Ginzburg-Landau equation: A nonlinear model for the radiation field of a free-electron laser

S. Y. Cai and A. Bhattacharjee
Department of Applied Physics, Columbia University, New York, New York 10027
(Received 15 February 1990; revised manuscript received 31 January 1991)

It is shown that the nonlinear dynamics of the radiation field in a free-electron laser can be modeled by the Ginzburg-Landau equation. The refractive index of the electron beam in the nonlinear regime and the saturation intensity of the radiation field are obtained from WKB theory. Although the Ginzburg-Landau equation does not permit soliton solutions, it is shown that certain types of solitary-wave solutions have a strong resemblance to spikes observed in simulations and experiments.

Free-electron-laser (FEL) theory has been very successful in describing FEL operation in the linear regime, and various predictions of linear theory have been shown to be in good agreement with experiments.[1] However, in the regime of saturated growth, which is strongly influenced by nonlinear effects, FEL physics has been studied largely in connection with specific problems such as efficiency enhancement techniques[2] and trapped-particle phenomena. The latter includes the important sideband instability,[3] which is a consequence of the synchrotron oscillation of the electrons trapped in potential wells of the electron bunches, together with the finite slippage of the optical wave with respect to the moving electrons. A basic understanding of these nonlinear effects has been developed through analytical studies in idealized models. While these analyses provide qualitative results and insights for specific problems, the burden of a general nonlinear description has rested on computer simulations, which have played a very important role both in the design of FEL's and the interpretation of experiments.

This paper is motivated in part by an interesting and somewhat less studied phenomenon called "spiking," which has been observed in computer simulations and more recently, in experiments carried out by the Warren, Goldstein, and Newnam,[4] Dodd and Marshall,[5] and Richman, Madey, and Szarmes.[6] "Spikes" generally occur in the high-power, saturated-signal regime, and are narrow, high-intensity radiation pulses that are generated spontaneously. A qualitative physical mechanism has been outlined by Warren and co-workers, who attribute the generation of spikes to the growth of sidebands.[4]

In this paper, we propose a simple, yet fairly general, model for the nonlinear evolution of the radiation field in a FEL. This model enables us to understand the spatial and temporal structure of the radiation field in the nonlinear regime. Preliminary results of this work were presented elsewhere.[7] We show that the radiation field obeys approximately the Ginzburg-Landau equation (GLE),

$$i\frac{\partial A}{\partial z'}-\frac{\alpha}{2}\frac{\partial^2 A}{\partial t'^2}+\lambda_0 A+\beta|A|^2 A=0\ , \tag{1}$$

where A is the amplitude of the radiation field, $z'=z$ is the distance along the undulator axis, $t'=t-z/v_g$ is the retarded time for a radiation pulse propagating with group velocity v_g, and α, β, and λ_0 are complex constants to be given later. The appearance of the GLE in a model of FEL nonlinear dynamics is less surprising than may appear at first glance. After all, there exists a useful analogy between an optical fiber and an electron beam,[8] and in certain types of dielectric fibers, it is known that the radiation field obeys a nonlinear Schrödinger equation,[9] which is nothing but a special case of the GLE. In order to strengthen further the analogy between an electron beam and a fiber, we derive an approximate expression for the refractive index of the electron beam in the nonlinear regime from a WKB theory. This is one of the important results of the present theory.

The GLE is a nonintegrable partial differential equation, and does *not* have soliton solutions. We are, therefore, led to the conclusion that it is not possible to generate optical solitons spontaneously from FEL dynamics. We note that solitons can still be created in principle, as they are in conventional lasers, by propagating the radiation output from a FEL through a dielectric fiber,[10] but this is not the subject of the present paper.

Though the GLE does not admit soliton solutions, it has solitary-wave solutions,[11] which can be obtained by sophisticated variants of the Painlevé analysis.[12,13] We show that a class of these solutions exhibits "spiking" behavior with characteristic widths that appear to be in fairly good agreement with experimental measurements[4-6] and numerical solutions.

