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# Energy sharing in a chaotic multimode laser

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Energy sharing among the globally coupled cavity modes of a chaotic laser is characterized by probability distributions of the output intensity. While the distribution for an individual mode intensity is highly non-Gaussian, the distribution for the total intensity is determined to be approximately Gaussian. Numerical integration of the nonlinear coupled differential equations that describe the laser dynamics yields results that agree very well with experimental observations.

Many complex physical, chemical, and biological systems can be accurately modeled as a collection of globally coupled nonlinear oscillators. Essential dynamical aspects of Josephson-junction arrays,[1] chemical turbulence,[2] and heartbeat rhythms,[3] for example, have been explained using such models. One particularly crucial feature of these coupled oscillator systems is the dynamics of energy sharing among the oscillators.[4]

When a driven, dissipative, globally coupled oscillator system is in a chaotic state, a complex energy-sharing process may occur among the oscillators. Probability distributions can be used to characterize the fluctuations of the total energy of the system as well as the energy of an individual oscillator. Kaneko has recently used probability distributions to study $N$ globally coupled logistic maps as a mean-field-type extension of coupled-map lattices.[5] He has shown the interesting result that mean-field fluctuations for these logistic maps are approximately Gaussian distributed despite the fact that they are coupled; parameter-dependent deviations from a Gaussian distribution were observed in computations even for large $N$. Kaneko also showed that for large $N$ the two-point mutual information is extremely small but remains finite and is responsible for the breakdown of the law of large numbers. This implies that any two individual maps are not statistically independent in their fluctuations.

In this paper we use probability distributions to analyze the energy sharing among the globally coupled axial modes of a chaotic neodymium-doped yttrium aluminum garnet (Nd:YAG) laser system.[6–9] However, unlike the mean-field approximation, global coupling is a natural description for a multimode laser, and not a computationally expedient approximation. A characterization of the energy sharing is provided by the probability distributions of the total and individual mode intensities as well as by an examination of their time evolution. Integration of a numerical model describing the laser dynamics yields predictions for the probability distributions and time traces that are remarkably similar to the experimental results. We find that even though the probability distribution of an individual mode intensity is highly non-Gaussian, the distribution of the total intensity is approximately Gaussian. This result is reminiscent of Kaneko's study of globally coupled maps; it is, however, observed for the case of a few globally coupled laser modes which are strongly statistically dependent, as will be shown.

A schematic of the laser experiment is shown in Fig. 1. When the laser is pumped several times above threshold, many longitudinal modes can be active simultaneously. Chaotic fluctuations in the output intensity are induced by the potassium titanyl phosphate (KTP) crystal, which nonlinearly couples the modes through sum-frequency generation. Each cavity mode has one of two independent linear polarizations, which we label as $x$ and $y$. We suppose that there are $N$ modes in all, with $m$ having $x$ polarization, and $n=N-m$ having $y$ polarization. We can vary $N$ from 1 to $\sim 10$, with a number of accompanying polarization combinations, by varying the pump excitation above threshold and by careful rotational alignment of the Nd:YAG and KTP crystals. Chaos is observed only for $N \geq 3$.[10] The experiments described here were performed on a laser with five modes ($N=n+m=5$) in all, one ($m=1$) $x$-polarized mode and four ($n=4$) $y$-polarized modes. The differential equations used to describe this system are[6–9]

$$\tau_c \frac{dI_k}{dt} = \Big[ G_k - \alpha_k - g\epsilon I_k - 2\epsilon \sum_{j\,(\neq k)} \mu_j I_j \Big] I_k \,,$$
$$\tau_f \frac{dG_k}{dt} = \gamma - \Big[ 1 + I_k + \beta \sum_{j\,(\neq k)} I_j \Big] G_k \,, \qquad (1)$$

where $\tau_c$ ($=0.2$ nsec) and $\tau_f$ ($=240$ $\mu$sec) are the cavity round trip time and fluorescence time, respectively; $I_k$ and $G_k$ are, respectively, the intensity and gain associated with the $k$th longitudinal mode; $\alpha_k$ is the cavity loss parameter, $\gamma$ ($=0.05$) is the gain parameter, $\beta$ ($=0.7$) is the cross-saturation parameter, $\epsilon$ ($=5.0\times10^{-6}$) is a parameter

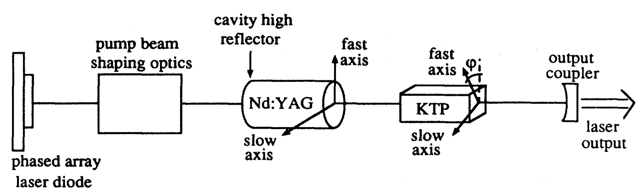


FIG. 1. Schematic of diode pumped Nd:YAG laser with intracavity KTP doubling crystal. The laser cavity is highly reflecting for the fundamental 1064-nm wavelength, but transmits the doubled green light at 532 nm. The relative angular orientation of the Nd:YAG and KTP crystals is adjusted to obtain a given combination of orthogonally polarized modes.

