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Failure of some theories of state reduction

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Several recent theories involve a modification of the Schrödinger equation, which dynamically suppresses coherent superpositions of macroscopically different states and so avoids Schrödinger's cat paradox. It is shown that these theories violate energy conservation and are incompatible with the existence of equilibrium and steady states. The reasons for these troubles are pointed out.

I. INTRODUCTION

It is a characteristic feature of quantum mechanics that the theory not only permits, but in certain situations demands, coherent superpositions of macroscopically distinct states. This was first pointed out in Schrödinger's cat paradox.[1] A more realistic example occurs in the theory of measurement, where the measurement of some quantum-mechanical variable usually leads to a final state of the total system (object plus apparatus) that is a coherent superposition of macroscopically distinct pointer positions of the measuring instrument.[2,3] Such superpositions, although a natural consequence of the quantum formalism, are nevertheless difficult to accept intuitively, and several responses to them have been proposed.

The earliest response was to introduce to the theory an additional *ad hoc* assumption, the *projection postulate,* according to which the coherent superposition state is somehow *reduced* into a more intuitive state. This *ad hoc* postulate is rather unsatisfactory because it conflicts with the Schrödinger equation of motion,[4] and it does not always yield correct results.[5]

A second approach is to adopt an interpretation of the quantum *state* concept that does not find macroscopic coherent superpositions to be embarassing.[2,3,6,7] In this view the state vector is not an attribute of an individual system, but only a device from which probabilities can be calculated. This view is often distinguished from the older view by the assertion that the state function describes an *ensemble* of similarly prepared systems, rather than being a complete description of an *individual* system. For the purposes of this paper, the question of an individual versus an ensemble interpretation is relevant only to the extent that the former requires a process *state reduction,* while the latter does not.

A third approach, which is the subject of this paper, is to modify the Schrödinger equation of motion to include a mechanism for spontaneous state reduction. It would cause long-range coherence to decay, and would make pure states spontaneously evolve into mixed states. Several theories of this type have been proposed. Ghirardi, Rimini, and Weber[8] (GRW) postulate that the wave function undergoes a sequence of spontaneous random jumps that tend to localize it. The basic postulate of this theory and the choice of values for its parameters are rather speculative and arbitrary. Diosi[9,10] considers the alternative possibility that quantum mechanics might be modified by gravitational effects. Averaging over an assumed universal fluctuation in the gravitational field (of a magnitude suggested by the Bohr-Rosenfeld analysis of the limits on field measurements) leads to the replacement of the Schrödinger by a nonunitary master equation. Joos and Zeh[11] (JZ) make no radical assumptions, but rather attempt to derive a master equation for the statistical operator of a system that interacts with an external environment.

These theories all lead to similar equations for the statistical operator (or density matrix), and hence they can be judged on the basis of their results, regardless of the considerable differences in their physical postulates and philosophies.[12] The fact that such different starting points have led to essentially the same equation of motion is interesting, and makes that equation worthy of study in its own right. Unfortunately our conclusion is negative—that these theories are not satisfactory.

II. ENERGY NONCONSERVATION

The simplest generalization of the Schrödinger equation that has been proposed has the form

$$\frac{d\rho}{dt}=\frac{-i}{\hbar}[H,\rho]-\frac{\gamma}{4}[R,[R,\rho]] \tag{1}$$

for a one-particle system. Here ρ is the statistical operator and H is the Hamiltonian. The first term on the right-hand side yields the usual quantum-mechanical evolution of the state, while the last term is responsible for spontaneous state reduction. In the R representation this term takes the form $-(\gamma/4)(r-r')^2\langle r|\rho|r'\rangle$, which clearly leads to a spontaneous decay of the nondiagonal elements of the density matrix $\langle r|\rho|r'\rangle$, provided the parameter γ is positive. Thus coherent superpositions of different R values will be suppressed. In the theory of (JZ) (Ref. 11) the state-reducing operator R is the position operator; in the theory of Diosi[9,10] it is the mass density. Both choices lead to the spontaneous destruction of coherent superpositions of macroscopically distinct states. But in addition to this intended effect, there are other consequences.

