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Generation of the Schrödinger-cat state by continuous photodetection

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Nonunitary time evolution of an initially highly squeezed state under continuous photon-number measurement is investigated using the quasiprobability distribution. It is found that the measured quantum state bifurcates into two macroscopically distinguishable states upon the detection of a single photon, while preserving purity by feeding back real-time readout information.

The Schrödinger-cat paradox is among the oldest puzzles in the interpretation issue of quantum mechanics. This paradox has attracted a great deal of interest[1] because it poses a serious problem of whether quantum mechanics—laws governing the microscopic world—can be applied to the macroscopic world. Like the Einstein-Podolsky-Rosen paradox, the Schrödinger-cat paradox has long been regarded as untestable, partly because it seems extremely difficult to experimentally produce a quantum-mechanical superposition of macroscopically distinguishable states, namely "the Schrödinger-cat state."[2]

Yurke and Stoler[3] have examined an anharmonic-oscillator model, showing that an initially coherent state evolves into a coherent superposition of quantum states via amplitude dispersion. Mecozzi and Tombesi[4] proposed another scheme for generating a similar state via nonlinear birefringence. Recently, Yurke, Schleich, and Walls[5] considered the generation of this type of state by quantum nondemolition measurement in nonlinear optical media. These schemes involve unitary evolution of a system followed by conditional measurement, where the unitary evolution is used to correlate two relevant modes, while the nonunitary conditional measurement prepares the quantum superposition state.

In this Rapid Communication, we propose an alternative scheme for generating a Schrödinger-cat state. Our scheme does not utilize any optical nonlinearities, but is based only on *nonunitary* state evolution of the measured photon field under *continuous photon-number measurement.*[6-12] The unique feature of our scheme is that it does not use unitary evolution to correlate two photon modes, but that the cat states can be generated from a *single-mode* photon field by continuous measurement of photon number. The temporal developments of the density operator of the measured photon field and its Q representation (quasiprobability distribution) are calculated during the photon counting to illustrate the temporal formation of a cat state. Furthermore, it is shown that the pure-state character of the measured field is preserved *during* the measurement process.

Suppose that the continuous photon-number measurement[6-11] of the photon field, $\rho(0)$, starts at $t=0$ and that m photons are registered at times $\tau_j \in [0,T]$ ($j=1$ $2,\ldots,m$) with no further photons registered in the measurement period ($0 \le t \le T$). Such a process is referred to as the quantum photodetection process of forward recurrence times (QPF).[6,7] The density operator of the photon field, $\rho_m^{\mathrm{QPF}}(T)$, immediately after the QPF is given by[6-8]

$$\rho_m^{\mathrm{QPF}}(T)=\frac{S_{T-\tau_m}JS_{\tau_m-\tau_{m-1}}J\cdots S_{\tau_1}\rho(0)}{\mathrm{Tr}[S_{T-\tau_m}JS_{\tau_m-\tau_{m-1}}J\cdots S_{\tau_1}\rho(0)]}=\frac{\exp[-(i\omega+\lambda/2)a^\dagger aT]a^m\rho(0)(a^\dagger)^m\exp[(i\omega-\lambda/2)a^\dagger aT]}{\mathrm{Tr}[\rho(0)(a^\dagger)^m\exp(-\lambda a^\dagger aT)a^m]}, \quad (1)$$

where λ represents the coupling constant between the field and the photodetector, and the superoperators J and S_τ, which stand for the one-count and no-count processes, respectively, are defined as $J\rho(t)\equiv\lambda a\rho(t)a^\dagger$ and

$$S_\tau\rho(t)\equiv\exp[-(i\omega+\lambda/2)a^\dagger a\tau]\times\rho(t)\exp[(i\omega-\lambda/2)a^\dagger a\tau]$$

(see Refs. 6–11).

