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ARTICLES

Quantum Zeno effect and quantum Zeno paradox in atomic physics

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Itano and co-workers [Wayne M. Itano, D. J. Heinzen, J. J. Bollinger, and D. J. Wineland, Phys. Rev. A **41**, 2295 (1990)] have recently reported the experimental verification of the quantum Zeno effect, which is the inhibition of a quantum transition by frequent measurements. In this article, we offer an alternative interpretation of the quantum Zeno effect. We show that an analysis of the dynamics of the full three-level system gives the same result. There is no need to assume explicitly that the wave function has collapsed, nor even to assume that an ideal measurement has been made. In addition, we differentiate between what has been referred to as the quantum Zeno effect and what has been termed the quantum Zeno paradox. The former is the inhibition of induced transitions, and the latter is the, as yet experimentally unobserved, inhibition of spontaneous decay. Our interpretation, which emphasizes the "measurement"-induced interruption of atomic-state coherences as the cause of inhibited quantum transitions, suggests a resolution to the quantum Zeno paradox. The theoretical limit of continuous observation is discussed.

In a recent paper, Itano *et al.* [1] demonstrate the quantum Zeno effect, which is the inhibition of quantum transitions by frequent measurements. The authors consider the probability that a transition from an atomic ground state (level 1) to an excited metastable state (level 2) is induced by a resonant "π" pulse (signal pulse), a pulse that, in the absence of any other applied fields and neglecting spontaneous decay, produces unit probability of excitation of level 2. The atom is simultaneously subjected to a number of additional pulses, equally spaced in time, which strongly couple the ground state to a second level (level 3), from which there is spontaneous decay to level 1. (See Fig. 1.) The claim is that these short pulses constitute true measurements on the 1-2 system [1]. If photons are seen to scatter by spontaneous decay from level 3, then it can be deduced that the system is in state 1 prior to the short pulse. If, however, no scattered photons are produced, then the system must be in the metastable state 2. The authors compute the final transition probability to state 2 by assuming that each strong pulse in this way constitutes an ideal measurement. The wave function is said to "collapse" to one of the unperturbed levels, 1 or 2, hence the coherence that describes the degree of superposition of these states is identically zero. In this way the transition probability is seen to decrease as the number of "measurements" increases.

The purpose of this article is twofold. In the first part, we carry out a direct calculation of $\rho_{22}(T)$, the probability to be in the metastable state, following the pulse sequence described above. It is assumed that the system evolves unobserved up to time T (as in the actual experiment [1]) and that, a time T, the population $\rho_{22}(T)$ is measured by the application of a probe pulse. The results of Itano *et al.* [1] are recovered, but our interpretation of the results differs from theirs. They argue that the fluorescence photons emitted from state 3 (which bring the atom back to state 1) are recorded in the electromagnetic field and constitute a measurement whether or not they are actually observed. We adopt the point of view that a measurement occurs only at the time some irreversible process has been used to record the state of the system. In the second section of the paper, we discuss the related but not identical quantum Zeno paradox. The quantum Zeno paradox, as originally discussed in relation to bubble-chamber experiments, refers to the seemingly contradictory fact that the "continuous" observation of an unstable particle in no way modifies the particle's lifetime. We introduce an atomic analog to this bubble-chamber experiment and are able to resolve the paradox by a careful examination of the atomic-state coherence. This key role played by atomic-state coherence also provides a link between the quantum Zeno effect and the quantum Zeno paradox.

The full density-matrix equations of motion for this three level atom interacting with two resonant cw fields are

$$\dot{\rho}_{11}=\frac{-i\chi_1}{2}(\tilde{\rho}_{21}-\tilde{\rho}_{12})-\frac{i\chi_2}{2}(\tilde{\rho}_{31}-\tilde{\rho}_{13})+\gamma_3\rho_{33}+\gamma_2\rho_{22}\ , \tag{1a}$$

$$\dot{\rho}_{22}=\frac{-i\chi_1}{2}(\tilde{\rho}_{12}-\tilde{\rho}_{21})-\gamma_2\rho_{22}\ , \tag{1b}$$

$$\dot{\rho}_{33}=\frac{-i\chi_2}{2}(\tilde{\rho}_{13}-\tilde{\rho}_{31})-\gamma_3\rho_{33}\ , \tag{1c}$$

$$\dot{\tilde{\rho}}_{12}=\frac{-i\chi_1}{2}(\rho_{22}-\rho_{11})-\frac{i\chi_2}{2}(\tilde{\rho}_{32})-\gamma_{12}\tilde{\rho}_{12}\ , \tag{1d}$$

$$\dot{\tilde{\rho}}_{23}=\frac{i\chi_2}{2}(\tilde{\rho}_{21})-\frac{i\chi_1}{2}(\tilde{\rho}_{13})-\gamma_{23}\tilde{\rho}_{23}\ , \tag{1e}$$