We begin our analysis with the well-known one-dimensional equations for a Compton FEL:[2,14]

$$\frac{d\gamma_j}{dz}=-\frac{k_s a_s a_w}{\gamma_j}\sin(\psi_j+\phi)\ , \tag{2a}$$

$$\frac{d\psi_j}{dz}=k_w\left[1-\frac{\gamma_r^2}{\gamma_j^2}\right]+\frac{k_s a_w a_s}{\gamma_j^2}\cos(\psi_j+\phi)\ , \tag{2b}$$

$$\frac{du}{dz}=i\frac{a_w\omega_p^2}{2k_s c^2}\left\langle\frac{\exp(-i\psi)}{\gamma}\right\rangle\ , \tag{2c}$$

where z is the direction of propagation of the electron and optical beams and also coincides with the undulator axis, $\psi_j+\phi$ is the relative phase of the electron (of rest mass m) with respect to the radiation field, and $\gamma_j mc^2$ is its energy; the vector potential $\mathbf{A}=\mathbf{A}_w+\mathbf{A}_s$, $a_w=eA_w/mc^2$, $a_s=eA_s/mc^2$; k_w, $k_s=\omega/c$ are the wave numbers of the undulator and radiation fields, respectively; ω is the carrier frequency, $u=a_s\exp(i\phi)$ is the complex amplitude of the radiation field, and $\gamma_r=[k_s(1+a_w^2)/2k_w]^{1/2}$ is the resonant energy. It has been shown in Ref. 15 that Eqs. (2) can be modeled by an approximate reduced set of equations that involve only the collective variables $x\equiv\langle\exp[-i(\psi_j-\psi_r)]\rangle$, $y\equiv\langle[(\gamma_j-\gamma_0)/\gamma_0]\exp[-i(\psi_j-\psi_r)]\rangle$, and $A\equiv u\exp(i\psi_r)$, where ψ_r is defined by the relation $d\psi_r/dz=k_w(1-\gamma_r^2/\gamma_0^2)$, and γ_0 is the energy of the initially monoenergetic electron beam. In our notation, these reduced equations can be written as

$$\frac{dA}{dz}=i\delta A+igx\ , \tag{3a}$$

$$\frac{dx}{dz}=-ihy\ , \tag{3b}$$

$$\frac{dy}{dz}=ifA-2ihy_0(y-xy_0)+2i\delta xy_0-2if(x^*A+A^*x)x\ , \tag{3c}$$

where $\delta\equiv k_w(1-\gamma_r^2/\gamma_0^2)$, $f\equiv k_s a_w/(2\gamma_0^2)$, $g\equiv\omega_p^2a_w/(2k_s\gamma_0c^2)$, $h\equiv k_s(1+a_w^2)/\gamma_0^2$ are parameters, and $y_0\equiv\langle(\gamma_j-\gamma_0)/\gamma_0\rangle$. For a detailed derivation of Eqs. (3), the reader is referred to Ref. 15. We note that Eqs. (3) admit an energy conservation law, given explicitly by $y_0+(f/g)(|A|^2-|a_0|^2)=0$, where $a_0=A\ (z=0)$ is the initial amplitude of the radiation field and is usually small compared with A in the saturated regime.

Equations (3a) and (3b) can be rewritten, respectively, as $x=(\dot A-i\delta A)/(ig)$ and $y=i\dot x/h=(\ddot A-i\delta\dot A)/(gh)$, where the overdot denotes d/dz. Substituting expressions for x and y in Eq. (3c), we obtain a third-order differential equation for A:

$$\dddot A-(i\delta-2i\epsilon hy_0)\ddot A+2\epsilon hy_0(2\delta-\epsilon hy_0)\dot A+\frac{2\epsilon fh}{ig}(\dot AA^*-\dot A^*A)(\dot A-i\delta A)-ifghA+2i\epsilon y_0\delta(\epsilon hy_0-\delta)A=0\ , \tag{4}$$

in which we have introduced a small parameter ϵ to tag all terms containing y_0, which is a small quantity for most FEL's. We will eventually set ϵ to unity.

