Nonlocal theory of rf suppression of current-driven ion cyclotron waves in a Q machine

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The suppression of current-driven ion cyclotron waves is studied in finite geometry. The mode structure equation for the interacting waves, which includes the nonlinear correction terms due to ponderomotive effects, is solved using the perturbation technique. From the resultant dispersion relation, expressions for real and imaginary parts of the frequency are obtained. It is found that the lower-hybrid pump causes a downward shift in the frequency of the cyclotron wave and enhances the ion cyclotron damping, thereby stabilizing it.

I. INTRODUCTION

Large-amplitude ion-cyclotron modes have been observed in many experiments during the past decade.[1–6] A general formalism for studying the dispersion characteristic of an electrostatic ion-cyclotron wave in a nonuniform plasma has been studied by Sperling and Perkins.[7] Many workers have shown that the lower-hybrid pump wave is effective in suppressing the drift waves, ion-cyclotron waves, and ion-acoustic waves.[6,8–13] In a Q machine electrostatic ion-cyclotron waves driven unstable by parallel currents are seen to be parametrically influenced by the presence of a radio frequency field.[6]

In this paper, we study the nonlinear coupling of current-driven ion-cyclotron and lower-hybrid modes in a cylindrical plasma column by considering a realistic pump of finite wave number. The coupling between the waves is via the ponderomotive force exerted by the high-frequency wave. The frequency shift induced by the presence of lower-hybrid waves is negative, shifting the frequency toward the cyclotron frequency and thus increasing the cyclotron damping.

In Sec. II the dispersion relation is derived for the current-driven ion-cyclotron waves. In Sec. III the nonlinear coupling of ion-cyclotron waves with lower-hybrid waves is studied, and the results are discussed in Sec. IV.

II. LINEAR THEORY

Consider an inhomogeneous plasma, embedded in a uniform magnetic field B oriented along the z direction of a cylindrical system in which particle density varies as $n_0(r)=n_0\exp(-r^2/r_0^2)$. Current flowing in the direction of the magnetic field is given by

$$\mathbf{j}=n_e e\mathbf{v}_d\times\hat{\mathbf{z}} ,$$

where v_d is the drift velocity. When v_d is greater than the parallel phase velocity v_{11}, the ion-cyclotron waves

$$\phi=\phi(r)\exp[-i(\omega t-l\theta-k_z\hat{\mathbf{z}})]$$

are excited, which perturbs the plasma equilibrium. The adiabatic response of electrons and ions due to this perturbation is written as

$$n_e=\frac{k^2}{4\pi e}\chi_e\phi , \tag{1}$$

$$n_i=-\frac{k^2}{4\pi e}\chi_i\phi , \tag{2}$$

where χ_e and χ_i are given by[14]

$$\chi_e=\frac{2\omega_p^2}{k^2v_{\text{th}}^2}(1+i\beta),\quad \beta=\sqrt{\pi}\frac{\omega-k_zv_d}{k_zv_{\text{th}}} ; \tag{3}$$

$$\chi_i=\frac{2\omega_{pi}^2}{k^2v_{\text{th}i}^2}\left[1-I_0\exp(-b_i)-\frac{\omega}{\omega-\omega_{ci}}I_1\exp(-b_i)\right]\times\left[1-i\alpha\frac{\omega-\omega_{ci}}{\omega}\right] ,$$

$$\alpha=\frac{\omega-\omega_{ci}}{k_zv_{\text{th}i}}\exp\left[\frac{-(\omega-\omega_{ci})^2}{k_z^2v_{\text{th}i}^2}\right] , \tag{4}$$

$$b_i=\frac{k_\perp^2v_{\text{th}i}^2}{2\omega_{ci}^2} .$$

$k_\perp$, k_z, v_{th}, $v_{\text{th}i}$, and ω_{ci} are parallel, perpendicular wave numbers, electron, ion thermal velocities, and ion-cyclotron frequency, respectively. $I_0(b_i)$ is the modified Bessel function of order 0, argument b_i.

