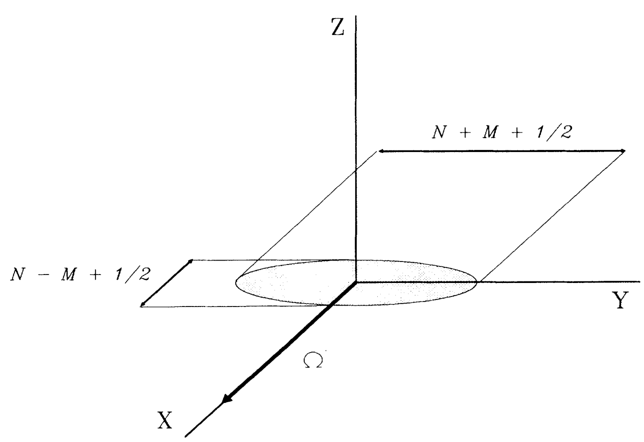

FIG. 1. Bloch space representation of a two-level atom interacting with a driving field and damped by a squeezed reservoir. The phase of the external field has been chosen in order to match with the maximally squeezed quadrature of the electromagnetic reservoir.

for all the products involving field and atomic operators, and we will keep this choice in the following.

The source terms $E^s_{x,y}$ can only carry one photon in whatever mode and therefore the spectrum of the radiated part will be the cavity mode density, i.e., a function centered at the central cavity frequency (which we assume to coincide with the atomic transition frequency ω_0) with a width Γ and a height $\frac{1}{2}\gamma_c$, where Γ^{-1} is the response time of the reservoir and γ_c the contribution of the cavity modes to the spontaneous emission rate when $\omega = \omega_0$.

On the other hand, the spectrum of the free part E_x (E_y) is given by a function of height $\gamma_c N - \gamma_c M$ ($\gamma_c N + \gamma_c M$) and width of the order of the minimum between the squeezing bandwidth along the X (Y) direction and the bandwidth of the cavity mode density function Γ. Note that the fluctuation intensities of the free part are not $\gamma_c(N \pm M + 1/2)$ but $\gamma_c(N \pm M)$, since, as mentioned above, we have chosen normal ordering (in fact, the last term $+\frac{1}{2}$ is contained in the radiation part).

Notice that when the reservoir is in a vacuum state $\langle \sigma E^f_{x,y}\rangle = 0$ due to the normal order chosen, and, therefore, only the radiated parts of the field contribute to the spontaneous emission. In the following we concentrate in an ideal and strong squeezing ($N \gg 1$) since this is the situation where most important features arise. With all the previous discussion, the following observations are in order.

(a) The evolution of the dressed population inversion is given by

$$\frac{d}{dt}\langle \pi_x\rangle = \langle \sigma_z E^f_y\rangle + \langle \sigma_z E^s_y\rangle. \tag{2.3}$$

In the free space ($\Gamma \longrightarrow \infty$) and for infinite-band squeezing, the spectrum of $E_y = E^f_y + E^s_y$ has a height of $(N + M + \frac{1}{2})\gamma_c$ and infinite bandwidth. Thus the right-hand side of Eq. (2.3) tends to zero for M negative (E_y squeezed) and therefore the decay of $\langle \pi_x\rangle$ is inhibited [1, 2]. For decreasing values of the squeezing bandwidth, much smaller than the Rabi frequency Ω, this last frequency lies outside the spectrum of E^f_y. As in the absence of damping, σ_z undergoes Rabi oscillations at frequency Ω, the term $\langle \sigma_z E^f_y\rangle$ becomes negligible, and therefore $\langle \pi_x\rangle$ decays as if it were damped by a vacuum state. Hence the effect of the squeezing vanishes for this component of the Bloch vector [6].

On the other hand, for decreasing values of Γ (good cavity limit) the Rabi frequency lies outside the spectrum of $E_y = E^f_y + E^s_y$ and therefore the decay of $\langle \pi_x\rangle$ is inhibited. This last phenomenon is the same reported by Lewenstein, Mossberg, and Glauber [10] when the reservoir is in a vacuum state.

(b)The evolution of the dressed polarizations may be expressed as

$$\frac{d}{dt}\langle \pi_\pm\rangle = \pm i(\Omega\langle \pi_\pm\rangle + \langle \pi_\pm E^f_x\rangle + \langle \pi_\pm E^s_x\rangle - \langle \pi_x E^f_y\rangle - \langle \pi_x E^s_y\rangle)\ . \tag{2.4}$$

In the free space and for infinite-band squeezing, again the terms with E_y tend to zero for M negative. In this case, the dressed polarizations decay due only to the term E_x. In the absence of damping, π_+ oscillates at the Rabi frequency Ω and then, for values of the squeezing bandwidth smaller than this frequency, $\langle \pi_x E_y^f \rangle \longrightarrow 0$ since now Ω lies out of the spectrum of E_y^f. Besides, when E_x is squeezed (M positive),

$$\frac{d}{dt}\langle \pi_+ \rangle \simeq i\Omega \langle \pi_+ \rangle + i\langle \pi_x E_y^s \rangle,$$

so it decays more slowly than in a vacuum state. In the good cavity limit ($\Omega \gg \Gamma$) also $\langle \pi_x E_y^s \rangle \longrightarrow 0$ for the same reason, and therefore this dressed polarization is undamped. Since as it has been shown above, $\langle \pi_x \rangle$ decay is also inhibited in this case, the atom is decoupled from the cavity. This behavior coincides with that predicted by Parkins [5] when the squeezed reservoir has infinite bandwidth. However, for strong, infinite-band squeezing, the spectrum of the E_y quadrature evaluated at the Rabi frequency can be relevant even for $\Omega \geq \Gamma$, which imposes a very intense field in order to decouple the atom from the cavity. As we show below, this condition is relaxed when the squeezing has finite bandwidth.