It is easy to show from (1) that the normalization $\mathrm{Tr}\rho=1$ is preserved. However, the average energy

$\langle H\rangle=\mathrm{Tr}(H\rho)$ is not conserved. Using the formal identity $\mathrm{Tr}(A[B,C])=\mathrm{Tr}([A,B]C)$ we obtain $d\langle H\rangle/dt=\mathrm{Tr}(Hd\rho/dt)=-(\gamma/4)\mathrm{Tr}([H,R][R,\rho])$. It is apparent that if the reduction operator R does not commute with the Hamiltonian, then there will be states for which $\langle H\rangle$ is not conserved, and indeed this nonconservation was pointed out by GRW.[8]

Specializing to a free particle in one dimension with $H=P^2/2m$, where P is the momentum operator, and R is taken to be the position operator Q, we obtain

$$\frac{d}{dt}\langle H\rangle=\frac{\gamma}{4}\frac{\hbar^2}{m}\ . \qquad (2)$$

Thus the system steadily gains energy at a constant rate, independent of the state. This result is also obtained in three dimensions, and is unaltered by the inclusion of vector and scalar potentials.

Other theories of this type lead to the following equation for the density matrix in coordinate representation, which we write only for the simplest case of a free particle in one dimension:

$$\frac{\partial}{\partial t}\rho(x,x',t)=\frac{i\hbar}{2m}\left[\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial x'^2}\right]\rho(x,x',t) -\lambda f(x-x')\rho(x,x',t)\ . \qquad (3)$$

The qualitative form of the function $f(x-x')$, which governs the damping of coherence, is shown in Fig. 1. In the theory of GRW (Ref. 8) it is equal to $f_{\mathrm{GRW}}(x-x')=1-\exp[-(\alpha/4)(x-x')^2]$. In the limit $\alpha\rightarrow 0$, $\lambda\rightarrow\infty$, with $\alpha\lambda=\gamma$ fixed, this yields $\lambda f(x-x')\rightarrow(\gamma/4)(x-x')^2$, which is equivalent to Eq. (1) with R replaced by the position operator. However, our results do not depend on that specific form, but only on the properties: $f(0)=0$, $f'(0)=0$, $f''(0)>0$. The rate of change of the average energy is now given by

$$\frac{d}{dt}\langle H\rangle=\mathrm{Tr}\left[\frac{P^2}{2m}\frac{d\rho}{dt}\right] =\frac{\lambda\hbar^2}{2m}\int\left[\frac{\partial^2}{\partial x^2}f(x-x')\rho(x,x',t)\right]_{x'=x}dx\ , \qquad (4)$$

the only nonvanishing contribution being given by the last term of (3). Now $(\partial/\partial x)^2 f(x-x')\rho=f''(x-x')\rho+2f'(x-x')\partial\rho/\partial x+f(x-x')\partial^2\rho/\partial x^2$. The second and third terms vanish for $x'=x$, hence Eq. (4) yields

$$\frac{d}{dt}\langle H\rangle=\frac{\hbar^2}{2m}\lambda f''(0) \qquad (5)$$

which, after appropriate identification of parameters, is identical with the result (2).

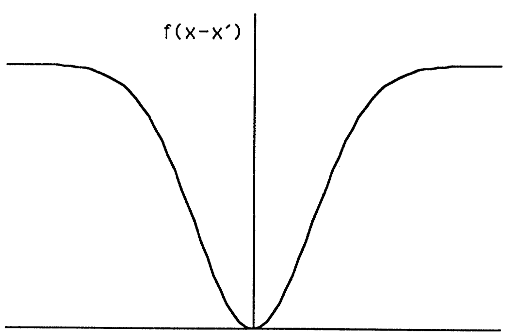

FIG. 1. The coherence damping function, introduced in Eq. (3).

Using the value $\gamma=\alpha\lambda=10^{-7}$ cm^{-2} s^{-1} suggested by GRW, the predicted rate of energy gain is about 10^{-45} W per particle, which is insignificant, even on astronomical time scales. However the physical origin of the reduction term [the last term in (3)] is unspecified in their theory, and so the value of γ is arbitrary. Their suggested value was deliberately chosen with an eye to avoid a contradiction of observations.