Equation (4) can be solved by a WKB method. We write $A=A_0\exp[(\sum_{n=0}^{\infty}\epsilon^{n-1}S_n)$, where A_0 is a constant and $S_n=S_n(z)$ are slowly varying functions of z. We assume that $\dot S_n=\epsilon\dot S_n(n=0,1,2,\dots)$ and $\ddot S_0=\epsilon^2\ddot S_0$. Using the conservation relation $y_0+(f/g)(|A|^2-|a_0|^2)=0$ and the assumption $|A|\gg|a_0|$, we can then solve Eq. (5) by equating coefficients of ϵ^n. To $O(\epsilon^0)$, we obtain

$$\dot S_0^3-i\delta\dot S_0^2-ifgh=0\ , \tag{5}$$

which reduces to the well-known linear cubic eigenvalue equation[16] $\lambda_0^3-\delta\lambda_0^2+fgh=0$ if we set $\dot S_0=i\lambda_0$. To $O(\epsilon)$, Eq. (4) gives

$$\ddot S_1+(3\dot S_0-i\delta)\dot S_1+(3\dot S_0^2-2i\delta\dot S_0)\dot S_1=Qy_0\ , \tag{6}$$

where $Q\equiv-\dot S_0^2+2i\delta\dot S_0-(\dot S_0-\dot S_0^*)\dot S_0+i\delta(\dot S_0-\dot S_0^*)+\delta^2$. We now transform from z to y_0 as the new independent variable by writing $d/dz=(d|A|^2/dz)(d/d|A|^2)$, whereupon using the WKB representation and the energy conservation relation we obtain $d/dz\simeq(\dot S_0+\dot S_0^*)y_0(d/dy_0)$ and $d^2/dz^2\simeq(\dot S_0+\dot S_0^*)^2y_0(d/dy_0)$ to the lowest order in ϵ. Equation (6) then reduces to

$$ay_0\frac{d\dot S_1}{dy_0}+b\dot S_1=Qy_0\ , \tag{7}$$

where $a\equiv(\dot S_0+\dot S_0^*)(4\dot S_0+\dot S_0^*-i\delta)$ and $b\equiv3\dot S_0^2-2i\delta\dot S_0$. Equation (7) can be integrated with the initial condition $\dot S_1=0$ when $y_0=0$ (the linear regime). The solution is $\dot S_1=[Q/(a+b)]y_0$. To $O(y_0)$, we thus obtain the index of refraction of the electron beam in the nonlinear regime, $n=1+(\dot S_0+\dot S_1)/ik_s\equiv1+\lambda/k_s$, where $\lambda=\lambda_0+\beta|A|^2$, and

$$\beta=-\frac{2fh}{g}\frac{(\lambda_0-\delta)(\lambda_0+2\,\mathrm{Re}\lambda_0-\delta)}{2\,\mathrm{Im}\lambda_0[2\,\mathrm{Im}\lambda_0-i(3\lambda_0-\delta)]-3\lambda_0^2+2\delta\lambda_0}\ . \tag{8}$$

We remark that the quadratic dependence of n on $|A|$ means that in the saturation regime, even when gain is negligible, refractive optical guiding can, in principle, aid the confinement of radiation in a FEL. This effect has been observed in numerical simulations,[7,17] but remains to be verified experimentally.[18]

In Fig. 1, we compare the numerical results obtained by integrating the original FEL equations (2) with those obtained from the reduced equations (3) for a typical set of parameters. On the same plot, we show $A(z)=A_0\exp(i\lambda z)$ obtained from the WKB analysis.

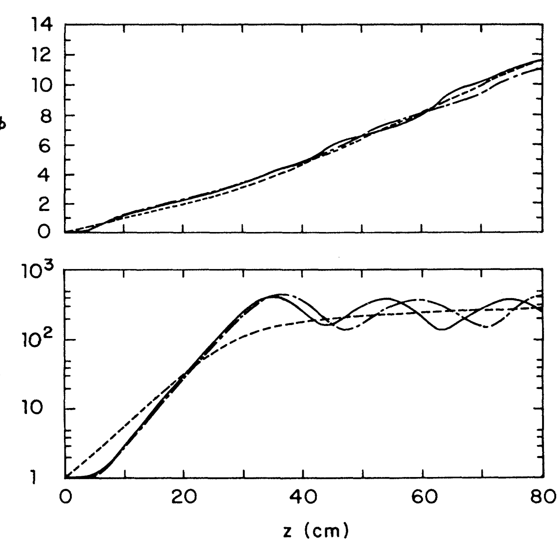

FIG. 1. Comparison of $|A|$ and ϕ calculated from the original equations (2) (solid lines), reduced equations (3) (dashed-dotted lines), and WKB analysis (dotted lines).