In conclusion, when an atom is placed in a good quality cavity the effects of the shape of the cavity-mode density function strongly modifies the behavior of the atom. When besides, the cavity modes are squeezed, the decay of the atom could be, in principle, inhibited. The finite bandwidth of the squeezing also modifies this behavior.

III. MODIFIED BLOCH EQUATIONS

The Hamiltonian for the model considered in the preceding section reads, in the rotating-wave approximation, as

$$H = \tfrac{1}{2}\omega_0\sigma_3 + \int d\omega\, \omega a_\omega^\dagger a_\omega + \int d\omega [g(\omega)\sigma^+ a_\omega + g(\omega)^* a_\omega^\dagger \sigma^-] + \frac{\Omega}{2}(\sigma^+ e^{-i\omega_0 t} + \sigma^- e^{i\omega_0 t}), \quad (3.1)$$

where a_ω and $a_\omega^\dagger$ are the annihilation and creation operators for the cavity mode of frequency ω. The two former terms give the free evolution of the atom and the cavity modes, while the third and the last terms give the interaction of the atom with the cavity modes and with the laser field, respectively. Here $g(\omega)$ characterize the cavity-mode density and Ω is the Rabi frequency for the laser field.

We are interested in the evolution equations for the mean values of the atomic operators. Using the Heisenberg equations for the Hamiltonian (3.1) and eliminating the equations for the evolution of the annihilation and creation operators we find

$$\frac{d}{dt}\langle \sigma^-(t)\rangle = i\frac{\Omega}{2}\langle \sigma_z(t)\rangle + i\int d\omega\, g(\omega)e^{i(\omega_0-\omega)t}\langle \sigma_z(t)a_\omega(0)\rangle + \int d\omega \mid g(\omega)\mid^2 \int_0^t dt' e^{i(\omega_0-\omega)(t-t')}\langle \sigma_z(t)\sigma^-(t')\rangle,$$
$$\frac{d}{dt}\langle \sigma_z(t)\rangle = \left(i\Omega\langle \sigma^-(t)\rangle - 2i\int d\omega\, g(\omega)e^{i(\omega_0-\omega)t}\langle \sigma_+(t)a_\omega(0)\rangle - 2\int d\omega \mid g(\omega)\mid^2 \int_0^t dt' e^{i(\omega_0-\omega)(t-t')}\langle \sigma^+(t)\sigma^-(t')\rangle \right) + \text{H.c.}, \quad (3.2)$$

where H.c. stands for Hermitian conjugate. These equations contain contributions from two-time correlation functions, which are connected to other three-time correlation functions and so on. In order to obtain a finite hierarchy of equations we must cut off in some order the infinite set of obtained equations. To do this we consider a series expansion in terms of $\mid g(\omega)\mid^2$ up to first order (Born approximation), which, in principle, is valid for short times. The validity of this approximation is contrasted in the Appendix. We need certain correlation functions for the reservoir modes at the initial time. For a squeezed vacuum they are

$$\langle a_\omega(0)\rangle = 0,$$
$$\langle a_\omega^\dagger(0)a_{\omega'}(0)\rangle = N(\omega)\delta(\omega-\omega'), \quad (3.3)$$
$$\langle a_\omega(0)a_{\omega'}(0)\rangle = M(\omega)\delta(2\omega_0-\omega-\omega').$$

where we have assumed that the frequency around which the squeezing has been produced coincides with the atomic transition frequency. $N(\omega)$ and $M(\omega)$ are parameters characterizing the squeezing and depend on the specific mechanism to squeeze the vacuum. In the following we consider the squeezing obtained at the output of a degenerate parametric amplifier (DPA) [3], which has proved to be one of the most successful ways to achieve it. In this case

$$N(\omega) = \frac{\lambda^2-\mu^2}{4}\left(\frac{1}{\mu^2+(\omega_0-\omega)^2} - \frac{1}{\lambda^2+(\omega_0-\omega)^2}\right),$$
$$M(\omega) = \frac{\lambda^2-\mu^2}{4}\left(\frac{1}{\mu^2+(\omega_0-\omega)^2} + \frac{1}{\lambda^2+(\omega_0-\omega)^2}\right), \quad (3.4)$$

where $\lambda = \frac{1}{2}\gamma_p \pm \epsilon$, $\mu = \frac{1}{2}\gamma_p \mp \epsilon$, γ_p is the damping constant of the DPA, and ϵ its amplification constant (the upper sign stands for M positive and the lower for M negative). Maximum squeezing is achieved at threshold of the cavity at $\epsilon = \frac{1}{2}\gamma_p$. Now the modified Bloch equations become