The quadrature-amplitude squeezed state, $|\alpha,r\rangle\equiv S(r)D(\alpha)|0\rangle$, is chosen as an initial state, $\rho(0)=|\alpha,r\rangle\langle\alpha,r|$, where $D(\alpha)$ is the displacement operator and $S(r)$ is the squeezing operator with a squeezing parameter r, i.e.,

$$D(\alpha)\equiv\exp(\alpha a^\dagger-\alpha^* a), \quad (2a)$$

$$S(r)\equiv\exp\left[\frac{r}{2}(aa-a^\dagger a^\dagger)\right]. \quad (2b)$$

Let us confine ourselves to the case where α is real and the squeezing parameter is positive, $r\ge 0$. The photon density operator (1) is then given as[8]

$$\rho_m^{\mathrm{QPF}}(t)=\frac{1}{N_m(t)}\sum_{k,n=0}^{\infty}\frac{1}{(k!n!)^{1/2}}\left(\frac{\tanh r}{2}\right)^{(k+n)/2}\exp\left(-\frac{\lambda}{2}(k+n)t\right)H_{k+m}\left(\frac{\alpha}{(\sinh 2r)^{1/2}}\right)H_{n+m}\left(\frac{\alpha}{(\sinh 2r)^{1/2}}\right)|k\rangle\langle n|, \quad (3)$$

where

$$N_m(t)\equiv 2^m\left\{\frac{\partial^m}{\partial y^m}\left[\frac{1}{(1-y^2)^{1/2}}\exp\left[\frac{2y}{1+y}\frac{\alpha^2}{\sinh 2r}\right]\right]\right\}_{y=e^{-\lambda t}\tanh r}, \tag{4a}$$

$$H_n(z)\equiv(-1)^n\exp(z^2)\frac{d^n}{dz^n}\exp(-z^2), \tag{4b}$$

for $m,n=0, 1, 2, \ldots$. Here, the rapidly oscillating term $e^{-i(k-n)\omega t}$ is omitted because we are interested only in the envelope function.

In order to investigate the effects of the one-count and no-count processes on the photon field and to illustrate visually the temporal formation of a quantum-mechanical superposition state of two macroscopically distinguishable states, the Q representation of the field is used. The Q representation of the field at time t in the QPF is given, from Eq. (3), as

$$Q_m^{\rm QPF}(\beta_1,\beta_2;t)\equiv\frac{1}{\pi}\langle\beta|\rho_m^{\rm QPF}(t)|\beta\rangle=\frac{e^{-|\beta|^2}}{\pi N_m(t)}\sum_{\mu=0}^{\infty}\frac{(\gamma^*)^\mu}{\mu!}H_{\mu+m}(z)\sum_{\nu=0}^{\infty}\frac{\gamma^\nu}{\nu!}H_{\nu+m}(z)$$
$$=\frac{e^{-|\beta|^2}}{\pi N_m(t)}\left[\frac{\partial^m}{\partial y^m}\exp(2yz-y^2)\right]_{y=\gamma^*}\left[\frac{\partial^m}{\partial y^m}\exp(2yz-y^2)\right]_{y=\gamma}, \tag{5}$$

where $\beta=\beta_1+i\beta_2$ is a complex variable (β_1 and β_2 are real and correspond to the amplitudes of two quadrature phases of the field), $|\beta\rangle$ is the coherent state, and

$$\gamma\equiv\beta\left(\tfrac{1}{2}e^{-\lambda t}\tanh r\right)^{1/2}, \tag{6a}$$

$$z\equiv\frac{\alpha}{(\sinh 2r)^{1/2}}>0. \tag{6b}$$

This is a general expression of the time-developed Q representation for an initially squeezed state in the QPF.