By substituting Eq. (3) in Eq. (1), and Eq. (4) in Eq. (2), n_e and n_i can be written as follows:

$$n_e=\frac{n_0e}{T}[1+i\beta]\phi ,$$

$$n_i=-\frac{n_0e}{T_i}\left[1-I_0\exp(-b_i)-I_1\exp(-b_i)\frac{\omega}{\omega-\omega_{ci}}(1-i\alpha)\right] .$$

By quasineutrality condition, the mode structure equation can be obtained as

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\left[k_\perp^2-\frac{l^2}{r^2}\right]\phi=0 , \tag{5}$$

where

$$k_\perp^2=-\frac{2}{\rho_i^2}\frac{T_i}{T}\frac{1}{1-\dfrac{\omega(1-i\alpha-i\beta)}{2(\omega-\omega_{ci})}} ,\quad \rho_i=\frac{v_{\text{th}i}}{\omega_{ci}} , \tag{6}$$

and c_s is the ion-acoustic speed. Equation (5) is a Bessel equation with well-known solutions

$$\phi=AJ_l(k_\perp r) . \tag{7}$$

At the boundary of the plasma $r=r_0$ we impose the metallic boundary condition, i.e., ϕ vanishes,

$$J_l(k_\perp r_0)=0 .$$

The dispersion relation can be modified by substituting β in Eq. (6), and also by employing $\omega=\omega+i\gamma$ one obtains

$$\omega=\omega_{ci}\left[1+\frac{(b_i/2)}{(T_i/T)+b_i}\right] \tag{8}$$

and

$$\gamma=\frac{\sqrt{\pi}\omega_{ci}(b_i/2)}{(T_i/T)+b_i}\times\left[\frac{v_d}{v_{\text{th}}}-\frac{\omega}{k_z v_{\text{th}}}-i\frac{\omega-\omega_{ci}}{k_z v_{\text{th}i}}\exp\left[\frac{-i(\omega-\omega_{ci})^2}{k_z^2 v_{\text{th}i}^2}\right]\right] . \tag{9}$$

The dispersion relation for lower-hybrid waves ($\omega_0\ll\omega_{\text{LH}}$) in this plasma column is given by

$$\omega_0^2=\frac{\omega_P^2 k_{0z}^2}{k_0^2} .$$

The mode structure equation can be written as

$$\nabla^2\phi_0+\frac{\omega_P^2 k_{0z}^2}{k_0^2}\phi_0=0 . \tag{10}$$

By approximating the density profile as $\omega_P^2=\omega_{p0}^2(1-r^2/r_0^2)$, the mode structure equation (10) can be modified

$$\frac{\partial^2\phi_0}{\partial r^2}+\frac{1}{r}\frac{\partial\phi_0}{\partial r}+\left[\frac{\omega_{p0}^2 k_{0z}^2}{k_0^2}-\frac{l_0^2}{r^2}-\frac{\omega_{p0}^2 k_{0z}^2}{\omega_0^2 r_0^2}r^2\right]\phi_0=0 . \tag{11}$$

By introducing a new variable $\xi=r(\omega_{p0}k_{0z}/\omega_0 r_0)^{1/2}$, the mode structure Eq. (11) reduces to

$$\frac{\partial^2\phi_0}{\partial\xi^2}+\frac{1}{\xi}\frac{\partial\phi_0}{\partial\xi}+\left[\lambda-\frac{l_0^2}{\xi^2}-\xi^2\right]\phi_0=0 , \tag{12}$$

with solutions $\phi_0=BL_n^{(l_0)}(\xi)$ and the eigenvalues

$$\lambda=\frac{\omega_{p0}k_{0z}r_0}{\omega_0}=2(2n+|l_0|-1),\quad n=0,1,2.\ldots \tag{13}$$

III. NONLINEAR COUPLING

A high-amplitude lower-hybrid pump wave $\phi_0\exp[-i(\omega_0-k_{0z}z-l_0\theta)]$ couples with a low-frequency ion-cyclotron mode $\phi\exp[-i(\omega t-k_z z-l\theta)]$ and two lower-hybrid sidebands $\phi_{1,2}\exp[-i(\omega_{1,2}t-k_{1,2z}z-l_{1,2}\theta)]$. The phase matching conditions demand $\omega_{1,2}=\omega\mp\omega_0$, $k_{1,2z}=k_z\mp k_0$, and $l_{1,2}=l\mp l_0$. The mode structure equation of the sideband waves are similar to that of the pump. The response of electrons to the high-frequency waves can be given by

$$\mathbf{v}_{j\perp}=\frac{-e\nabla\phi_j\times\boldsymbol{\omega}_c}{m\omega_c^2} , \tag{14}$$

$$v_{jz}=\frac{-ek_{jz}\phi_j}{m\omega_j},\quad j=0,1,2 . \tag{15}$$

From the low-frequency ponderomotive force, arising due to the high-frequency field, the ponderomotive potential ϕ_P can be obtained. Only the parallel component of this force is effective (since $\omega<k_z v_{\text{th}}$, $\omega\ll\omega_c$),

$$\phi_P=\frac{e}{2im\omega_c^2\omega_0}[(\nabla\phi_0\times\boldsymbol{\omega}_c)\cdot\nabla\phi_1-(\nabla\phi_0^*\times\boldsymbol{\omega}_c)\cdot\nabla\phi_2] . \tag{16}$$

