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Noise quenching in lasers and masers by strong coherent pumping

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An intense single-mode correlated-spontaneous-emission laser or maser can be realized by driving the active atoms coherently with an injected external field. The scheme involves single-photon transitions unlike the two-photon or quantum-beat correlated-emission lasers that utilize correlations between successive photon-cascade emissions or between simultaneous emissions into two modes of the field. Quenching of both the amplitude and phase noise and, in certain cases, squeezing of the amplitude fluctuations are found.

The reduction of quantum noise and the enhancement of coherence in lasers and masers has been a longstanding goal in quantum optics. One method of accomplishing this involves the introduction of correlations between pairs of photons emitted simultaneously in certain devices. We have proposed several schemes for achieving correlated-spontaneous-emission laser (CEL) operation in two-mode systems,[1–4] and in single-mode two-photon systems.[5] The correlation arises from the initial coherent preparation of the active atoms. Atomic coherence leads to amplitude noise quenching even in an ordinary single-mode, single-photon-transition laser (polarization CEL) where, far above threshold, this is accompanied with a significant reduction of the linewidth.[6,7]

Reduced pump fluctuations also improve the noise performance in, e.g., semiconductor lasers[8] and micromasers,[9] but their impact on ordinary laser operation was unclear. Recent efforts to incorporate the effect of pump fluctuations[10,11] into the quantum theory of the laser have indicated that it is significant in the far-above-threshold regime of operation. This conclusion changes drastically when atomic coherence is involved. We have found that sub-Poissonian pump fluctuations lead to squeezing of the amplitude fluctuations already around threshold in this case.[12]

Here we report on a scheme which is perhaps the simplest one from the point of view of experimental feasibility. We consider lasers and masers in which the active atoms, in addition to being pumped into the upper level of the lasing transition by the usual incoherent pump mechanism, are also driven by an external field which is injected into the resonator from the side. The external field changes the gain of the lasing mode and gives rise to phase-sensitive noise. This, in turn, leads to various noise-quenching effects.

Consider an optical or microwave transition between two levels. The excited level $|a\rangle$ is coupled to the ground level $|b\rangle$ by the laser transition and by the mode into which the coherent field is injected. This latter mode differs from the lasing mode, e.g., in the direction of its $\mathbf{k}$ vector (the external field is injected from the side into the cavity). Assuming an intense classical injected field, we obtain the following effective Hamiltonian in the interaction picture:

$$H_1 = \hbar g(a|a\rangle\langle b| + a^\dagger |b\rangle\langle a|) - \hbar[(\Omega/2)|a\rangle\langle b| + (\Omega^*/2)|b\rangle\langle a|]\,. \qquad (1)$$

It is assumed that the laser is resonant with the atomic transition, $\nu = \omega_a - \omega_b$, where $\hbar\omega_a$ ($\hbar\omega_b$) is the energy of the upper (lower) level and ν is the frequency of the laser field and the injected field. a and $a^\dagger$ are annihilation and creation operators for the laser mode, g is the coupling constant between the atom and the lasing mode, and Ω is the complex Rabi frequency of the injected field, $\Omega = |\Omega|\exp(i\phi_0)$. We shall discuss the following cases: (a) maser models, where the excited atoms are injected into the cavity and interact with the fields for a time equal to the transit time τ_0, assumed shorter than the lifetime τ of the atoms ($\tau_0 < \tau$) so that atomic decay can be neglected; and (b) laser models, where atoms are injected into the cavity in their upper states or are inside the cavity and are being pumped into the excited state by incoherent pump mechanism. The interaction time is longer than the lifetime of the atoms, which is assumed to be the same for both levels, $\tau_a = \tau_b = \tau = \Gamma^{-1}$ where Γ is the atomic-decay constant.

The reduced density matrix ρ for the laser field, including the arrival times statistics of the active atoms, satisfies the following master equation[10]

$$\frac{\partial\rho}{\partial t} = (r/p)\ln[1 + p(M-1)]\rho + L_{\rm cav}\rho\,. \qquad (2)$$

The last term on the right-hand side accounts for the cavity losses, its explicit form is given below, in Eq. (4). r is the injection rate for the atoms. p is a parameter, $0 < p < 1$, describing the effect of pump statistics (i.e., arrival times statistics of the atoms). Namely, $p = 0$ corresponds to Poissonian pump statistics, i.e., the number of excited atoms at any time t is random with Poissonian fluctuations around its mean [notice that for this case Eq. (2) reduces to the usual master equation[13] for the quantum theory of the laser]. $p = 1$ corresponds to regular pumping, i.e., the number of excited atoms at time t is fixed. M is an operator describing the effect of a single ac-

TABLE I. The parameters a_i ($i=1,\ldots,4$) and s of Eq. (4) for masers (left column) and lasers (right column). $a_m=r(g\tau_0)^2$ is the maser gain expression and $a_l=2r(g/\Gamma)^2$ is the corresponding expression for laser gain, without injected signal. $x_m=|\Omega|\tau_0$ is a dimensionless intensity parameter for the external field for the maser case, $x_l=|\Omega|/\Gamma$ is a similar parameter for the laser case. In the expressions below $p=0$ corresponds to Poissonian-pump statistics, $p=1$ to regular pumping.