Whereas the analytical solution is remarkably good in predicting the phase shift, the prediction for $|A|$ is less accurate. In particular, the analysis predicts the approximate average saturation level for $|A|$, but cannot reproduce the oscillations caused by synchrotron motion. [The average intensity of the saturated radiation field can be estimated by simply setting $\mathrm{Im}(n)=0$, and is given by $|A|^2=-\mathrm{Im}\lambda_0/\mathrm{Im}\beta$.] In order to account for the oscillations at saturation, higher-order WKB calculations are needed. Note, however, that neglect of these higher-order terms in the nonlinear analysis for $|A|$ does not mean that the physics of synchrotron oscillations has been eliminated altogether. These oscillations are implicitly contained in the electron dynamical equations that involve Eqs. (3b) and (3c).

We have considered so far the single-frequency FEL equations. In the presence of multiple frequencies, it is extremely difficult to obtain a description of FEL dynamics in terms of a reduced set of equations involving collective variables. Instead, we exploit the analogy between the electron beam and an optical fiber to obtain heuristically an equation for the radiation field. We consider an optical signal of the form $\mathcal{A}(z,t)=A(z,t)\exp(ikz-i\omega_0 t)$, where $k_0=\omega_0/c$, and $A(z,t)$ is a slowly varying amplitude. Note that by including the time dependence in the amplitude $A(z,t)$, we have allowed for the excitation of multiple modes. At a given point $z=z_0$, the Fourier transformation of this signal is defined by the relation

$$\tilde{\mathcal{A}}(z_0,\Delta\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}dt'\exp(ik_0z_0+i\Delta\omega t')A(z_0,t')\ , \tag{9}$$

where $\Delta\omega=\omega-\omega_0$. Each frequency component is advanced along z according to the relation

$$\tilde{\mathcal{A}}(z_0+dz,\Delta\omega)=\tilde{\mathcal{A}}(z_0,\Delta\omega)\exp\{i[\omega/c+\lambda(\omega)]dz\}\ .$$

The inverse Fourier transform of $\mathcal{A}(z_0+dz,\Delta\omega)$ in the limit $dz\rightarrow 0$ then gives

$$\frac{\partial A(z,t)}{\partial z}=\frac{i}{2\pi}\int_{-\infty}^{+\infty}d\Delta\omega\int_{-\infty}^{+\infty}dt'A(z,t')\times\exp[i\Delta\omega(t'-t)]\times\left[\frac{\Delta\omega}{c}+\lambda(\omega)\right]. \tag{10}$$

For a FEL, $\lambda(\omega)$ usually has a narrow bandwidth around the maximum linear gain frequency ω_0. We can, therefore, expand $\lambda(\omega)$ in a Taylor series around ω_0, and write

$$\frac{\Delta\omega}{c}+\lambda(\omega)\simeq\lambda_0(\omega_0)+\beta|A|^2+v_g^{-1}\Delta\omega+\tfrac{1}{2}(\alpha_1+i\alpha_2)\Delta\omega^2+\cdots, \tag{11}$$

where $v_g=c/(1+c\partial\lambda/\partial\omega)$ is the group velocity of the signal, $\alpha_1+i\alpha_2=\partial^2\lambda_0/\partial\omega^2|_{\omega=\omega_0}$, α_1 is the group velocity dispersion and α_2 is the gain dispersion, and β is the complex coefficient defined by Eq. (8). Substituting Eq. (11) in (10) yields

$$\frac{\partial A}{\partial z}=i\lambda_0(\omega_0)A-v_g^{-1}\frac{\partial A}{\partial t}-\frac{i(\alpha_1+i\alpha_2)}{2}\frac{\partial^2 A}{\partial t^2}+i\beta|A|^2A, \tag{12}$$

which can be reduced to the GLE (1) by transforming to variables $z'=z$, $t'=t-z/v_g$, $\alpha\equiv\alpha_1+i\alpha_2$.