By including the ponderomotive potential ϕ_P in Eq. (1), the mode structure equation of the low-frequency wave is modified as

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\left[-\frac{2}{\rho_i^2}\frac{T_i}{T}\frac{1}{1-\dfrac{\omega(1-i\alpha-i\beta)}{2(\omega-\omega_{ci})}}-\frac{l^2}{r^2}\right]\phi=\frac{2}{\rho_i^2}\frac{T_i}{T}\frac{1}{1-\dfrac{\omega}{2(\omega-\omega_{ci})}}\phi_P . \tag{17}$$

The high-frequency nonlinear response can be obtained as

$$n_1^{\text{NL}}=\frac{n_0 e k_\perp^2}{4iM\omega_1\omega_{ci}^2}\left[1-\frac{\omega}{2(\omega-\omega_{ci})}\right]\frac{e}{m\omega_c^2}(\nabla\phi_0^*\times\boldsymbol{\omega}_{ci})\cdot\nabla\phi \tag{18}$$

and

$$n_2^{\text{NL}}=\frac{n_0 e k_\perp^2}{4iM\omega_2\omega_{ci}^2}\left[1-\frac{\omega}{2(\omega-\omega_{ci})}\right]\frac{e}{m\omega_c^2}[\nabla\phi_0\times\boldsymbol{\omega}_{ci}]\cdot\nabla\phi , \tag{19}$$

where M is the ionic mass.

By substituting Eqs. (18) and (19) into the Poisson equation the mode structure equation of sideband waves is modified as

$$\frac{\partial^2\phi_1}{\partial r^2}+\frac{1}{r}\frac{\partial\phi_1}{\partial r}+\left[\frac{\omega_{p0}^2k_{1z}^2}{\omega_1^2}-\frac{l_1^2}{r^2}-\frac{\omega_{p0}^2k_{1z}^2r^2}{\omega_1^2r_0^2}\right]\phi_1 =+\frac{\omega_p^2k_\perp^2e}{4i\omega_1\omega_c^2\omega_{ci}^2M}\left[1-\frac{\omega}{2(\omega-\omega_{ci})}\right](\nabla\phi_0^*\times\boldsymbol{\omega}_c)\cdot\nabla\phi \; . \tag{20}$$

Similarly,

$$\frac{\partial^2\phi_2}{\partial r^2}+\frac{1}{r}\frac{\partial\phi_2}{\partial r}+\left[\frac{\omega_{p0}^2k_{2z}^2}{\omega_2^2}-\frac{l_2^2}{r^2}-\frac{\omega_{p0}^2k_{2z}^2r^2}{\omega_2^2r_0^2}\right]\phi_2 =+\frac{\omega_p^2k_\perp^2e}{4i\omega_2\omega_c^2\omega_{ci}^2M}\left[1-\frac{\omega}{2(\omega-\omega_{ci})}\right](\nabla_0\times\boldsymbol{\omega}_c)\cdot\nabla\phi \; . \tag{21}$$

In the absence of coupling, the solution of Eq. (21) is written as

$$\phi_1=\Psi_{n_1,l_1}^{(1)}\left[r\left[\frac{\omega_{p0}k_{1z}}{(\omega_0-\omega)r_0}\right]^{1/2}\right]e^{il_1\theta}$$

and the eigenvalue is

$$\lambda_1=\frac{\omega_{p0}k_{1z}r_0}{\omega_0-\omega}\equiv\lambda_{n_1,l_1} \; .$$

In the presence of nonlinear coupling, ϕ_1 may be expanded as[15]

$$\phi_1=\sum_{n,l}A_n^{(1)}\Psi_{n_1,l_1}^{(1)} \; . \tag{22}$$

Substituting Eq. (22) in Eq. (20), multiplying the resultant equation by $\Psi_{n_1,l_1}^{(1)}rdr$, and integrating over r, one obtains

$$\left[\frac{\omega_{p0}k_{1z}r_0}{\omega_0-\omega}-\lambda_{n_1,l_1}\right]A_n^{(1)} =\frac{ek_\perp^2\left[1-\frac{\omega}{2(\omega-\omega_{ci})}\right]}{4i\omega_1\omega_c^2\omega_{ci}^2M}\times\int_0^{r_0}r\,dr\,\omega_P^2(r)\Psi_{n_1,l_1}^*(\nabla\phi_0\times\boldsymbol{\omega}_c)\cdot\nabla\phi \; . \tag{23}$$