While no photons are detected ($m=0$; no-count process), the Q representation evolves as

$$Q_0^{\rm QPF}(\beta_1,\beta_2;t)=\frac{e^{-|\beta|^2}}{\pi N_0(t)}\exp\left[-\left[(\beta_1^2-\beta_2^2)e^{-\lambda t}\tanh r-\frac{2\alpha e^{-\lambda t/2}}{\cosh r}\beta_1\right]\right]. \tag{7}$$

For larger r, the Q representation has a squeezed Gaussian form situated along a line parallel to the β_2 axis [see Fig. 1(a)].[13] As time proceeds, this state approaches the vacuum state whose Q representation is a symmetric Gaussian distribution around zero,

$$\lim_{t\to\infty}Q_0^{\rm QPF}(\beta_1,\beta_2;t)=\frac{e^{-|\beta|^2}}{\pi}. \tag{8}$$

It is remarkable that the initially highly squeezed state, which has a large average photon number, reduces to the vacuum state in the QPF despite the fact that no photons were actually detected in the measurement process. This is a unique feature of continuous measurement. The physics underlying such an unexpected result is that the readout information of no counts having been registered during a time interval require us to modify the knowledge about the original photon density operator according to Eq. (1).[6-8,14] In other words, we are renormalizing the density operator of the field *every moment,* according to the real-time readout of the continuous measurement.

Throughout the no-count process, the Q representation keeps a single-peak character, and is squeezed in a Gaussian form, its cross section being an ellipse centered at

$$\left[\frac{\alpha e^{-\lambda t/2}}{(\cosh r)(1+e^{-\lambda t}\tanh r)},0\right]. \tag{9}$$

However, as soon as one photon is detected ($m=1$), the Q representation abruptly changes into

$$Q_1^{\rm QPF}(\beta_1,\beta_2;t)=2e^{-\lambda t}(\tanh r)\frac{N_0(t)}{N_1(t)}\times\left[\left[\beta_1-\frac{\alpha e^{\lambda t/2}}{\sinh r}\right]^2+\beta_2^2\right]Q_0^{\rm QPF}(\beta_1,\beta_2;t). \tag{10}$$

Thus, we find from the β_2^2 term in the right-hand side that the quasiprobability distribution just after the one-count process is suppressed around $\beta_2=0$ and enhanced around a large value of $|\beta_2|$ compared to the quasiprobability distribution before the one-count process [see Eq. (7)], resulting in a double-peak structure. This corresponds to the fact that matrix elements ρ_{mn} with large m and n [i.e., $(m+1)(n+1)>\langle n(t)\rangle^2$] are enhanced by the one-count process.[8] In the case of small α, large r, and small λt, regions around a large circle centered at the origin in the phase space (β_1,β_2) are especially enhanced by the one-count process, which leads to an increase in the photon-number fluctuations.[15]

Figures 1(a) and 1(b) illustrate the Q representations (a) just before and (b) just after a one-count process. From these figures, we find that the single-peaked Q representation changes instantaneously to a double-peaked one when a single photon is detected. The double-peaked structure indicates super-Poissonian statistics because the variance of the distance from the origin becomes large.

The subsequent one-count process further emphasizes the two-peaked character of the Q representation [see Fig. 1(c)]. In fact, the Q representation after two one-count processes is given by

$$Q_2^{\rm QPF}(\beta_1,\beta_2;t)=4\frac{N_0(t)}{N_2(t)}q_2(\beta_1,\beta_2;t)Q_0^{\rm QPF}(\beta_1,\beta_2;t)\,, \tag{11}$$

where $q_2(\beta_1,\beta_2;t)$ is an enhancement factor for the $m=2$ case and is given by

$$q_2(\beta_1,\beta_2;t)=1+|\beta|^4e^{-2\lambda t}\tanh^2 r+\frac{4a^2e^{-\lambda t}}{\cosh^2 r}|\beta|^2+\frac{4a^2}{\sinh 2r}\left[\frac{a^2}{\sinh 2r}-1\right]$$
$$+2\left[\frac{2a^2}{\sinh 2r}-1\right]e^{-\lambda t}(\tanh r)(\beta_1^2-\beta_2^2)+\frac{4ae^{-\lambda t/2}}{\cosh r}\left[1-\frac{2a^2}{\sinh 2r}-|\beta|^2e^{-\lambda t}\tanh r\right]\beta_1\,. \tag{12}$$

Note that the fifth term of Eq. (12) is essential to yield two peaks of the Q function.