$$\begin{aligned}\frac{d}{dt}\langle\sigma^-(t)\rangle &= i\frac{\Omega}{2}\langle\sigma_z(t)\rangle - i\int_0^t dt'\langle\sigma_z(t')\rangle\sin\Omega(t-t')[N(t-t')-M(t-t')] \\ &\quad -2\int_0^t dt'\langle\sigma^-(t')\rangle\cos\Omega(t-t')N(t-t') \\ &\quad -2\int_0^t dt'\langle\sigma^+(t')\rangle\cos\Omega(t-t')M(t-t') - \frac{i}{2}\int_0^t dt'\sin\Omega(t-t')G(t-t'), \\ \frac{d}{dt}\langle\sigma_z(t)\rangle &= i\Omega[\langle\sigma^-(t)\rangle - \langle\sigma^+(t)\rangle] \\ &\quad -2\int_0^t dt'\langle\sigma_z(t')\rangle\mathrm{Re}\{N(t-t')+M(t-t')+\cos\Omega(t-t')[N(t-t')-M(t-t')]\} \\ &\quad +4\int_0^t dt'\langle\sigma^+(t')\rangle\sin\Omega(t-t')\mathrm{Im}[\langle\sigma^-(t')N(t-t')+\langle\sigma^+(t')\rangle M(t-t')] \\ &\quad -\int_0^t dt'\{1+\cos\Omega(t-t')\mathrm{Re}[G(t-t')]\},\end{aligned} \tag{3.5}$$

where

$$\begin{aligned}N(\tau) &= \int d\omega \mid g(\omega)\mid^2 [N(\omega)+\tfrac{1}{2}]e^{i(\omega_0-\omega)\tau}, \\ M(\tau) &= \int d\omega g(\omega)g(2\omega_0-\omega)M(\omega)e^{i(\omega_0-\omega)\tau}, \\ G(\tau) &= \int d\omega \mid g(\omega)\mid^2 e^{i(\omega_0-\omega)\tau},\end{aligned} \tag{3.6}$$

and $\langle\sigma^+(t)\rangle = \langle\sigma^-(t)\rangle^*$. Note first that these equations display memory effects through the time integrals, i.e., the evolution in a time t depends on the values taken in previous times. These memory effects are related to the non-Markovian coupling between the atom and the cavity modes. On the other hand, these equations are in principle valid for short times. However, when the characteristic decay time for the atom is much larger than the inverse of the bandwidth of $\mid g(\omega)\mid^2 N(\omega)$ (broadband limit) the fluctuations of the field are so rapid that the atom "sees" the field always as a squeezed vacuum and therefore the equations are valid for longer times.

In most of this work we will assume that the phase of the driving field in the DPA is chosen in order $N(\tau)$, $M(\tau)$, and $G(\tau)$ to be real quantities. We will analyze these cases since they represent the limits where most physical effects are found. In this case and in the same order of expansion, the modified Bloch equations become

$$\begin{aligned}\frac{d}{dt}\langle\pi_x\rangle &= -2\int_0^t d\tau\cos\Omega\tau[N(\tau)+M(\tau)]\langle\pi_x\rangle, \\ \frac{d}{dt}\langle\pi_+\rangle &= i\Omega\langle\pi_+\rangle - \left(2\int_0^t d\tau[N(\tau)-M(\tau)] + \int_0^t d\tau e^{-i\Omega\tau}[N(\tau)+M(\tau)]\right)\langle\pi_+\rangle \\ &\quad + \int_0^t d\tau e^{i\Omega\tau}[N(\tau)+M(\tau)]\langle\pi_-(t)\rangle - i\int_0^t d\tau(1+e^{i\Omega\tau})G(\tau),\end{aligned} \tag{3.7}$$

where $\langle\pi_x\rangle = \langle\sigma_x\rangle = \langle\sigma^+\rangle + \langle\sigma^-\rangle$ and $\langle\sigma_y\rangle = -i(\langle\sigma^+\rangle - \langle\sigma^-\rangle)$ are the first and second components of the Bloch vector, respectively, and $\langle\pi_\pm\rangle = \langle\sigma_y\rangle \pm i\langle\sigma_z\rangle)$ are the two dressed polarizations.

IV. DECAY IN FREE SPACE

In the free space the cavity-mode density is considered to be constant over the frequencies in which the atom interacts, and hence

$$g(\omega) = \sqrt{\frac{\gamma}{2\pi}}. \tag{4.1}$$

Let us first particularize the modified Bloch equations for the case in which the reservoir is in an infinite-band squeezed state. This corresponds to the usual white-noise limit, where the reservoir response time is zero. To obtain the evolution equations for the atomic operators we must take the limits $\lambda,\mu \longrightarrow \infty$ in (3.4), keeping μ/λ constant. The equations obtained from (3.7) become

$$\begin{aligned}\frac{d}{dt}\langle\pi_x\rangle &= -\gamma\left(N+M+\tfrac{1}{2}\right)\langle\pi_x\rangle, \\ \frac{d}{dt}\langle\pi_+\rangle &= i\Omega\langle\pi_+\rangle - \gamma\left(\tfrac{3}{2}(N+\tfrac{1}{2}) - \frac{M}{2}\right)\langle\pi_+\rangle \\ &\quad + \frac{\gamma}{2}(N+\tfrac{1}{2}+M)\langle\pi_-\rangle - i\gamma,\end{aligned} \tag{4.2}$$

with

$$N+\frac{1}{2}=\frac{1}{4}\left(\frac{\lambda^2}{\mu^2}+\frac{\mu^2}{\lambda^2}\right),$$
$$M=\frac{1}{4}\left(\frac{\lambda^2}{\mu^2}-\frac{\mu^2}{\lambda^2}\right), \tag{4.3}$$

which coincide with the results of Ref. [2]. In this limit, the decay of $\langle\pi_x\rangle$ is inhibited when the E_y quadrature is strongly squeezed, i.e., $N \gg 1$ with $M < 0$.