The GLE (1), with complex coefficients, is not integrable in general.[12] This rules out the possibility of spontaneous soliton formation during the nonlinear evolution of a FEL. However, special solitary wave solutions can be constructed following the methods described in Refs. 11 and 12. These solutions are

$$A(z',t')=\frac{q\exp(-i\Omega z')}{[\exp(Kt')+\exp(-Kt')]^{1+i\sigma}}\ , \tag{13}$$

where $\lambda_0=\zeta-i\chi$, $\beta=\beta_r+i\beta_i$,

$$|q|^2=\frac{8\chi(\alpha_1+i\alpha_2)(1+i\sigma)(2+i\sigma)}{\beta[\alpha_2(1-\sigma^2)+2\sigma\alpha_1]}\ , \tag{14a}$$

$$\Omega=\frac{\chi[\alpha_1(1-\sigma^2)-2\sigma\alpha_2]}{\alpha_2(1-\sigma^2)+2\sigma\alpha_1}+\zeta\ , \tag{14b}$$

$$K=\left[\frac{2\chi}{\alpha_2(\sigma^2-1)-2\alpha_1\sigma}\right]^{1/2}, \tag{14c}$$

and σ satisfies the quadratic equation

$$\sigma^2-3\frac{\alpha_1\beta_r+\alpha_2\beta_i}{\alpha_2\beta_r-\alpha_1\beta_i}\sigma-2=0\ . \tag{15}$$

Equation (15) has two real roots, and produces two families of solitary-wave solutions. In order to obtain simple analytical formulas for these solutions, we make certain approximations. First, we note that for a FEL operating near the maximum linear gain frequency ω_0, $\mathrm{Re}(n)$ is approximately a linear function of ω.[17] Therefore, we set $\alpha_1\simeq 0$ in Eq. (15), which reduces to $\sigma^2-3(\beta_i/\beta_r)\sigma-2\simeq 0$. For the case $|\beta_i|>|\beta_r|$, which is satisfied by the parameters of the FEL's considered here, the two roots obey the inequalities $\sigma_1^2\gg 1$ and $\sigma_2^2\ll 1$. When $\sigma^2=\sigma_1^2\gg 1$, $K_1\simeq[2\chi/(\alpha_2\sigma_1^2)]^{1/2}$ is real, and Eq. (13) gives

$$|A|^2=\frac{|q|^2}{4\cosh^2(K_1t')}\ . \tag{16}$$

The solution (16) for $|A|^2$ has a single peak at $t'=0$ ($z=v_gt$) with the half-width $\Delta t_1\simeq K_1^{-1}$. In the case $\sigma=\sigma_2$, $K_2\simeq i\sqrt{2\chi}/\alpha_2$, and Eq. (13) gives

$$|A|^2=\frac{|q|^2}{4\cos^2(K_2t')}\ . \tag{17}$$

The solution (17) exhibits periodic, finite-time singularities, as shown in Fig. 2. The width of each peak is given by $\Delta t_2\simeq K_2^{-1}$, and the separation between neighboring peaks is $\Delta T\simeq\pi K_2^{-1}$. Clearly, at the singularities, our model beaks down, and higher-order terms in the expansion of $\lambda(\omega)$ become significant.

We now show that the frequency of the periodic solution (17) has the same parametric dependence and is of the same order of magnitude as the amplitude oscillation frequency expected from sideband theory.[3] To see this, we note that the frequency of solution (17) is $\Delta\omega=\sqrt{-2\chi/\alpha_2}$. Since at saturation $|A|=\sqrt{\chi/\mathrm{Im}\beta}$, we write $\Delta\omega=[(-2a_s/\alpha_2)\sqrt{\chi\,\mathrm{Im}\beta}]^{1/2}$. From the cubic equation $\lambda_0^3-\delta\lambda_0^2+fgh=0$, assuming $\delta\simeq 0$, we obtain $\lambda_0^3\simeq -fgh$, which gives $\zeta=-\frac{1}{2}(fgh)^{1/3}$ and $\chi=(\sqrt{3}/2)(fgh)^{1/3}$. Also, from the cubic equation, it follows by straightforward algebra that $\alpha_2 \equiv \mathrm{Im}(\partial^2\lambda_0/\partial\omega^2)\simeq[-\sqrt{3}/(36\omega^2)][h^2/(fgh)^{1/3}]$. From Eq. (8), we get $\mathrm{Im}\beta\simeq(36\sqrt{3}/241)(fh/g)$. Using these expressions for α_2, χ, and $\mathrm{Im}\beta$, we then obtain

$$\Delta\omega\simeq 0.9\left[2\omega\left[\frac{a_s a_w}{1+a_w^2}\right]^{1/2}\right]. \qquad (18)$$