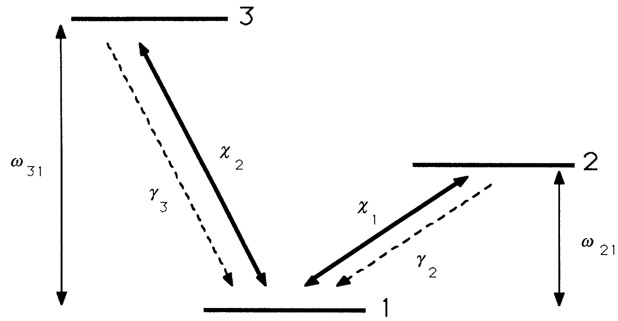

FIG. 1. The three-level configuration considered by Itano *et al.* Levels 3 and 2 have resonance frequencies ω_{20} and ω_{10} and spontaneous decay rates γ_3 and γ_2, respectively. The Rabi frequencies χ_2 and χ_1 are proportional to the amplitudes of the applied fields. We assume that $\chi_2,\gamma_3 \gg \chi_1,\gamma_2$.

$$\dot{\tilde{\rho}}_{13}=\frac{-i\chi_1}{2}(\tilde{\rho}_{23})-\frac{i\chi_2}{2}(\rho_{33}-\rho_{11})-\gamma_{13}\tilde{\rho}_{13} , \tag{1f}$$

where we have transformed to a field interaction representation,

$$\begin{aligned} \tilde{\rho}_{12}&=e^{-i\omega_1 t}\rho_{12} ,\\ \tilde{\rho}_{13}&=e^{-i\omega_2 t}\rho_{13} ,\\ \tilde{\rho}_{23}&=e^{-i(\omega_2-\omega_1)t}\rho_{23} , \end{aligned} \tag{2}$$

and the usual rotating-wave approximation has been made. The Rabi frequencies for the weak and strong transitions are χ_1 and χ_2, the spontaneous decay rates from states 2 and 3 are γ_2 and γ_3, $\gamma_{ij}=(\gamma_i+\gamma_j)/2$, and ω_1 and ω_2 are the laser frequencies, which are equal to the level separations ω_{10} and ω_{20}. (In the experiment described by Itano *et al.* [1], the 1-2 transition is in the rf rather than in the optical range; however, this does not change the level dynamics of the problem). In the closed system considered here, there are five independent equations, characterized by four parameters. A full, analytical solution is not easily obtained. However, the fact that

$$\chi_2,\gamma_3 \gg \chi_1,\gamma_2 \tag{3}$$

leads to a great simplification. Treating χ_1 as a perturbation, and using an adiabatic elimination procedure to reexpress density-matrix elements which evolve rapidly to a quasisteady state, one obtains simple rate equations for the level populations. This derivation is in analogy with that of Kimble, Cook, and Wells [2], who solve the resulting equations in steady state in order to describe intermittent atomic fluorescence. Here we retain the time-dependent solutions for the populations $\rho_{11}(t)$, $\rho_{22}(t)$, and $\rho_{33}(t)$, and for the coherence $\tilde{\rho}_{12}(t)$.

If we consider time scales long compared with γ_3^{-1} but short compared with χ_1^{-1}, we may solve for the density-matrix elements describing the strong transition in steady state, to zeroth order in χ_1. In terms of the density-matrix elements representing the strong transition alone, this solution is [2]

$$\begin{aligned} \tilde{\rho}_{13}^{(0)}&=\frac{i\chi_2\gamma_3}{2\chi_2^2+\gamma_3^2}S_{33,11}^{(0)}(t) ,\\ W_{33,11}^{(0)}&=\frac{-\gamma_3^2}{2\chi_2^2+\gamma_3^2}S_{33,11}^{(0)}(t) , \end{aligned} \tag{4}$$

where $W_{33,11}=\rho_{33}-\rho_{11}$ and $S_{33,11}=\rho_{33}+\rho_{11}$. Equation (1e) for the coherence $\tilde{\rho}_{23}$ to first order in the signal field is

$$\dot{\tilde{\rho}}_{23}^{(1)}=\frac{i\chi_2}{2}\tilde{\rho}_{21}^{(1)}-\frac{i\chi_1}{2}\tilde{\rho}_{13}^{(0)}-\frac{\gamma_3}{2}\tilde{\rho}_{23}^{(1)} . \tag{5}$$