In the case of finite-band squeezing (but broad enough in order for the Born approximation to be valid) Eqs. (3.7) may be expressed as

$$\frac{d}{dt}\langle\pi_x\rangle=-\gamma_x(t)\langle\pi_x\rangle,$$
$$\frac{d}{dt}\langle\pi_+\rangle=-A(t)\langle\pi_+\rangle+B(t)\langle\pi_-\rangle-i\gamma, \tag{4.4}$$

where now

$$\gamma_x(t)=\frac{\gamma}{2}\left[1+\frac{\lambda^2-\mu^2}{\mu^2+\Omega^2}\times\left(1-\frac{e^{-\mu t}}{\mu}(\mu\cos\Omega t-\Omega\sin\Omega t)\right)\right],$$
$$B(t)=\frac{\gamma}{4}\left(1+\frac{\lambda^2-\mu^2}{\mu(\mu-i\Omega)}(1-e^{-\mu t}e^{i\Omega t})\right), \tag{4.5}$$
$$A(t)=i\Omega-\frac{\gamma}{2}\left(\frac{\mu^2}{\lambda^2}+\frac{\lambda^2-\mu^2}{\lambda^2}e^{-\lambda t}\right)-B^*.$$

When the squeezing bandwidth fulfills $\gamma/2 \ll \mu^3/\lambda^2$, λ^3/μ^2 (see the discussion in the Appendix), we can replace $A(t)$, $B(t)$, and $\gamma_x(t)$ by their values at $t=\infty$. The decay constants are then given by

$$\gamma_x=\gamma_x(\infty)=\frac{\gamma}{2}\frac{\lambda^2+\Omega^2}{\mu^2+\Omega^2} \tag{4.6}$$

for $\langle\pi_x\rangle$, and

$$\gamma_R=-\text{Re}[A(\infty)]=\frac{\gamma}{2}\frac{\mu^2}{\lambda^2}+\tfrac{1}{2}\gamma_x \tag{4.7}$$

for the dressed polarizations. The most important feature of these decay constants is that, opposite to the infinite-band squeezing case, they depend on the Rabi frequency of the laser field. So, for $\Omega \gg \lambda$, μ, the decay constant γ_x tends to its value when the reservoir is in a vacuum state and, therefore, the effect of the squeezing on the first component of the Bloch vector is lost. This is depicted in Fig. 2, where it is shown the evolution of this component given by Eq. (4.4) with the definitions (4.5) (solid line) and those of (4.6) (dashed line) for $\lambda/\gamma=32$, $\mu/\gamma=8$, and $\Omega/\gamma=10$, 100. First note that the results predicted by both theories are very similar, which permits one to use the decay constants derived at $t \longrightarrow \infty$. For increasing values of Ω both results coincide and the evolution of $\langle\pi_x\rangle$ tends to the solution obtained in the vacuum (dotted line), as mentioned above.

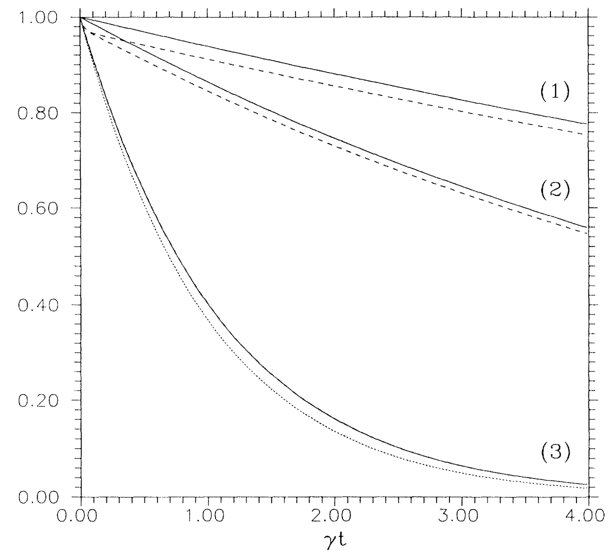

FIG. 2. Evolution of the dressed population inversion π_x in free space for $\lambda/\gamma=32$, $\mu/\gamma=8$, and (1) $\Omega/\gamma=1$, (2) $\Omega/\gamma=10$, (3) $\Omega/\gamma=100$. The solid lines represent the solution obtained from Eq. (4.4), the dashed lines that of Eq. (4.6), and the dotted line the result obtained in a vacuum reservoir.