In the theory of Diosi[9,10] the state-reduction mechanism is related to gravity, and so, unlike GRW, he does not have an arbitrary coupling constant. In Diosi's theory the reduction operator R in Eq. (1) is essentially the mass density. Assuming the particle to be a uniform sphere of radius a, Diosi obtains Eq. (3) with

$$\lambda f_D(x-x')=\hbar^{-1}[U(|x-x'|)-U(0)]\ , \qquad (6)$$

where $U(r)$ is the gravitational pair interaction energy of two (interpenetrating) spheres of mass m and radius a. Equation (6) also has the qualitative form shown in Fig. 1. From the classical expression for the gravitational interaction of two spheres $U(r)=-(Gm^2/a)[\frac{3}{2}-\frac{1}{2}(r/a)^2]$ for $r<a$; $U(r)=-Gm^2/a$ for $r>a$, we obtain from (5) a rate of energy gain of the form (2) with $\gamma=2Gm^2/\hbar a^3$, where G is the Newtonian gravitational constant. Taking m to be the proton mass, and the proton radius to be $a=10^{-13}$ cm, we obtain a rate of energy gain of about 10^{-26} W per particle, or 10^{-2} W per mole. This is too large a value to have escaped notice in low-temperature experiments.[13]

The state-reduction mechanism in the JZ (Ref. 11) theory is taken to be the ordinary interactions between the system and its environment. There is no universal formula for γ in this theory; the expressions given by JZ depending on temperature, scattering cross sections, and various approximations. However, the magnitude of the energy gain in their theory should be detectable, at least for some range of the various parameters.[14]

It is apparent from Eq. (5) that if $f''(0)=0$ then the average energy $\langle H\rangle$ would be constant. This could be achieved, for example, by taking $f(x-x')=1-\exp[-\alpha^2(x-x')^4]$. Note that the energy of the system would not be strictly conserved ($\langle P^2\rangle$ would be constant but $\langle P^4\rangle$ would not), but at least the energy would be conserved on the average. However, we shall show in Sec. III that even with such a modification the theory remains unsatisfactory. (It has been suggested that modifications of this sort may also jeopardize the nonnegative character of the density matrix.)

III. NONEXISTENCE OF EQUILIBRIUM

The theories that we have discussed are incompatible with the establishment of equilibrium or steady state, since $d\langle H\rangle/dt>0$ in those theories. We shall now show that even if the function $f(x-x')$ is modified to yield $d\langle H\rangle/dt=0$, the theories will still possess no steady states.

We seek the time-independent solutions of (3). With the substitutions $y=x-x'$ and $z=x+x'$, Eq. (3) becomes

$$\frac{\partial^2\rho(y,z)}{\partial y\,\partial z}+i\Lambda f(y)\rho(y,z)=0\ , \tag{7}$$

where we have introduced $\Lambda=\lambda m/2\hbar$. This equation can be solved by a separation of variables. We substitute $\rho(y,z)=\Upsilon(y)\Xi(z)$ and obtain

$$\frac{\Upsilon'}{\Upsilon(y)}\frac{\Xi'}{\Xi(z)}=-i\Lambda f(y)\ . \tag{8}$$

Since the right-hand side is independent of z, so must be the left-hand side. Hence Ξ'/Ξ must be a constant, which we write as ik. Thus we obtain $\Xi(z)=e^{ikz}$, with k being arbitrary. We must take k to be real, since an exponentially diverging solution is not physically acceptable. With $\Xi'/\Xi=ik$ the solution of (8) becomes $\ln\Upsilon(y)=-(\Lambda/k)\int f(y)dy$, and hence $\Upsilon(y)=\exp[-(\Lambda/k)F(y)]$, where $F(y)=\int f(y)dy$. Now $f(y)$ approaches a positive limit as $|y|\to\infty$ (see Fig. 1), so $F(y)$ becomes asymptotically proportional to y as $y\to\pm\infty$. Therefore, regardless of the sign of k, $\Upsilon(y)$ diverges exponentially in one direction. Thus there are no physically acceptable steady-state solutions to Eq. (3). This conclusion depends only upon $f(y)$ not going to zero as $y\equiv x-x'\to\infty$. Since this is the condition needed to ensure the destruction of coherence over large distances, this condition cannot be abandoned without destroying *raison d'etre* of the theory.