The expression in parentheses on the right-hand side of Eq. (18) is the same as the frequency separation of a sideband from the carrier signal predicted by standard sideband theory.[3] We also note that solutions resembling the solitary-wave solutions can be generated by the time-dependent FEL equations, which include the effect of slippage. We have used our computer code[19] to carry out simulations of the experiment in Ref. 5. The simulation shows "spikes" of width $\Delta t\simeq 25$ psec separated by periods of 80 psec. [See Fig. 3(a) of Ref. 5.] For the FEL described in Ref. 5, we calculate numerically the parameters $\chi=0.11$ cm^{-1} and $\alpha_2=5.3\times10^{-21}$ sec^2/cm. The analytical solutions (17) then predict periodic spikes of width $\Delta t_2\simeq 154$ psec. The experimentally measured width ~ 150 psec.

We now compare the analytical solutions with results from the experiment of Ref. 4, for which $\lambda_w=2.7$ cm, $a_w=0.8$, $\lambda_s=9.85$ μm, $I=39.3$ A, $r_b=0.1$ cm, and $\gamma_0=42.7$. From the linear cubic equation (5), we find $\chi=0.018$ cm^{-1} and $\alpha_2=2.0\times10^{-27}$ sec^2/cm; then Eq. (17) gives $\Delta t_2\simeq 0.24$ psec. The experimentally observed width is 0.2 psec.[4]

Finally, in the FEL experiment in Ref. 6, the pulse separation (not width) is measured by an autocorrelation technique and is confirmed by numerical simulations. By using the parameters given in Ref. 6 (except that the energy spread is taken to be zero and the electron-beam radius to be 0.02 cm) in the cubic equation, we get $\chi=0.026$ cm^{-1}, $\alpha_2=1.24\times10^{-24}$ sec^2/cm. The theoretically predicted pulse separation for the solution (17) is $\Delta T\simeq 0.22$ psec. The experimentally measured separation is 0.8 psec.

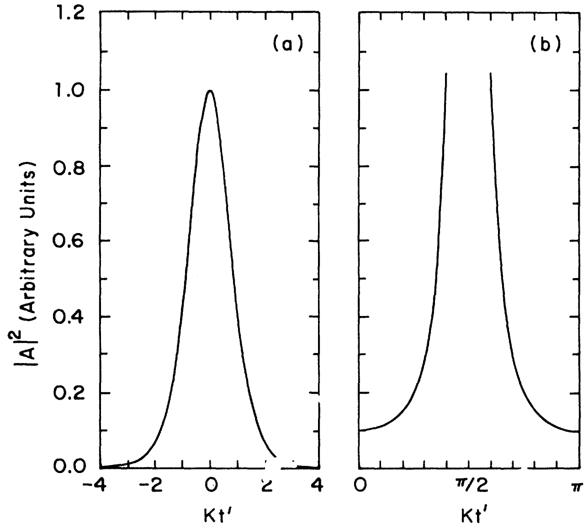

FIG. 2. Solitary-wave solutions of the Ginzburg-Landau equation. Solution (16) is plotted in (a) and solution (17) in (b).

We emphasize that our model is rather simple, and the remarkable accord with experimental results (from Refs. 4 and 5) should therefore be taken with a grain of salt. However, we believe that the essential idea of spikes as a form of solitary wave propagating in the "fiber," that is, the electron beam, is qualitatively supported by experimental results.

In comparing theory with experiments, we have so far used the singular, periodic solution (17). The solution (16), which is nonsingular but aperiodic, also predicts spike widths that are of the same order as those obtained from (17). Clearly, initial and boundary conditions, as well as issues of stability, will determine the dominance of particular solitary-wave solutions in a given experiment.