On the other hand, and in the same limit ($\Omega \gg \lambda$, μ), the decay constant for the dressed polarizations tends to $(\gamma/2)(1/2+\lambda^2/\mu^2)$. Hence, when E_x is strongly squeezed and $\Omega \gg \lambda$, this decay constant tends to half the one for standard spontaneous emission. Finally, it is worth mentioning that Eqs. (4.6) and (4.7) coincide with those of Ristch and Zoller [4] when the same limit is taken (first-order approximation in γ/μ and γ/λ in the decorrelation approximation).

V. DECAY IN A CAVITY

In this section we analyze the effects produced in the atomic decay when the atom is placed in a cavity. We assume the mode function of the cavity to be

$$g(\omega)=\sqrt{\frac{\gamma_c}{2\pi}}\frac{\Gamma}{\Gamma-i(\omega-\omega_c)}. \tag{5.1}$$

This corresponds to a cavity-mode-density modeled as a Lorentzian function with a maximum value at ω_c. Here γ_c is the contribution of the cavity modes to the spontaneous emission rate when $\omega=\omega_c$ and Γ^{-1} is the reservoir response time. In the infinite-band squeezing, we obtain, for the terms defined in Eq. (3.6),

$$N(\tau)=(N+\tfrac{1}{2})\frac{\gamma_c}{2}\Gamma e^{-\Gamma\tau}e^{-i\Delta\tau},$$
$$M(\tau)=M\frac{\gamma_c}{2}\frac{\Gamma^2}{\Gamma-i\Delta}e^{-\Gamma\tau}e^{-i\Delta\tau},$$
$$G(\tau)=\frac{\gamma_c}{2}\Gamma e^{-\Gamma\tau}e^{-i\Delta\tau},$$

where $\Delta = \omega_0 - \omega_c$ is the atomic transition-cavity frequency detuning and N and M have been defined in (4.3). First note that substituting these values in Eq. (3.5), and for a detuning much broader than that the cavity width Γ, the integrals become very small and therefore the atom remains decoupled from the cavity. This effect has been long studied, so we leave aside the problem of the detuning and in the following we concentrate in the resonant case ($\Delta = 0$). The modified Bloch equations become, in this case,

$$\frac{d}{dt}\langle\pi_x\rangle = -\gamma_x\langle\pi_x\rangle, \qquad \frac{d}{dt}\langle\pi_+\rangle = A\langle\pi_+\rangle + B\langle\pi_-\rangle + C, \tag{5.2}$$

where

$$\gamma_x = \gamma_c(N + M + \tfrac{1}{2})\frac{\Gamma^2}{\Gamma^2+\Omega^2}, \quad B = \frac{\gamma_c}{2}(N+M+\tfrac{1}{2})\frac{\Gamma}{\Gamma - i\Omega}, \quad A = i\Omega - \gamma_c(N - M + \tfrac{1}{2}) - B^*, \quad C = -i\frac{\gamma_c}{2}\left(1 + \frac{\Gamma}{\Gamma - i\Omega}\right). \tag{5.3}$$

Note that for $\Omega^2 \gg \Gamma^2$, $\Gamma^2(N + M + \frac{1}{2})$, one can dynamically suppress the decay of the $\langle\pi_x\rangle$ component. When E_y is strongly squeezed (M negative), the effect of the squeezing is added and the decay becomes still slower. On the other hand, the decay constant for $\langle\pi_+\rangle$ becomes, in the same limit,

$$\gamma_R = -\mathrm{Re}(A) = \frac{\gamma_c}{2}(N - M + \tfrac{1}{2}) + \tfrac{1}{2}\gamma_x, \tag{5.4}$$

which consists of two terms. In order for this component not to decay, it is necessary first that M be positive in the strong squeezing limit (the first term becomes as small as we please) and second, γ_x must be very small. If the first condition is fulfilled, then we need $\Omega^2 \gg \Gamma^2$, $\Gamma^2(N+ \mid M \mid + \frac{1}{2})$, which implies a very intense laser field and in the strong-squeezing limit ($N \gg 1$). All this results have already been reported in Ref. [5].

Let us concentrate on the effects of the finite-band squeezing in this decay. Introducing Eq. (3.4) in (3.7), when we substitute the upper limits in the integrals by infinity, we find Eq. (5.2) with

$$\begin{aligned} \gamma_x &= \frac{\gamma_c}{2}\left(\frac{\lambda^2+\Omega^2}{\mu^2+\Omega^2}\right)\frac{\Gamma^2}{\Gamma^2+\Omega^2}, \\ B &= \frac{\gamma_c}{4}\frac{\Gamma}{\Gamma - i\Omega} + \frac{\gamma_c}{4}(\lambda^2-\mu^2)\Gamma^2 \\ &\quad \times\{[\Gamma(\Gamma - i\Omega)(\mu^2-\Gamma^2)]^{-1} \\ &\quad +[\mu(\mu - i\Omega)(\Gamma^2-\mu^2)]^{-1}\}, \\ A &= i\Omega - \frac{\gamma_c}{2}\frac{\mu^2}{\lambda^2} - B^*, \\ C &= -i\frac{\gamma_c}{2}\left(1+\frac{\Gamma}{\Gamma - i\Omega}\right), \end{aligned} \tag{5.5}$$

and therefore

$$\gamma_R = -\mathrm{Re}A = \frac{\gamma_c}{2}\frac{\mu^2}{\lambda^2} + \frac{1}{2}\gamma_x. \tag{5.6}$$

As in Sec. IV the effect of the squeezing γ_x is disminished in this case in comparison with the infinite-band one. Hence, in order to decouple the atom from the cavity it is necessary that $\Omega \gg \Gamma, \lambda, \mu$ and a strong squeezing with M positive. Thus, in this case, a strong squeezing ($\mu^2/\lambda^2 \ll 1$) does not lead to a much higher value of the Rabi frequency Ω, i.e., a very intense laser field. The finite band of squeezing may be an advantage for the dynamical suppression of the atomic decay.