Similarly by Eq. (21)

$$\left[\frac{\omega_{p0}k_{2z}r_0}{\omega_0+\omega}-\lambda_{n_2,l_2}\right]A_n^{(2)} =\frac{ek_\perp^2\left[1-\frac{\omega}{2(\omega-\omega_{ci})}\right]}{4i\omega_2\omega_c^2\omega_{ci}^2M}\times\int_0^{r_0}r\,dr\,\omega_P^2(r)\Psi_{n_2,l_2}^*(\nabla\phi_0\times\boldsymbol{\omega}_c)\cdot\nabla\phi \; . \tag{24}$$

Similarly using Eq. (17), multiplying by $J_l(k_1r)rdr$, and integrating over r,

$$\left[\frac{\omega_{ci}^2}{c_s^2}\frac{1}{\left[\frac{(1-i\alpha-i\beta)}{2(\omega-\omega_{ci})}-1\right]}-k^2\right]A =\frac{\omega_{ci}^2\int_0^{r_0}J_l(k_\perp r)\phi_p r\,dr}{c_s^2\left[1-\frac{\omega}{2(\omega-\omega_{ci})}\right]} \; . \tag{25}$$

Substituting Eq. (16) in Eq. (25)

$$\left[\frac{\omega_{ci}^2}{c_s^2}\frac{1}{\left[\frac{\omega(1-i\alpha-i\beta)}{2(\omega-\omega_{ci})}-1\right]}-k^2\right]A=\frac{\omega_{ci}^2(e/2im\,\omega_c^2\omega_0)}{c_s^2\left[1-\frac{\omega}{2(\omega-\omega_{ci})}\right]}\left[\left[A_{n_1,l_1}^{(1)}\int_0^{r_0}r\,dr\,J_l(k_\perp r)(\nabla\phi_0\times\boldsymbol{\omega}_c)e^{il_1\theta}\right]\cdot(\nabla\Psi_{n_1,l_1}e^{il_1\theta})\right]$$
$$-\left[A_{n_2,l_2}^{(2)}\int_0^{r_0}r\,dr\,J_l(k_\perp r)(\nabla\phi_0^*\times\boldsymbol{\omega}_c)e^{il_2\theta}\right]\cdot(\nabla\Psi_{n_2,l_2}e^{il_2\theta}) \; . \tag{26}$$

If $l_0=0$, then $l_1=l_2$. Also, by setting $n_1=n_2$, Eq. (26) becomes

$$\frac{(\omega_{ci}^2/c_s^2)}{\left[\frac{\omega(1-i\alpha-i\beta)}{2(\omega-\omega_{ci})}-1\right]}-k^2=2\mu\frac{\left[\frac{\omega_{p0}k_zr_0}{\omega_0}-\lambda\right]}{\left[\frac{\omega_{p0}k_{1z}r_0}{\omega_0}-\lambda\right]^2-\left[\frac{\omega_{p0}k_{0z}r_0}{\omega_0}\right]^2} \; , \tag{27}$$

where

$$\mu=\frac{e^2k_\perp^2l[1-\omega/2(\omega-\omega_{ci})]\left|\int_0^{r_0}r\,dr\,\Psi_{n_1,l_1}(\partial\phi_0/\partial r)J_l(k_\perp r)\right|^2}{4\omega_c^2\omega_{ci}^2\omega\omega_0Mm} \; . \tag{28}$$

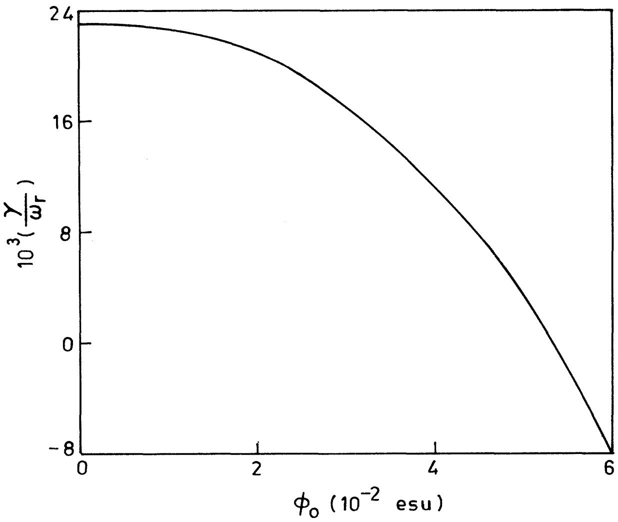

FIG. 1. Variation of growth rate (γ/ω_r) with the amplitude ϕ_0 for the pump frequency $\omega_0/\omega_{pi}=9.0$.