 

that depends on the nature of the second-harmonic-generating crystal, and $g$ ($=0.1$) is a geometrical factor whose value depends on the orientation of the YAG crystal relative to the KTP doubling crystal as well as the phase delays due to their birefringence.[9] Here, $\mu_j = g$ for modes having the same polarization as the $k$th mode, while $\mu_j = 1-g$ for modes having the opposite polarization. We have made the simplifying approximation that the gain $\gamma$ and cross-saturation parameter $\beta$ are the same for all modes. Cross-saturation of the active medium [represented by the $\beta I_j G_k$ terms in Eq. (1)] and sum-frequency generation in the intracavity nonlinear crystal [represented by the $2\epsilon\mu_j I_j I_k$ terms in Eq. (1)] introduce global coupling among the laser modes. The individual mode losses are assumed to differ only slightly, with $\alpha_k \sim 0.01$. The exact values of these parameters affect detailed aspects of the probability distributions. The parameter values given above represent typical experimental operating conditions.

A theoretical analysis of these equations has successfully predicted conditions for stable operation.[8,9] The equations also accurately describe periodic phenomena, such as antiphase states.[11] Here we use these equations to examine the chaotic dynamics of the laser.

The diode pumped Nd:YAG laser was carefully aligned to support a total of five axial modes, composed of one $x$-polarized and four $y$-polarized modes. The second harmonic was filtered from the laser output, and only the 1064-nm fundamental wavelength was incident upon a photodiode. The axial mode structure was monitored during the experiment with a confocal Fabry-Pérot interferometer. The photodiode signal was observed and stored on a digital oscilloscope interfaced to a microcomputer. The digitized signal was transferred to the computer and the total intensity probability distribution was calculated. Repeating this procedure allowed us to obtain a probability distribution accumulated from many time traces. A polarizing prism was then inserted before the photodiode, allowing us to obtain time traces and probability distributions of either the $x$- or the $y$-polarized intensity.

Time traces for the single $x$-polarized mode intensity, the $y$-polarized intensity and the total intensity are shown in Figs. 2(a)–2(c), respectively. The 1-msec time interval for measurement of these time traces is much longer than the typical time scale of chaotic fluctuations, as is clear from Fig. 2. Note that the experimental data was not obtained simultaneously. Figures 2(d)–2(f) show the corresponding results from an integration of the nonlinear Eq. (1) over 1 msec with a 10-nsec time step. The similarity in overall character of these time traces is quite remarkable. The numerical data has been scaled in order to account for the transmission of the prism. The International Mathematics and Scientific Library subroutine DGEAR was used to perform these computations. The largest Liapunov exponent calculated from this integration is $1.6\times10^4$ sec$^{-1}$, indicative of strongly chaotic behavior for this system. The time for separation of trajectories ($\approx 60$ $\mu$sec) is thus very short compared with the total time of measurement (fifteen trajectories of 1 msec) for accumulation of the probability distributions.

Figures 3(a)–3(c) show the experimental probability distributions accumulated from fifteen time traces of length 1 ms with 4000 samples each. The distribution for the total intensity is found to be approximately Gaussian.

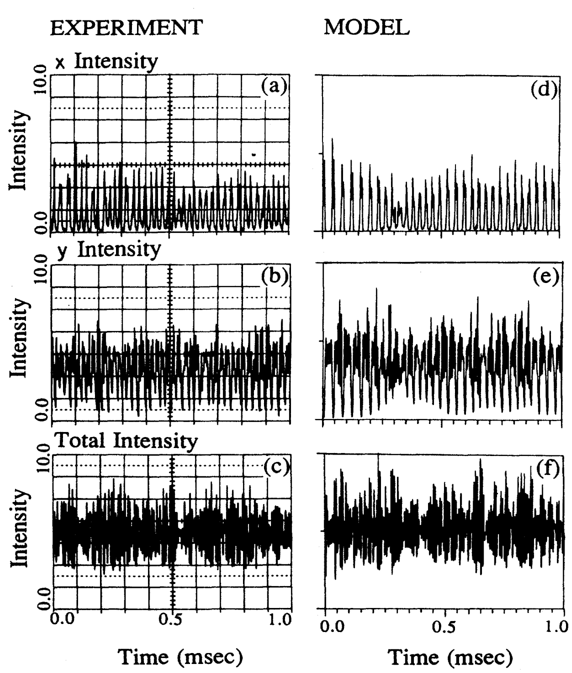


FIG. 2. (a)–(c) Experimental time traces for the $x$ polarized, $y$ polarized, and total intensities. (d)–(f) Corresponding time traces from numerical integration of Eq. (1).