Finally, to conclude this section, we study the effects of the inhibition of decay on the resonance spectrum. In order to measure the spectrum emitted by the atom, it is necessary to consider that the atom is not only coupled to the cavity modes but also to other modes (background modes), which we take to be in a vacuum state. We assume that the coupling between the atom and the background modes is much weaker than the coupling between the atom and the cavity modes, so the precedent study remains valid in this case. The spectrum of the light emitted by the atom in the background modes in the stationary state is given by the Fourier transform of the two-time dipole autocorrelation function

$$S(\omega) = k \lim_{t\to\infty} \mathrm{Re}\int_0^\infty d\tau\, e^{-i\omega\tau}\langle\sigma_+(t)\sigma_-(t+\tau)\rangle, \tag{5.7}$$

where k is a normalization constant that we take to be $k = 1$. The spectrum is usually calculated by using the quantum regression theorem, valid for Markovian systems [15]. However, it has been shown that in systems damped by a colored reservoir, one can use this approach only on the condition that the reservoir spectrum can be regarded as flat on each emission line [12]. In the case under study, despite that the bandwidths of the reservoir are finite, they have been assumed to be much broader than the damping rates in order for the upper limit t in the integrals of Eq. (3.7) could be substituted by infinity. Thus the quantum regression theorem can be used as long as this condition is fulfilled. In this case, the two-time dipole autocorrelation function obeys the equations

$$\begin{aligned} \frac{d}{d\tau}\langle\sigma_+(t)\pi_x(t+\tau)\rangle &= -\gamma_x\langle\sigma_+(t)\pi_x(t+\tau)\rangle, \\ \frac{d}{d\tau}\langle\sigma_+(t)\pi_+(t+\tau)\rangle &= A\langle\sigma_+(t)\pi_+(t+\tau)\rangle \\ &\quad + B\langle\sigma_+(t)\pi_-(t+\tau)\rangle \\ &\quad + C\langle\sigma_+(t)\rangle, \end{aligned} \tag{5.8}$$

where γ_x, A, B, and C are given in (5.5). These equations must be solved by taking as initial conditions ($\tau = 0$) the values obtained in the stationary state from the precedent analysis. To obtain the spectrum from these equations is a simple task, but we omit here the final results since they lead to involved expressions. We plot the results in Fig. 3, where we compare the infinite-band squeezing (solid line), the finite band (dashed line), and the free-space

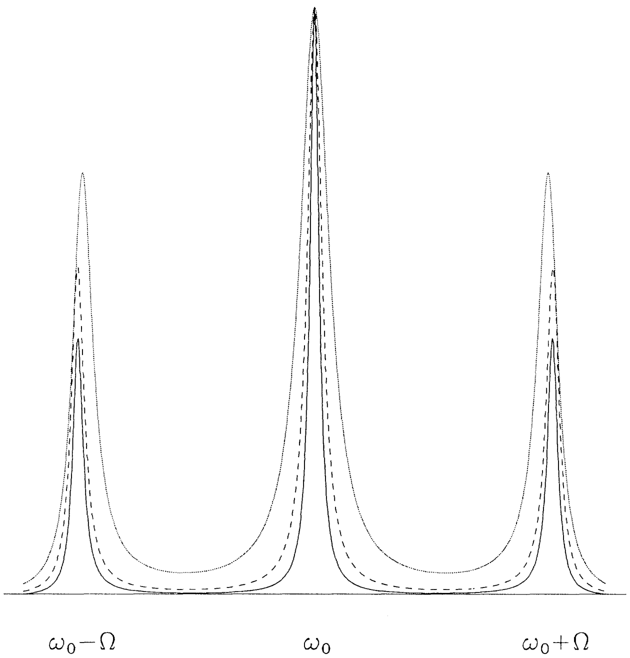

FIG. 3. Fluorescence spectrum of an atom damped by a squeezed reservoir with $\lambda/\mu = 1.5$ and (1) in free space and infinite bandwidth squeezing, (2) in a cavity ($\Gamma/\gamma_c = 30$) and infinite bandwidth squeezing, and (3) in the same cavity and finite bandwidth squeezing ($\lambda/\gamma_c = 30$). The spectra have been normalized in order that their maximum values coincide.

case (dotted line) for $\lambda/\mu = 4$. Note that Mollow's sidebands are much narrower for the finite-band case. This is not surprising since, as it is well known, the width of the central band is γ_x and the width of the sidebands is γ_R. As mentioned above, the effect of the finite-band squeezing improves the inhibition of the decay of the atom for M positive and, therefore, the decay constants γ_x and γ_R become smaller.