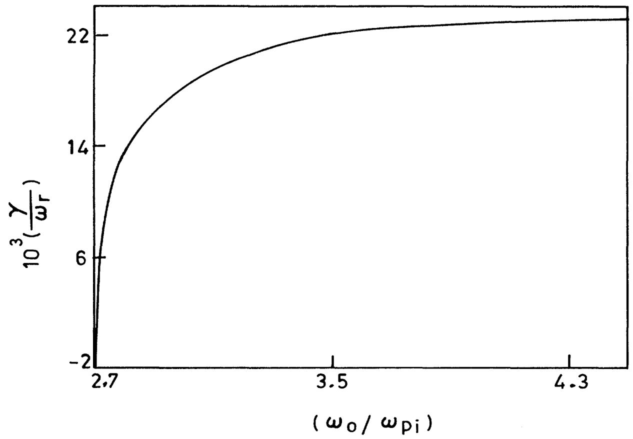

FIG. 2. Variation of growth rate (γ/ω_r) with pump frequency ω_0/ω_{pi} for the amplitude $\phi_0=5\times10^{-2}$ esu.

From Eq. (27) we get

$$\omega_r=\omega_{ci}\left[1+\frac{(b_i/2)}{(T_i/T)+b_i+R}\right] \tag{29}$$

and

$$\gamma=\frac{\sqrt{\pi}\omega_{ci}(b_i/2)}{(T_i/T)+b_i+R}\times\left[\frac{v_d}{v_{\rm th}}-\frac{\omega}{k_z v_{\rm th}}-\frac{\omega-\omega_{ci}}{k_z v_{{\rm th}i}}\exp\left[\frac{-i(\omega-\omega_{ci})^2}{k_z^2 v_{{\rm th}i}^2}\right]\right], \tag{30}$$

where

$$R=2\mu\left[\frac{\left[\dfrac{\omega_{p0}k_z r_0}{\omega_0}-\lambda\right]}{\left[\dfrac{\omega_{p0}k_z r_0}{\omega_0}-\lambda\right]^2-\left[\dfrac{\omega_{p0}k_{0z}r_0}{\omega_0}\right]^2}\right].$$

IV. RESULTS AND DISCUSSIONS

Computations were made for the following set of parameters, typical of a Q machine:[6] $n=10^8$ cm^{-3}, $B=1.8$ kG, $T_e\simeq T_i\simeq0.2$ eV, the density scale length $r_0=1$ cm, the axial extent of the plasma column (L_{11})$\simeq$70 cm, and the electron drift velocity $v_d=0.15v_{\rm th}$. From the first zero of the Bessel function, $k_{\perp n}$ is found to be 3.85 cm^{-1}. The parallel wave number k_z is taken as $5/L_{11}$. The effect of the pump wave amplitude (ϕ_0) on the growth rate of the ion-cyclotron instability was studied for a range of values of $\omega_0/\omega_{pi}\simeq6.58$ to 16.33 and for the radial mode number $n=5$. The pump was assumed to be azimuthally symmetric ($l_0=0$). For both the ion-cyclotron wave and the sideband, the $l=1$ mode was considered.

Figure 1 shows the variation of γ/ω_r with ϕ_0 for $\omega_0/\omega_{pi}=9$. The minimum value of ϕ_0 required to suppress the ion-cyclotron instability is 0.05 esu. The variation of ω/ω_r with ω_0/ω_{pi} for $\phi_0=0.05$ esu is shown in Fig. 2. It may be noted that the instability is suppressed over a wide range of pump frequencies.

As $r_0\to\infty$ our results do not converge to those of local theory. As in a bounded plasma the radial modes are standing waves, whereas in an unbounded plasma these are the traveling waves. Furthermore, as $r_0\to\infty$ the frequency separation between different eigenmodes is small and the assumption of a single mode number fails. Nevertheless, if one assumes the mode structures of all interacting waves to be uniform, the growth rate turns out to be the same as that obtained by local theory, where the plasma is large and homogeneous. It is concluded that the current-driven ion-cyclotron waves can be suppressed by a radio frequency pump field of moderate strength.

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