Integrating by parts, we obtain the solution

$$\tilde{\rho}_{23}^{(1)}=\frac{i\chi_2}{\gamma_3}\tilde{\rho}_{21}^{(1)}(t)-\frac{i\chi_1}{\gamma_3}\tilde{\rho}_{13}^{(0)}(t) -\frac{2i\chi_2}{\gamma_3^2}\dot{\tilde{\rho}}_{21}^{(1)}+\frac{2i\chi_1}{\gamma_3^2}\dot{\tilde{\rho}}_{13}^{(0)}+\cdots , \tag{6}$$

where the terms dependent on time derivatives of $\tilde{\rho}_{21}$ and $\tilde{\rho}_{13}$ are very small as $\tilde{\rho}_{21}$ and $\tilde{\rho}_{13}$ are, on the time scales considered, seen to be in the quasistatic regime. The quantity of interest is $\rho_{22}(t)$, the population, of the metastable state, to second order in the signal field. From Eqs. (1) this clearly depends on the coherence $\tilde{\rho}_{12}$, which in turn depends on the coherence $\tilde{\rho}_{23}$.

To first order in χ_1, the equation for $\tilde{\rho}_{12}$ is

$$\dot{\tilde{\rho}}_{12}^{(1)}=\frac{-i\chi_1}{2}(\rho_{22}^{(0)}-\rho_{11}^{(0)})-\left[\frac{\gamma_2}{2}+\frac{\chi_2^2}{2\gamma_3}\right]\tilde{\rho}_{12}^{(1)} -\frac{i\chi_2^2\chi_1}{2(\gamma_3^2+2\chi_2^2)}S_{33,11}^{(0)} , \tag{7}$$

where Eq.(6) or $\tilde{\rho}_{23}$ has been substituted into Eq. (1d). The key point is that $\tilde{\rho}_{12}$ now relaxes with the effective rate

$$\Gamma_{\text{eff}}=\frac{1}{2}\left[\gamma_2+\frac{\chi_2^2}{\gamma_3}\right] , \tag{8}$$

which is much greater than the usual transverse rate $\gamma_2/2$. From the above analysis it is clear that the rapid transverse decay enters from the coupling of the two transitions through $\tilde{\rho}_{23}$. This fast transverse relaxation fulfills the consistency requirement that $\dot{\tilde{\rho}}_{21}$ terms may be neglected in Eq. (6) for the solution for $\tilde{\rho}_{23}(t)$. Finally, the quasi-steady-state solution for $\tilde{\rho}_{12}$ is substituted into Eqs. (1a)–(1c), and the rate equations governing the evolution of the populations are found to be

$$\dot{\rho}_{22}=-R_1\rho_{22}+R_2S_{33,11} ,$$
$$\dot{S}_{33,11}=+R_1\rho_{22}-R_2S_{33,11} , \tag{9}$$

where the rates are

$$R_1=\left[\frac{\chi_1^2}{2\Gamma_{\text{eff}}}+\gamma_2\right] ,$$
$$R_2=\left[\frac{1}{2\Gamma_{\text{eff}}}\left[\frac{\gamma_3^2\chi_1^2}{(2\chi_2^2+\gamma_3^2)}\right]\right] . \tag{10}$$

This is in agreement with Kimble, Cook, and Wells [2] in the limit of zero detuning. The time-dependent solutions for the populations are

$$\rho_{22}(t)=\left[\frac{R_2+R_1e^{-(R_1+R_2)t}}{R_1+R_2}\right]\rho_{22}(0)+\left[\frac{R_2-R_2e^{-(R_1+R_2)t}}{R_1+R_2}\right]S_{33,11}(0) ,$$
$$S_{33,11}(t)=\left[\frac{R_1-R_1e^{-(R_1+R_2)t}}{R_1+R_2}\right]\rho_{22}(0)+\left[\frac{R_1+R_2e^{-(R_1+R_2)t}}{R_1+R_2}\right]S_{33,11}(0) , \tag{11}$$

and the corresponding time-dependent equations for the coherences $\tilde{\rho}_{12}$ and $\tilde{\rho}_{21}$ are easily obtained since, for $\Gamma_{\text{eff}}>>\chi_1,\gamma_2$, they adiabatically follow the populations.

In the free-evolution periods, during which the signal field only acts, the development of $\rho_{22},\rho_{11},\tilde{\rho}_{12}$ is most easily stated in terms of the rotation of the Bloch vector $\mathbf{B}(u,v,w)$ in state space, where

$$u=\tilde{\rho}_{12}+\tilde{\rho}_{21} ,$$
$$v=i(\tilde{\rho}_{12}-\tilde{\rho}_{21}) , \tag{12}$$
$$w=\rho_{22}-\rho_{11} .$$

The solution is simply $\mathbf{B}(t)=\underline{M}\mathbf{B}(t_0)$, where

$$\underline{M}=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\chi_1(t-t_0) & \sin\chi_1(t-t_0)\\ 0 & -\sin\chi_1(t-t_0) & \cos\chi_1(t-t_0)\end{bmatrix} . \tag{13}$$

The rate γ_2 is so small that it is neglected over these time scales. To solve the general problem posed by this experiment in which a number n of equally spaced strong pulses resonant with the 1-3 transition are superimposed on the long π pulse resonant with the 1-2 transition, we simply consider the two solutions above, and match boundary conditions.