Note added in proof. After the completion of this work, we found that for a small-gain oscillator, a Ginzburg-Landau equation (of rather different form than has been presented in this paper) has been given by Colson and Ride.[20]

We are deeply indebted to Professor T. C. Marshall for many stimulating discussions, and to Mr. J. Cao for his involvement and interest. This work is supported by the Office of Naval Research, Grant No. N0014-89-J-1652 and the National Science Foundation, Grant No. ECS-89-122581.

[1]See, for instance, T. C. Marshall, *Free Electron Lasers* (MacMillan, New York, 1985), and the recent review paper by C. W. Roberson and P. Sprangle, Phys. Fluids B **1**, 3 (1989).

[2]N. M. Kroll, P. L. Morton, and M. N. Rosenbluth, IEEE J. Quantum Electron. **QE-17**, 1436 (1981); A. T. Lin and J. M. Dawson, Phys. Rev. Lett. **42**, 1670 (1979); P. Sprangle, C.-M. Tang, and W. M. Mannheimer, *ibid.* **43**, 1932 (1979).

[3]N. M. Kroll and M. N. Rosenbluth, in *Physics of Quantum Electronics* (Addison-Wesley, Reading, MA, 1980), Vol. 7, p. 147.

[4]R. W. Warren, J. C. Goldstein, and B. E. Newnam, Nucl. Instrum. Methods Phys. Res. A **250**, 104 (1986).

[5]J. W. Dodd and T. C. Marshall, IEEE Trans. Plasma Sci. **18**, 447 (1990).

[6]B. A. Richman, J. M. J. Madey, and E. Szarmes, Phys. Rev. Lett. **63**, 1182 (1989).

[7]A. Bhattacharjee, J. Cao, and S. Y. Cai, in *Proceedings of the International Conference on Free Electron Lasers,* 1989, Naples, Florida (IEEE Lasers and Electro Optics Society, Naples, 1989), p. 35; see also J. Cao, A. Bhattacharjee, and S. Y.

Cai, Bull. Am. Phys. Soc. **34**, 1983 (1989).

[8]E. T. Scharlemann, A. M. Sessler, and J. S. Wurtele, Phys. Rev. Lett. **54**, 1925 (1985).

[9]See, for instance, A. E. Siegman, *Lasers* (University Science, Mill Valley, CA, 1986), p. 395.

[10]A. Hasegawa and F. Tappert, Appl. Phys. Lett. **23**, 142 (1973); **23**, 171 (1973).

[11]Here, the term "soliton" describes "a pulselike nonlinear wave (solitary wave) which emerges from a colision with a similar pulse having unchanged shape and speed" [A. C. Scott, F. Y. F. Chu, and D. W. McLaughlin, Proc. IEEE **61**, 1443 (1973)].

[12]F. Cariello and M. Tabor, Physica D **39**, 77 (1985).

[13]N. Bekki and I. Nozaki, Phys. Lett. **110**, 133 (1985).

[14]I. Gjaja and A. Bhattacharjee, Opt. Commun. **58**, 201 (1986); **62**, 39 (1987), and references cited therein.

[15]R. Bonifacio, F. Casagrande, and L. De Salvo Souza, Phys. Rev. A **33**, 2836 (1986).

[16]R. Bonifacio, C. Pellegrini, and L. M. Narducci, Opt. Commun. **50**, 373 (1984).

[17]S. Y. Cai, A. Bhattacharjee, and T. C. Marshall, IEEE J. Quantum Electron. **QE-23**, 1651 (1987).

[18]A. Bhattacharjee, S. Y. Cai, S. P. Chang, J. W. Dodd, A. Fruchtman, and T. C. Marshall, Phys. Rev. A **40**, 5081 (1989).

[19]S. Y. Cai, A. Bhattacharjee, S. P. Chang, J. W. Dodd, and T. C. Marshall, Phys. Rev. A **40**, 3127 (1989).

[20]W. B. Colson and S. K. Ride, in *Physics of Quantum Electronics* (Ref. 3), Vol. 7, p. 377.