This double-peak distribution has two important features: (i) It is clear from Figs. 1(b) and 1(c) that the two peaks shown in the Q representation have different quadrature phases. If we measure such a state using a phase-sensitive detection scheme, the result will yield values sharply distributed around the two peaks. Therefore we can conclude that two macroscopically distinguishable states are generated from the single-mode photon field in a highly squeezed state by the continuous photodetection. The larger the squeezing parameter r is, the more distinguishable these two peaks become. However, here we can conclude only that the obtained state is either a superposition or a mixture of two macroscopically distinguishable states. (ii) The obtained state is found to be a quantum superposition state; that is, the double-peaked photon state is a *pure* state. This is evident from the fact that the density operator of the measured photon state, Eq. (3), satisfies the idempotency condition,

$$[\rho_m^{\rm QPF}(t)]^2=\rho_m^{\rm QPF}(t)\,, \tag{13}$$

for arbitrary time $t\geq 0$ and for any number of photocounts $m=0, 1, 2, \ldots$. Thus, we can conclude that the obtained state is the Schrödinger-cat state. It is remarkable that this idempotency holds generally, provided that a pure state is initially prepared. The proof is as follows. Any pure state can be written as $\rho=|\psi\rangle\langle\psi|$. Then the square of the density operator after the photodetection becomes

$$[\rho(T)]^2=\frac{\exp[-(i\omega+\lambda/2)a^\dagger aT]a^m|\psi\rangle\langle\psi|(a^\dagger)^m e^{-\lambda a^\dagger aT}a^m|\psi\rangle\langle\psi|(a^\dagger)^m\exp[(i\omega-\lambda/2)a^\dagger aT]}{\{{\rm Tr}[|\psi\rangle\langle\psi|(a^\dagger)^m e^{-\lambda a^\dagger aT}a^m]\}^2}\,, \tag{14}$$

where Eq. (1) is used. From Eq. (14) and

$${\rm Tr}[|\psi\rangle\langle\psi|(a^\dagger)^m e^{-\lambda a^\dagger aT}a^m]=\langle\psi|(a^\dagger)^m e^{-\lambda a^\dagger aT}a^m|\psi\rangle\,, \tag{15}$$

we obtain $[\rho(T)]^2=\rho(T)$.

This result can be intuitively understood from the following consideration: Since we utilize all readout information concerning the results of measurement to renormalize the initial density operator (such a measurement process is referred to as the referring measurement process[6-8]), there is no room for dissipation of information in the measurement process. Hence, we have Eq. (13). In other words, an initially squeezed state evolves nonunitarily in the referring measurement process, but it remains a pure state, even though photon counting is a second-kind measurement. From the two features described above, we can conclude that the double-peaked state is the Schrödinger-cat state (i.e., the quantum-mechanical superposition state of macroscopically distinguishable states). Here we note that this state is not a quantum superposition of two *coherent* states, but of two quadrature-amplitude squeezed states, which can also be confirmed by Figs. 1(b) or 1(c), where each peak is squeezed. This marks another distinction from Refs. 3-5.