At the end of a free-evolution period of length $\pi/n\chi_1$, the density-matrix elements computed with the use of Eq. (13) are given by

$$\rho_{22}=[1-\cos(\pi/n)]/2 ,$$
$$\rho_{11}=[1+\cos(\pi/n)]/2 , \tag{14}$$
$$\tilde{\rho}_{12}=[-i\sin(\pi/n)]/2 ,$$

where we have assumed that the atom is initially in its ground state, $\mathbf{B}(t_0=0)=(0,0,-1)$. After the first "measurement pulse" of length τ_p we have, for the populations,

$$\rho_{22}(\tau_p+\pi/n\chi_1)=\left[\frac{R_2+R_1e^{-(R_1+R_2)\tau_p}}{R_1+R_2}\rho_{22}(\pi/n\chi_1)\right]+\left[\frac{R_2-R_2e^{-(R_1+R_2)\tau_p}}{R_1+R_2}S_{33,11}(\pi/n\chi_1)\right] . \tag{15}$$

Taking the following as plausible values of the parameters consistent with Eq. (3), $\gamma_3=10^8$ sec^{-1}, $\chi_1=10$ sec^{-1}, $\chi_2=10^9$ sec^{-1}, $\gamma_2\sim0$, and $\tau_p=10^{-3}$ sec, we have $(R_1+R_2)\tau_p\cong\chi_1^2\tau_p/\Gamma_{\text{eff}}\cong\chi_1^2\gamma_3\tau_p/\chi_2^2\cong10^{-11}$. Clearly the populations do not evolve significantly over τ_p, and Eq. (15) can be simply rewritten as

$$\rho_{22}(\tau_p+\pi/n\chi_1)\cong\rho_{22}(\pi/n\chi_1)+O\left[\frac{\chi_1^2\tau_p}{\Gamma_{\text{eff}}}\right] . \tag{16}$$

The coherences $\tilde{\rho}_{12}$ and $\tilde{\rho}_{21}$ are obtained from the quasi-steady-state solution of Eq. (7),

$$\tilde{\rho}_{12}(t)=\tilde{\rho}_{12}(\pi/n\chi_1)e^{-\Gamma_{\text{eff}}\tau_p}-\frac{i\chi_1}{2\Gamma_{\text{eff}}}\rho_{22}(t)+\frac{i\chi_1}{2\Gamma_{\text{eff}}}\left[\frac{\gamma_3^2}{2\chi_2^2+\gamma_3^2}\right]S_{33,11}(t)+\cdots , \tag{17}$$
$$\tilde{\rho}_{21}(t)=\text{c.c.} ,$$

where $t=\pi/n\chi_1+\tau_p$. The last two terms reflect the slow adiabatic following of the populations. These terms are small, of order $\chi_1/\Gamma_{\text{eff}}$. The important result is in the first term; any coherence that had developed so far is rapidly damped in the short time Γ_{eff}^{-1}. After the next free-evolution time, the solution for $\rho_{22}(\tau_p+2\pi/n\chi_1)$ is

$$[1-\cos^2(\pi/n)]/2+O(\chi_1^2\tau_p/\Gamma_{\text{eff}}) . \tag{18}$$

It is clear that after any number of such pulses, the solution is $[1-\cos^n(\pi/n)]/2+\cdots$, which is the solution derived by Itano *et al.* [1], up to small terms of the order $\chi_1^2\tau_p/\Gamma_{\text{eff}}$.

Although the results are essentially identical to those obtained by Itano *et al.* [1], this does not necessarily support their contention that the "measurement" pulses result in wave-function collapse. We prefer to interpret the results solely in terms of the dynamics of the atom plus field system. Rather than wave-function collapse, it is the rapid decay of the coherence $\tilde{\rho}_{12}$ that is caused by each strong pulse which results in the dramatic reduction in population for state 2 following the π pulse.

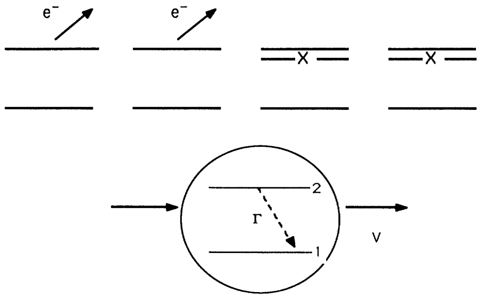

FIG. 2. The atomic analog of a bubble chamber. An initially excited atom moving with velocity v passes through an array of "detectors", which are atoms excited very near to ionization. The detectors eject photoelectrons as the excited atom passes them, but not if a ground-state atom passes.