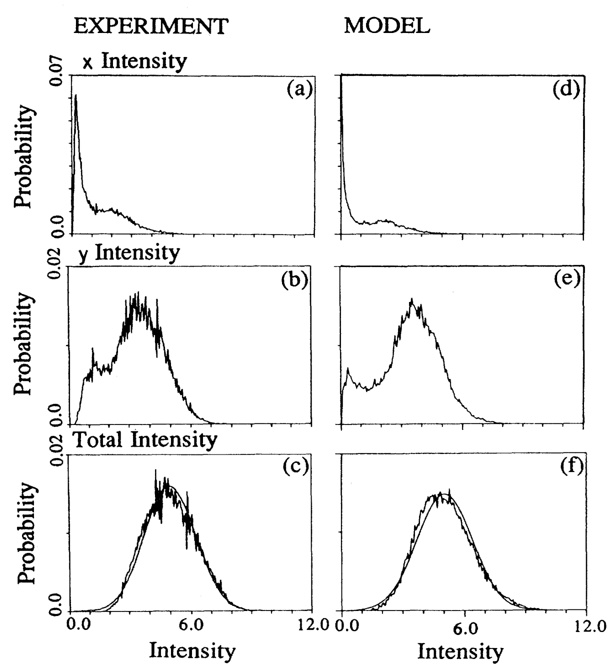


FIG. 3. (a)–(c) Experimental probability distributions for the $x$ polarized, $y$ polarized, and total intensities. (d)–(f) Corresponding probability distributions from numerical integration of Eq. (1).

To facilitate this comparison, a Gaussian calculated from the mean and variance of the total intensity fluctuations is included in Fig. 3(c). Figures 3(a) and 3(b) are the distributions for the single $x$-polarized mode intensity and the $y$-polarized intensity, respectively. Note that both these distributions are highly non-Gaussian in character.

Corresponding probability distributions calculated from the differential Eq. (1) integrated over 4.5 msec are shown in Figs. 3(d)–3(f). To enable direct comparison between the theoretical and experimental results, the bin widths relative to the mean have been set equal for corresponding distributions. The resemblance of the distributions is striking. To our knowledge, such close correspondence between experiment and theory for the statistical measures that characterize chaotic dynamics is very difficult to achieve. It can be seen that the experimental and theoretical probability distributions for the total intensity, though approximately Gaussian, show very similar deviations from the Gaussian fits.

The distributions for the $x$ and $y$ polarizations (Fig. 3) are seen to be double peaked. A polarizing beam splitter was used to separate the two polarization components and obtain simultaneous measurements of their intensities. Figure 4 shows a strong anticorrelation between the experimentally observed $x$- and $y$-polarized intensities. The sum of the anticorrelated intensities has a distribution which is single peaked and approximately Gaussian [Figs. 3(c) and 3(f)].

The central limit theorem does not usually apply to strongly dependent random variables.[12] The modes in the laser described here are coupled, and are strongly dependent random variables. Yet, the total intensity of the laser is approximately Gaussian distributed. The results presented in this paper are not unique to the laser operating with five modes. We have made similar observations when the laser is operated in a chaotic state with three to ten lasing modes over a large range of pump excitation and mode polarizations. We cannot, of course, rule out the possibility of a non-Gaussian distribution of the total intensity for some other laser configuration.

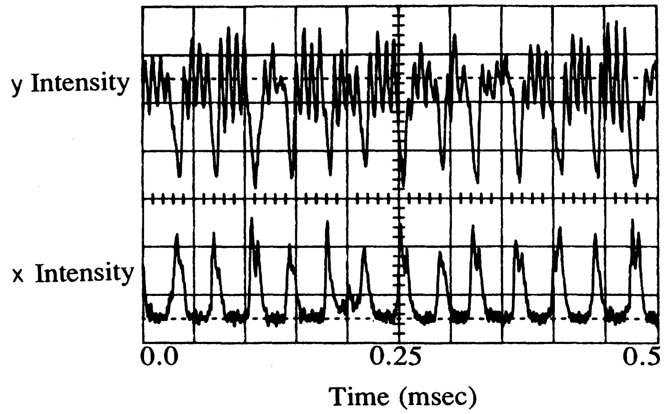


FIG. 4. Experimental time traces from simultaneously measured $x$-polarized and $y$-polarized intensities. Note the anticorrelation between the orthogonally polarized intensities.

It is worthwhile to examine the statistics of chaotic physical systems and identify general trends that are exhibited by appropriate statistical measures, in order to develop a physical intuition for such systems. The probability distribution for the fluctuations of the system energy is an extremely useful measure of the chaotic fluctuations in many practical situations, such as the chaotic laser examined in this paper. Despite the extensive literature on experimental observations of chaos, it appears to us that there have been few, if any, attempts to determine statistical measures of the fluctuations of a chaotic physical system and compare these directly with theoretical predictions. Here, we report the results of such a study; we hope to stimulate further statistical investigations of fluctuations in chaotic physical systems.

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