The absence of physically acceptable steady-state solutions of (3) is surprising. The familiar spreading of a wave packet, induced by the first term on the right-hand side of (3) and leading to long-range coherence, is countered by the spontaneous localization caused by the last term. It may be thought that an equilibrium would occur when the two effects balance, and indeed one can easily estimate a characteristic "coherence length" by equating the orders of magnitude of the two terms.[9] Such an argument is misleading since there is, in general, no persistent structure on such a length scale. [See JZ (Refs. 11 and 12) for an example illustrating the detailed time dependence of a solution.]

IV. DISCUSSION

We have examined a class of theories of spontaneous state reduction, and have shown that they are incompatible with the attainment of equilibrium, since they imply that the system must continually gain energy. To assess the significance of this result, we need to briefly examine the aims of the various theories and the assumptions on which they are based.

The most radical theory is that of Ghirardi, Rimini, and Weber,[8] who sought to create a new fundamental theory that would replace the Schrödinger equation for an isolated system. The nonconservation of energy is a very unattractive feature in a fundamental theory, even if its magnitude is so small as to be impractical to detect.

The theories of Diosi[9,10] and of Joos and Zeh[11] do not treat the system as isolated, and so its energy need not be conserved. However energy should be gained in some states and lost in others, depending upon the relative temperatures of the system and its environment, whereas these theories predict an inexorable energy gain, independent of the state.

The JZ theory does not introduce any new physical postulates. JZ treat the ordinary interactions between a system and its environment by means of ordinary quantum mechanics, and they attempt to derive a master equation for the density matrix of the system. This approach is familiar in nonequilibrium statistical mechanics, where the effect of the environment (usually characterized as a thermal reservoir) is to provide damping and to ensure the approach of the system to equilibrium. But according to the JZ theory, instead of providing damping, the environment would act as an inexhaustible energy source, making equilibrium impossible.

A clue to the origin of the trouble can be found in a recent paper by Unruh and Zurek[15] (UZ). They consider a harmonic oscillator coupled to a massless scalar field, which acts as an environment for the oscillator. From the exact solution of this model, they deduce a master equation for the density matrix of the oscillator. Their master equation [UZ, Eq. (3.20)] contains more terms than does (3), and it has time-dependent coefficients which reach steady limiting values after a short transient. In the high-temperature limit their master equation simplifies to a form [UZ, Eq. (4.7)] closer to (3), but containing an additional term of the form $-\frac{1}{2}\epsilon^2(x-x')(\partial\rho/\partial x-\partial\rho/\partial x')$, and with the parameter λ in (3) being proportional to the temperature, $\lambda=\epsilon^2 T$. Here ϵ is the strength of the coupling between the oscillator and its environment, the scalar field. This additional term causes energy loss, which can balance the energy gain produced by the last term of (3), and hence can lead to equilibrium of the system.

If we now take the limit $T\to\infty$, $\epsilon\to 0$, with $\epsilon^2 T=\lambda$const, we will obtain Eq. (3). The continuous energy gain predicted by Eq. (3) can now be understood as due to the environment being so much hotter than the system. Presumably the theory of JZ (Ref. 11) is valid only in such a limit, although the details of their approximations are not transparent.

UZ also show that treating the environmental fluctuations as pure white noise is equivalent to a high-temperature approximation. In effect, it treats the influence of the environment on the system but neglects the effect of the system on the environment. Diosi[9,10] explicitly assumed a white-noise spectrum, which is evidently the reason for the energy gain in his theory.

Finally we remark that the quest to obtain state reduction as a real dynamical process is not a fundamental problem of quantum mechanics, but rather is only a problem that arises within one interpretation of quantum mechanics. There is the alternative of adopting an interpretation of quantum mechanics in which the notion of state reduction is not needed.[7]

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[13]The effect of spontaneous energy production would be indistinguishable from heat leakage into a cryostat. A standard manufactured He cryostat with a capacity of 6.5 liters will boil off about 2 liters of liquid He per day, corresponding to a heat input of about 2×10^{-4} W/mol. The heat leakage into cryostats designed to work at mK temperatures is much smaller. This performance would be impossible if energy were spontaneously produced at the rate of 10^{-2}W/mol.
[14]Table 2 of Ref. 11 lists values of $\gamma/4$ (denoted Λ by JZ) for several different scattering mechanisms. For a large molecule these range from 10^{-12} to 10^{30} $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$. These may be compared with the GRW value of $\gamma=10^{-7}$ (unobservably small), and the value from Diosi's theory of $\gamma=10^{12}$ (easily observable).
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