Having obtained this dynamical solution it is instructive to examine the limiting case as intermittent measurements go over to continuous observation. Misra and Sudarshan [3] have defined the appropriate limit for observing the Zeno effect as a succession of instantaneous measurements in the limit that the intervals between measurements approach zero. In other words, this is the limiting case in which periods of free evolution are separate from periods of measurement. Applying our result that the strong field causes a fast relaxation of $\tilde{\rho}_{12}$ at each strong pulse, it is clear that the population of the metastable state does remain zero so long as the measurement field is strong enough.

One might expect that the limit of continuous observation can also be reached discontinuously, that is, when both the measurement and signal fields are applied for the duration of the π pulse. This certainly seems plausible from a qualitative standpoint, assuming that observations of fluorescence continuously collapse the wave function; however, such a conclusion is not warranted. If the inhibition of induced transitions were caused by wave-function collapse, then for two cw fields, this inhibition should persist indefinitely. Yet, it is shown in "quantum jump" experiments [4] performed in similar three-level configurations, that in steady state the metastable level may be occupied for a significant fraction of the time. Kimble, Cook, and Wells [2] have shown that a steady-state population ρ_{22} equal to $\frac{1}{3}$ can be achieved in this limit [5].

The quantum Zeno effect refers to the inhibition of induced transitions by repeated measurement, whereas the quantum Zeno paradox refers to the analogous inhibition of spontaneous decay. In order to discuss the quantum Zeno paradox and its relation to the quantum Zeno effect, we construct an atomic analog of the bubble-chamber experiment analyzed by Misra and Sudarshan. Consider the "bubble chamber" to consist of an array of "detector" atoms excited very nearly to ionization. We assume that the atom under investigation has two levels separated in frequency by ω_0, and that it is initially prepared in excited state 2 with some translational kinetic energy. This atom decays to state 1 owing to its interaction with the vacuum field. As the excited-state particle moves by the detectors it provides the small amounts of energy required to ionize the detector atoms and the energy of state 2 is perturbed. (See Fig. 2.) We assume that interaction with the ground state is not sufficient to cause ionization. The analogy with a particle in a bubble chamber is clear. The excited atom corresponds to an unstable particle, and spontaneous emission to the ground state corresponds to particle decay into its stable by-products.

The Hamiltonian describing the system is

$$H = H_{\text{atom}} + H_{\text{field}} - \boldsymbol{\mu}\cdot\mathbf{E}_{\text{vac}} + V_{\text{int}} , \tag{19}$$

where H_{atom} is the free-atom Hamiltonian, H_{field} is the free-field Hamiltonian, $-\boldsymbol{\mu}\cdot\mathbf{E}_{\text{vac}}$ is the atom-vacuum (field interaction) Hamiltonian, and V_{int} is the Hamiltonian describing the interaction of the atom with the detectors. We model this interaction, which affects only the upper state, as

$$V_{\text{int}} = \sum_n V_{22}(n) f(t - t_n) , \tag{20}$$

where $f(\tau)$ is sharply peaked at $\tau=0$, $V_{22}(n)$ is some arbitrary function of n, and atom-detector collisions occur at instants t_n, which are separated by some average interval T.

The probability amplitudes b_2 for the atom to be in state 2 with no photons in the field and b_{1k} for the atom to be in state 1 with a photon of type k present in the field obey the evolution equations in the interaction representation

$$\dot{b}_2 = (i\hbar)^{-1} \sum_{\mathbf{k}} H_{2;1k} e^{-i(\omega_k - \omega_0)t} \times \exp\left[-i\int V_{\text{int}}(t')dt'\right] b_{1k} , \tag{21a}$$

$$\dot{b}_{1k} = (i\hbar)^{-1} H_{1k;2} e^{i(\omega_k - \omega_0)t} \times \exp\left[i\int V_{\text{int}}(t')dt'\right] b_2 . \tag{21b}$$

We first solve Eqs. (21) in the limit $V_{\text{int}}=0$. So long as the emitted photons do not act back on the system an approach such as that of Weisskopf and Wigner [6] can be used to obtain the approximate solution $b_2(t) = e^{-\Gamma_2 t}$. Inserting this result in Eq. (21b) and integrating,

$$b_{1k}(t) = (i\hbar)^{-1} \int_0^t H_{1k;2} e^{i(\omega_k - \omega_0)t'} e^{-(\Gamma/2)t'} dt' , \tag{22}$$

where the initial conditions are $b_2(0)=1$ and $b_{1k}(0)=0$. The emission probability $I(t)$ is found by squaring this amplitude and summing over all photon modes:

$$I(t) = \sum_{\mathbf{k}} |b_{1k}|^2 = \sum_{\mathbf{k}} \frac{1}{\hbar^2} |H_{1k;2}|^2 \int_0^t dt' e^{i(\omega_k - \omega_0)t'} e^{-(\Gamma/2)t'} \int_0^t dt'' e^{-i(\omega_k - \omega_0)t''} e^{-(\Gamma/2)t''} . \tag{23}$$