VI. CONCLUSIONS

In this work we have studied the behavior of a single two-level atom driven by a very intense laser field and damped by a squeezed reservoir in a cavity. We have shown that the effects of the finite bandwidth of the reservoir and the cavity produce strong modifications in comparison with the case where the bandwidths are infinite. These modifications can be predicted by decomposing the electromagnetic field inside the cavity in free and source part, and studying the spectrum of fluctuations of both parts. To check these predictions we have derived a model accounting for finite (but broad) bandwidths. We have shown that in free space, the effect of the finite band of the squeezing in the dressed population inversion is to decrease the effect of the squeezing; we have also shown that the decay of the dressed polarizations can be reduced to half that obtained in the normal vacuum. In the case where the atom is placed in a cavity, it is shown that one can decouple the atom from the cavity by squeezing strongly the E_x quadrature of the electromagnetic field and pumping the atom with a very strong laser field. However, in this case, the first condition imposes very intense laser fields. We show that for finite values of the squeezing bandwidth, this last condition is relaxed.

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APPENDIX: COMPARISON WITH THE WORK OF PARKINS AND GARDINER

In this appendix we compare, for a particular case, the results given by Eq. (3.5) with the exact numerical ones

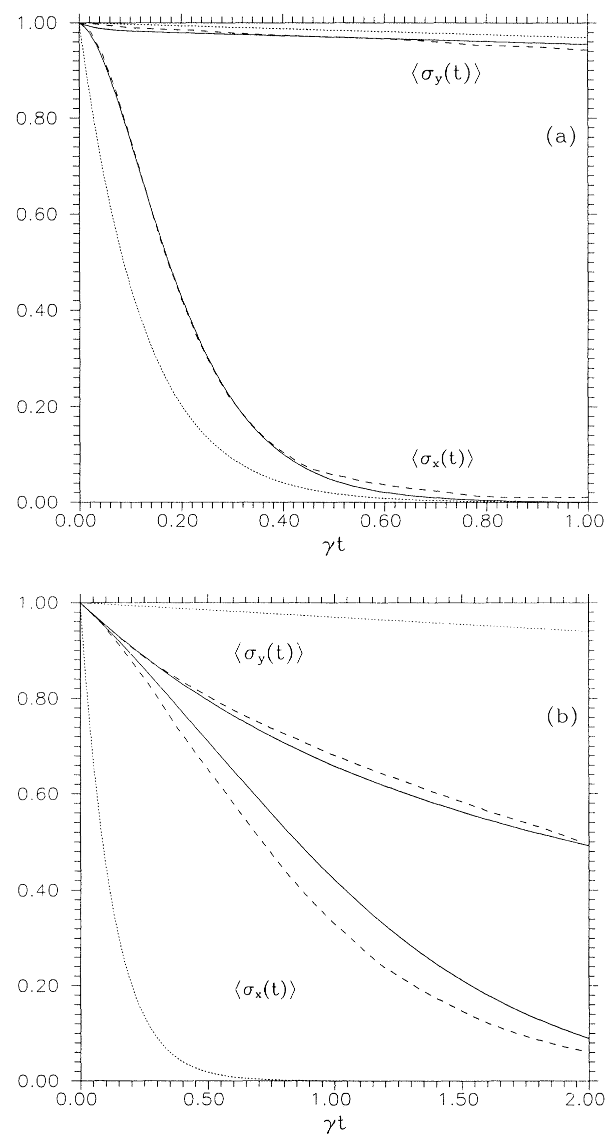

FIG. 4. Decay of the atomic mean values for (a) $\lambda/\gamma = 32$, $\mu/\gamma = 8$ and (b) $\lambda/\gamma = 0.8$, $\mu/\gamma = 0.2$ given Eq. (A2) (solid line), Eq. (A4) (dotted line), and the numerical simulation of Ref. [6] (dashed line).

given by Parkins and Gardiner [6]. In their work they study the interaction in the free space of a single two-level atom with a finite-band squeezed reservoir in the absence of a driving field. For this case Eq. (3.5) gives

$$\frac{d}{dt}\langle\sigma_x\rangle = -2\int_0^t d\tau[N(\tau)+M(\tau)]\langle\sigma_x\rangle,$$
$$\frac{d}{dt}\langle\sigma_y\rangle = -2\int_0^t d\tau[N(\tau)-M(\tau)]\langle\sigma_y\rangle, \tag{A1}$$
$$\frac{d}{dt}\langle\sigma_z\rangle = -4\int_0^t d\tau N(\tau)\langle\sigma_z\rangle - 2\int_0^t d\tau G(\tau).$$

Substituting $g(\omega)$ given in (4.1) in this equation we obtain

$$\frac{d}{dt}\langle\sigma_x\rangle = -\gamma_x(t)\langle\sigma_x\rangle,$$
$$\frac{d}{dt}\langle\sigma_y\rangle = -\gamma_y(t)\langle\sigma_y\rangle, \tag{A2}$$
$$\frac{d}{dt}\langle\sigma_z\rangle = -[\gamma_y(t)+\gamma_x(t)]\langle\sigma_z\rangle - \gamma,$$

where

$$\gamma_x(t) = \frac{\gamma}{2}\left(\frac{\lambda^2}{\mu^2}(1-e^{-\mu t}) + e^{-\mu t}\right),$$
$$\gamma_y(t) = \frac{\gamma}{2}\left(\frac{\mu^2}{\lambda^2}(1-e^{-\lambda t}) + e^{-\lambda t}\right). \tag{A3}$$