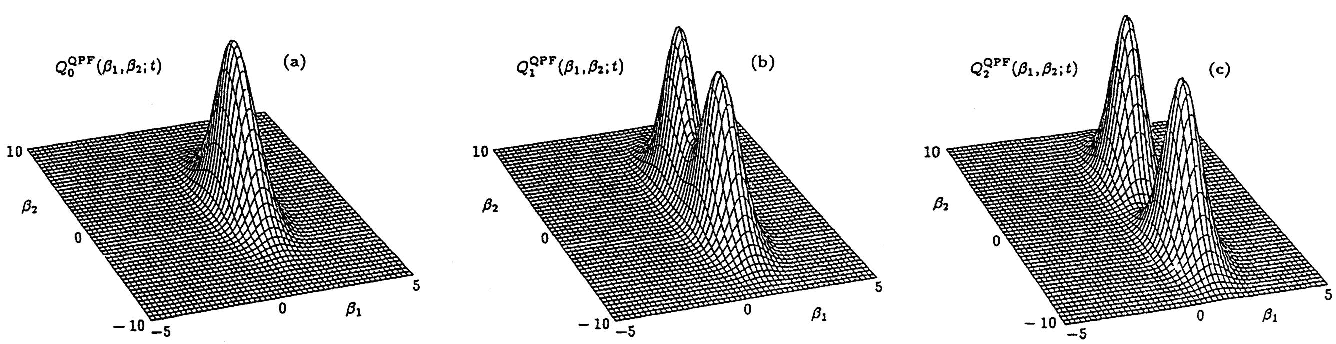

FIG. 1. The quasidistributions of the measured photon state (a) just before and (b) just after the one-count process and (c) just after two photoelectrons are detected. The initial field is chosen to be the quadrature-amplitude squeezed state with $a=10.0$ and $r=1.5$. The quasidistributions of (b) and (c) have two distinct peaks, indicating the Schrödinger-cat state.

There are two important requirements in our scheme for generating the cat state. One is the preparation of a suitable initial state. To put this method into practice, we must prepare a highly squeezed state whose statistical properties are completely known. The other requirement concerns the times at which photons are detected. Photoelectrons should be counted at times long before the photon state approaches the vacuum state; that is, for effective formation of the double-peaked distribution, the photons should be detected as soon as the measurement starts. This varies with each experiment. If photons are detected at suitable times, we should switch off the photodetector. Otherwise, the two peaks that are formed will again become confluent, approaching the origin as time passes.

We have proved the idempotency relation [Eq. (13)] to distinguish *theoretically* a quantum superposition state from a statistical mixture. It is also worth noticing an *experimentally detectable* signature of a pure superposition state. To this end, the quadrature-phase measurement proposed by Yurke and Stoler[3] is applicable to our system. That is, an interference between the two macroscopically distinguishable states of the superposition can be observed as fringes in the probability distribution for the homodyne-detector output current. In this experiment, $|\alpha|$ should be small, the phase of the local-oscillator light is chosen properly,[3] and this measurement must be made in a short time in comparison with λ^{-1} which is a characteristic time of the photon-state evolution.[16] Then we can distinguish experimentally the superposition from a mixture to check whether a quantum superposition has in fact been generated.

In practical experiments, the quantum efficiency of photodetectors is much lower than 100%. In this paper, however, we confine ourselves to the case of perfect quantum efficiency (100%) in order to propose this phenomenon and to avoid some complexities. Needless to say, we should pay attention to effects arising from the imperfect quantum efficiency when our formalism is applied to practical problems. This is left for future study.

Finally, we mention the uncertainty relations of the photon field. When no photon is detected ($m=0$), the uncertainty product, that is, the variances of two quadrature-phase components, always satisfies the minimum uncertainty relation ($\langle[\Delta a_1(t)]^2\rangle\langle[\Delta a_2(t)]^2\rangle=\frac{1}{16}$). However, detecting at least one photoelectron destroys the minimality, thus increasing the quantum fluctuations of the measured field in spite of the a_1 variance being reduced. Therefore the obtained Schrödinger-cat state is not the minimum uncertain state with respect to the quadrature-phase amplitudes.

Quantum measurement modifies quantum fluctuations of the measured photon state because of the measurement back action. Making appropriate use of this effect provides an alternative scheme for generating an additional quantum state. If we can prepare a well-defined highly squeezed state as an initial field, it changes according to the experimental results, and thus we get another state when the measurement is finished. The effect of the one-count processes on the state characteristic is favorable for generating a quantum superposition of two macroscopically distinguishable states (the Schrödinger-cat state). As shown in Fig. 1, the Q representation becomes double peaked after photoelectrons are detected. Our proposed method generates a quantum superposition state generated from a single-mode photon field using only quantum measurement without any optical nonlinearities.

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[16]Characteristics of an intracavity field at time t can be measured, at least in principle, by means of, for example, (i) inserting a phase-sensitive detector, whose coupling with the photon field is very strong, into the cavity instantly at t, or (ii) attaching the detector with a large aperture to a cavity end just after an end mirror is opened all over the end surface. Thus, in principle, we can observe the quasiprobability remaining inside the cavity.