Replacing the summation over k by an integral, we have, after some rearrangement

$$I(t)=\frac{1}{\hbar^2}\int_0^{\infty} d\omega_k \mathcal{D}(\omega_k)|H_{1k;2}(\omega_k)|^2 \int_0^t dt' e^{-\Gamma t'} \int_{t'-t}^{t'} d\tau\, e^{i(\omega_k-\omega_0)\tau} e^{(\Gamma/2)\tau} , \tag{24}$$

where $\mathcal{D}(\omega_k)$ is the density of final states $V\omega_k^2/(\pi^2 c^3)$, and where $t>t'>0$. As in the usual Weisskopf-Wigner approach [7], we assume that $\mathcal{D}(\omega_k)$ and $H(\omega_k)$ vary little about ω_0 for which the integral is large, giving

$$I(t)=\frac{1}{\hbar^2}\mathcal{D}(\omega_0)|H_{1k;2}(\omega_0)|^2 \int_0^{\infty} d\omega_k \int_0^t dt' e^{-\Gamma t'} \int_{t'-t}^{t'} d\tau\, e^{-i(\omega_k-\omega_0)\tau} e^{(\Gamma/2)\tau} . \tag{25}$$

Performing the frequency integral first, one finds a delta function in time $\delta(\tau)=\delta(t'-t'')$ and integrating over time, one obtains the emission probability $I(t)=(1-e^{-\Gamma t})$.

It is instructive to interpret Eq. (23) in terms of double-sided Feynman diagrams [8]. The atom in some sense "sees" all possible paths in which it jumps from the excited state to the ground state at some instant t. The integrand in expressions (23)–(25) represents the superposition of all possible pairs of such paths [see Fig. 3(a)]. In evaluating Eq. (25) we have a δ function in time which shows that only diagrams of the form $t'=t''$ actually contribute [see Fig. 3(b)]. There is no observable effect on the upper state lifetime, arising from interference between different Feynman paths. This is a mathematical manifestation of the quantum jump nature of spontaneous decay. If we now include the perturbative effects of the detectors in the atomic bubble chamber we can show that the lifetime is still unaffected so long as the atom-detector interaction time is greater than the correlation time between the excited state and the vacuum. Typically, this vacuum correlation time $\tau_{c\,\mathrm{vac}}$, is extremely short; it is at most $1/\omega_0$, where $\omega_0 \simeq 10^{14}\ \mathrm{sec}^{-1}$ is the optical transition frequency of the atom. By including V_{int}, Eq. (23) is modified to read

$$I(t)=\sum_{\mathbf{k}} \frac{1}{\hbar^2}|H_{1k;2}|^2 \int_0^t dt' e^{i(\omega_k-\omega_0)t'} e^{-(\Gamma/2)t'} \exp\left[i\sum_n V_{22}(n)\int_0^{t'} f(t'''-t_n)dt''' \right] \times \int_0^t dt'' e^{-i(\omega_k-\omega_0)t''} e^{-(\Gamma/2)t''} \exp\left[-i\sum_n V_{22}(n)\int_0^{t''} f(t'''-t_n)dt''' \right] . \tag{26}$$

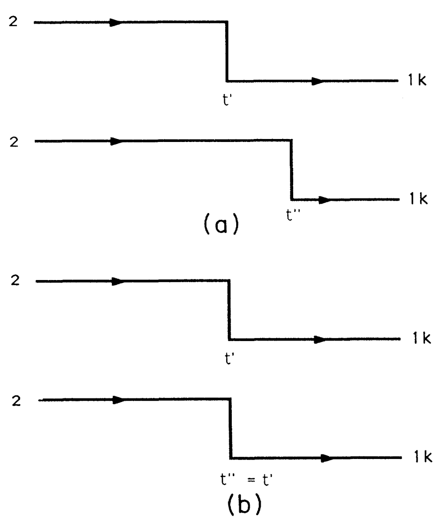

FIG. 3. Pairs of Feynman paths contributing to integrals of the form Eq. (23). Each path represents a jump, at some instant, from the state consisting of a two-level atom in its excited state with no photons in the field, to that in which the atom is in its ground state having emitted one photon of type k into the field. (a) Pairs of Feynman paths characterized by different jump times. (b) Only pairs of paths with equal jump times ($t''=t'$) contribute to the expression for the total fluorescence intensity. This is a mathematical manifestation of the quantum-jump nature of spontaneous decay.