In the infinite bandwidth case these equations become

$$\frac{d}{dt}\langle\sigma_x\rangle = -\gamma(N+M+\tfrac{1}{2})\langle\sigma_x\rangle,$$
$$\frac{d}{dt}\langle\sigma_y\rangle = -\gamma(N-M+\tfrac{1}{2})\langle\sigma_y\rangle, \tag{A4}$$
$$\frac{d}{dt}\langle\sigma_z\rangle = -\gamma(2N+1)\langle\sigma_z\rangle - \gamma,$$

where N and M have been defined in (4.3), in agreement with the results of Ref. [1].

Note first that for very short times ($\mu t, \lambda t \sim 0$) the effect of the squeezing is negligible and the atom behaves as if it were embedded in a vacuum state. For very long times ($t \gg 1/\mu$, $1/\lambda$) we recover the infinite bandwidth limit. However, before reaching this time, the state of the atom may have changed substantially and Eqs. (A4) give a wrong result. In order for these equations to be valid, it is necessary that $\gamma \ll \mu^3/\lambda^2$, λ^3/μ^2, i.e., the atom almost remains in its initial state after $t \sim \lambda^{-1}$, μ^{-1}. In this case the equations obtained are the same as in Eq. (A4).

In Fig. 4 we have plotted $\langle\sigma_x\rangle$ and $\langle\sigma_y\rangle$ given by (A2) (solid line), the simulation theory (dashed line), and the infinite-band squeezing theory (dotted line) for several values of λ/γ and μ/γ and $M < 0$. Note that we have not plotted $\langle\sigma_z\rangle$ since it decays with the sum of the decay constants for the x and y components [see Eq. (A2)]; and therefore the agreement of the simulation result with these components ensures the agreement with the $\langle\sigma_z\rangle$. For the moderate values of the squeezing bandwidth which have been used in Figs. 2 and 3 [Fig. 4(a)] it is clear that our approach is in very good, agreement with the exact result. The white-noise result gives a slight result but still qualitatively good. For smaller values of the squeezing bandwidth the agreement is still good though for the $\langle\sigma_x\rangle$ component, it is a little worse [Fig. 4(b)]. This is due to the fact that, for this component, the times for which the Born approximation is valid are shorter, and therefore for intermediate times there is a slight difference.

Finally, it is worth mentioning that Parkins and Gardiner also introduce several analytical approximations and compare them with the simulated result, finding good agreement. However, the main misfit comes from the different values predicted by their analytical approach and the simulation method for short times. In our approach it is precisely for these short times where the theory is best suited. For long times and in the limits mentioned above, it is expected that their results and ours coincide.

[1] C. W. Gardiner, Phys. Rev. Lett. **56**, 1917 (1986).
[2] H. J. Carmichael, A. S. Lane, and D. F. Walls, Phys. Rev. Lett. **58**, 2539 (1987); J. Mod. Opt. **34**, 821 (1987).
[3] M. J. Collet and C. W. Gardiner, Phys. Rev. A **30**, 1386 (1984).
[4] H. Ritsch and P. Zoller, Phys. Rev. Lett. **61**, 1097 (1988); Phys. Rev. A **38**, 4657 (1988).
[5] A. S. Parkins, Phys. Rev. A **42**, 4352 (1990).
[6] A. S. Parkins, and C. W. Gardiner, Phys. Rev. A **37**, 3867 (1988).
[7] C. W. Gardiner, A. S. Parkins, and M. J. Collet, J. Opt. Soc. Am. B **4**, 1683 (1987).
[8] E. M. Purcel, Phys. Rev. **69**, 681 (1946).
[9] D. Kleppner, Phys. Rev. Lett. **47**, 233 (1981).
[10] M. Lewenstein, T. W. Mossberg, and R. J. Glauber, Phys. Rev. Lett. **59**, 775 (1987); M. Lewenstein and T. W. Mossberg, Phys. Rev. A **37**, 2048 (1988). Note that there is a factor 2 between our definition of γ and theirs.
[11] P. Goy, J. M. Raimond, M. Gross, and S. Haroche, Phys. Rev. Lett. **50**, 1903 (1983); R. G. Hulet, E. S. Hilfer, and D. Kleppner, *ibid.* **55**, 2137 (1985); W. Jhem A. Anderson, E. A. Hind, D. Meschede, L. Moi, and S. Haroche, *ibid.* **58**, 666 (1987); D. Heinzen, J. J. Childs, J. E. Thomas, and M. S. Feld, *ibid.* **58**, 1320 (1987).
[12] J. M. Courty and S. Reynaud, Opt. Commun. **72**, 93 (1989), and references therein.
[13] J. M. Courty and S. Reynaud, Europhys. Lett. **10**, 237 (1989).
[14] J. Dalibard, J. Dupont-Roc, and C. Cohen-Tannoudji, J. Phys. (Paris) **43**, 1617 (1982); **45**, 637 (1984).
[15] C. W. Gardiner, *Handbook of Stochastic Methods* (Springer, Berlin, 1975).