Once again, we change the summation over photon modes to an integral and find that the integration over frequencies gives a δ function $\delta(t'-t'')$ in the Weisskopf-Wigner approximation. The result is that there is no modification of the lifetime since

$$\left[\exp\left[iV_{22}\int_0^{t'} f dt''' \right]\right] \left[\exp\left[-iV_{22}\int_0^{t'} f\, dt''' \right]\right] = 1 .$$

Again, this result can be linked to the fact that the correlation time for the vacuum field is much smaller than the atom-detector interaction time. In measuring the atomic decay time, one averages over all the modes of the vacuum field, which eliminates any coherence between states 2 and 1**k** in the Feynman diagrams of Fig. 3(a). For the measurements to modify the decay rate it would be necessary to have an atom-detector interaction time much less than the correlation time of the vacuum field. This conclusion has been reached by other authors [2,9,10], but from a somewhat different perspective.

Even though the upper-state lifetime is unaffected by the presence of the detectors the fluorescence *spectrum* of the radiation emitted by a test atom in the atomic bubble chamber may be modified. In terms of the spectrum, the coherences formed by overlapping Feynman paths do play a role. The spectrum is given by the probability to be in the lower atomic state with a photon of type k present. In the absence of atom-detector interactions, the spectrum is given by

$$I(t=\infty)=|b_{1k}(\infty)|^2=\frac{1}{\hbar^2}\int_0^\infty dt' H_{1k;2}e^{i(\omega_k-\omega_0)t'}e^{-(\Gamma/2)t'}\int_0^\infty dt'' H_{1k;2}e^{-i(\omega_k-\omega_0)t''}e^{-(\Gamma/2)t''}$$
$$=\frac{|H_{1k;2}|^2/\hbar^2}{(\Gamma/2)^2+(\omega_k-\omega_0)^2}\ , \tag{27}$$

giving the expected result that the spectrum is peaked at ω_0 with a width determined by the atomic lifetime Γ. The important point is that contributions to the integral Eq. (27) come from all pairs of paths of the form shown in Fig. 3(a). These coherences, arising from interference of paths characterized by different jump times, are affected by perturbations in the form of Eq. (20). In the presence of such perturbations, the fluorescence spectrum is broadened in a manner analogous to that of the traditional pressure broadening of spectral lines.

We may employ yet another approach in order to demonstrate the role of the ultrashort correlation time of vacuum fluctuations is removing the quantum Zeno effect from observations of spontaneous decay. In the usual application of quantum mechanics, the probability of finding that a system, initially prepared in some state $|\psi_0\rangle$, remains in that state at time t is given by

$$P_2(t)=|\langle\psi_0|e^{-iHt/\hbar}|\psi_0\rangle|^2\ , \tag{28}$$

where H is a Hermitian Hamiltonian. In particular for the bubble chamber example, $|\psi_0\rangle$ is the upper atomic state with zero photons in the field and H is the Hamiltonian described above. The probability that the system is found to remain in state $|\psi_0\rangle$ given that it is in the state $|\psi_0\rangle$ at n instants $(t_1,t_2,\ldots,t_n)$ is [11]

$$P_{2,n}(t)=|\langle\psi_0|e^{-iHt_1/\hbar}|\psi_0\rangle|^2 \times|\langle\psi_0|e^{-iH(t_2-t_1)/\hbar}|\psi_0\rangle|^2\cdots \times|\langle\psi_0|e^{-iH(t-t_n)/\hbar}|\psi_0\rangle|^2\ , \tag{29}$$

where we prefer to interpret this as a joint probability of observing a particular outcome, and on that the wave function has actually collapsed at each intermediate time. In general, P_2 does not equal $P_{2,n}$. Misra and Sudarshan [3] have rigorously proved that in the limit $n\to\infty$, the joint probability $P_{2,n}$ approaches 1, provided that H is a Hermitian and semibounded operator. This is a precise restatement of the quantum Zeno effect.

From this analysis it is tempting to conclude that a particle should be inhibited from decay if it is frequently observed. Certainly, the Hamiltonian governing the interaction of the excited-state atom with the vacuum field modes is Hermitian and has a spectrum bounded from below as required by the theorem of Misra and Sudarshan [3]. It would seem to follow that the probability to be in the undecayed state after passing n detectors is given by an expression of the form Eq. (29), indicating a longer lifetime for the "observed" particle, in contradiction to all experimental data on spontaneous decay.

The resolution to this seeming paradox is that, given the ultrashort correlation time of vacuum fluctuations, in making our observation of the system we have implicitly averaged over vacuum variables. The quantities we are actually measuring correspond to atomic states only. The equation of motion for the reduced density matrix which represents these states is

$$\dot\rho_{ij}=(i\hbar)^{-1}[H,\rho]_{ij}-\Gamma_{ij}\rho_{ij}\ , \tag{30}$$

where the Γ_{ij} are decay constants. The important point is that averaging over vacuum field modes has yielded these irreversible damping terms Γ_{ij} in the equations for the reduced density matrix. Evolution is no longer governed by a Hermitian Hamiltonian and the theorem of Misra nd Sudarshan [3] no longer applies. In the absence of an incident field the evolution of the excited-state probability is easily found from Eq. (30) to be $P_2(t)=e^{-\Gamma(t-t_0)}P_2(t_0)$, where Γ is the upper-state lifetime. What is uniquely true for exponential decay [11] is that

$$P_{2,n}=P_2(0)e^{-\Gamma t_0}e^{-\Gamma(t_1-t_0)}\cdots e^{-\Gamma(t-t_n)} =P_2(0)e^{-\Gamma t}=P_2(t)\ . \tag{31}$$

Therefore, so long as we have implicitly averaged over vacuum variables in making our observations of decay, the probability to remain undecayed is independent of the number of such observations we make. In other words, Eq. (31) shows that in this case we expect no manifestation of the quantum Zeno effect.

The quantum Zeno effect, as discussed by Itano *et al.* [1] differs from the problems posed by quantum jump paradox in that it is the induced rather than spontaneous transitions that are considered. The reason that induced transitions are inhibited, whereas spontaneous transitions are not, is again a matter of the time scales over which the measurements are performed. In the work of Itano *et al.* [1] the measurements, or more properly, the interruptions of the developing coherence, occur over times much shorter than the time over which induced transitions to the metastable state would occur. These interruptions result in the reduction in ρ_{22}. This is not unlike the situation in which active atoms undergo phase interrupting collisions with perturber atoms. Phase interrupting collisions are simply included in the Bloch equations by writing the phenomenological transverse decay rate as $\gamma_{12}=\Gamma/2+\gamma'$, where Γ is the spontaneous decay rate and γ' is the collision rate. Solving the Bloch equations approximately for strong fields gives that $\rho_{12}(t)$ is exponentially damped with rate γ', and hence the probability for induced transitions is reduced.

Itano *et al.* [1], note that collisional relaxation may constitute a "Zeno effect," but dismiss this with the claim that collisions represent an essentially different process from the one considered in their experiment. They point out that it is the ensemble average value of the coherence that decays, where the average is over collisional histories. In this experiment the relaxation of coherences occurs because of spontaneous decay. However, an ensemble average has been made implicitly, where the average is over the vacuum field modes; perhaps the two situations are not so dissimilar.

What has motivated our alternative approach is the desire to avoid any physical explanation that relies on the idea of wave-function collapse, with all its attendant difficulties. In the usual interpretation of quantum mechanics the probability of an outcome after a succession of measurements is given an appropriate correlation function or joint probability. There is usually no need to invoke the idea of a wave-function collapse. In this spirit we solve the full dynamical problem posed by Itano's experiment, and show that the inhibition of quantum transitions between two quantum states result from the disruption of coherence existing between these states, and where the time over which these interruptions occur plays a crucial role. Clearly, a "quantum Zeno effect" is manifested only if the interruptions occur during times much shorter than the time it takes for the signal field to induce complete inversion.

In addition, by considering the dynamics, and not the presumed consequences of measurement, we offer a resolution to the quantum Zeno paradox. The fact that interruptions occur over times long compared to $\tau_{c\,\mathrm{vac}}$ leads to the result that the observed lifetime of the excited state is unchanged. The analysis in terms of Feynman paths is particularly illuminating because it suggests that a detector measuring total fluorescence "sees" a quantum jump from the excited to the ground state at some instant. The detector is never in a superposition state.

This picture seems to clarify the completely analogous scenario of Schrödinger's cat [12]. This consists of a lethal dose of poison which kills the cat when its release is triggered by a particle undergoing spontaneous decay. If the cat is enclosed in a box, then it is asked if quantum mechanics demands the strange situation that the cat is in a superposition of being both alive and dead until someone looks in the box and thereby projects the system into one of these two possibilities. Yet, by our analysis the cat is, unfortunately, a 100% effective detector which records decay by changing state (quite irreversibly) from alive to dead. Anyone peering inside the box in no way affects the state of the cat. As in our previous example of an atomic bubble chamber, the detector jumps irreversibly from one state to the other without being in a superposition state. Likewise, owing to the ultrashort vacuum correlation time, our observing the cat through a window in the box will have no effect on the system. No matter how humanitarian our intentions, we will not succeed in postponing Kitty's sorry fate!

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