	Masers	Lasers
α_1	$\frac{\alpha_m}{2}\left[\frac{1}{4}+\frac{\sin x_m}{2x_m}+\frac{\sin^2(x_m/2)}{x_m^2}-p\frac{\sin^4(x_m/4)}{x_m^2}\right]$	$\frac{\alpha_l}{2}\left[\frac{1+3x_l^2/4+x_l^4/4-px_l^2/8}{(1+x_l^2)^2}\right]$
α_2	$\frac{\alpha_m}{2}\left[\frac{1}{4}-\frac{\sin x_m}{2x_m}+\frac{\sin^2(x_m/2)}{x_m^2}-p\frac{\sin^4(x_m/4)}{x_m^2}\right]$	$\frac{\alpha_l}{2}\left[\frac{3x_l^2/4+x_l^4/4-px_l^2/8}{(1+x_l^2)^2}\right]$
$\|\alpha_3\|$	$\frac{\alpha_m}{2}\left[\frac{1}{4}-\frac{\sin x_m}{2x_m}+\frac{\sin^2(x_m/2)}{x_m^2}+p\frac{\sin^4(x_m/4)}{x_m^2}\right]$	$\frac{\alpha_l}{2}\left[\frac{3x_l^2/4+x_l^4/4+px_l^2/8}{(1+x_l^2)^2}\right]$
$\|\alpha_4\|$	$\frac{\alpha_m}{2}\left[\frac{1}{4}+\frac{\sin x_m}{2x_m}-\frac{3\sin^2(x_m/2)}{x_m^2}+p\frac{\sin^4(x_m/4)}{x_m^2}\right]$	$\frac{\alpha_l}{2}\left[\frac{x_l^4/4-x_l^2/4+px_l^2/8}{(1+x_l^2)^2}\right]$
$\|s\|$	$-rg\tau_0\frac{\sin^2(x_m/2)}{x_m}$	$-r\frac{g}{2\Gamma}\frac{x_l}{1+x_l^2}$

tive atom on the field. For the maser models the equation

$$\rho(\tau)=\mathrm{Tr}_{\mathrm{atom}}[U(\tau)\rho_{\mathrm{atom+field}}(0)U^{-1}(\tau)]=M(\tau)\rho(0) \tag{3a}$$

defines the operator $M=M(\tau)$, where U is the evolution operator of the Hamiltonian (1). For the laser models $M=M(\Gamma^{-1})$, where

$$M(\Gamma^{-1})=\Gamma\int_0^\infty d\tau M(\tau)e^{-\Gamma\tau}, \tag{3b}$$

and the averaging with respect to the distribution $P(\tau)=\Gamma\exp(-\Gamma\tau)$ represents the effect of atomic decay.[13]

To study noise quenching it suffices to work out a linearized theory in terms of the cavity mode operators a and $a^\dagger$ [terms up to $O(g^2)$ in Eq. (2)]. However, the external field is treated to all orders. This is most easily accomplished if we introduce a second interaction picture where the external field (terms proportional to Ω) is eliminated from Eq. (1).[2] Thus, starting from Eq. (2), we obtain the master equation for the reduced density matrix for the cavity mode in the form:

$$\begin{aligned}\frac{\partial\rho}{\partial\tau}=&-\alpha_1(aa^\dagger\rho-a^\dagger\rho a)-(\alpha_2+\gamma)(a^\dagger a\rho-a\rho a^\dagger)\\&+\alpha_3(a^\dagger a^\dagger\rho-a^\dagger\rho a^\dagger)+\alpha_4(\rho a^\dagger a^\dagger-a^\dagger\rho a^\dagger)\\&+s[a^\dagger,\rho]+\mathrm{H.c.}\end{aligned} \tag{4}$$

This master equation will be valid to second order in g and to all orders in Ω. In Eq. (4) the losses through the cavity mirrors are represented by the coefficient γ, and H.c. stands for Hermitian conjugate terms. The parameters α_i ($i=1,\ldots,4$) and s are related to the properties of the atoms in Eq. (1) and the particular decay and pump mechanism. In general, they are given in terms of the two-time correlation functions of the atomic-dipole moment operators.[14] For our simple models of the maser and laser they can be brought to an explicit form by using standard methods of the quantum theory of the laser.[13] In particular, $\alpha_3=|\alpha_3|\exp(2i\phi_0)$, $\alpha_4=|\alpha_4|\exp(2i\phi_0)$, and $s=|s|\exp(i\phi_0)$. The resulting expressions are summarized in Table I.

In order to see the physical meaning of the various α parameters, it is convenient to transform master Eq. (4) into a Fokker-Planck equation for the Glauber-Sudarshan P representation.[13] The result is

$$\begin{aligned}\frac{\partial P}{\partial t}=&-\frac{\partial}{\partial\alpha}\{[s+(\alpha_1-\alpha_2-\gamma)\alpha+(\alpha_3-\alpha_4)\alpha^*]P\}\\&+\frac{\partial^2}{\partial\alpha^2}(\alpha_3P)+\frac{\partial^2}{\partial\alpha\partial\alpha^*}(\alpha_1P)+\mathrm{c.c.}\end{aligned} \tag{5}$$

Clearly, α_1 (α_3) and α_2 (α_4) correspond respectively to phase-insensitive (phase-sensitive) gain and loss. Besides the usual phase-insensitive diffusion, α_1P, we also have a phase-sensitive diffusion term, α_3P, in Eq. (5).[15] On writing $\alpha=r\exp(i\phi)$, Eq. (5) can be converted to a differential equation for the phase and amplitude variables from which it is easy to obtain the phase-locking condition.[15] It turns out that stable locking occurs for $\phi=\phi_0+\pi/2$. Under the phase-locking conditions the diffusion coefficients for the phase and amplitude are given by

$$D_{\phi\phi}=(\alpha_0/4\bar{n})(2\alpha_1/\alpha_0)[1+(|\alpha_3|/\alpha_1)] \tag{6}$$

and

$$D_{11}=(\alpha_0/4)(2\alpha_1/\alpha_0)[1-(|\alpha_3|/\alpha_1)]\,. \tag{7}$$

where $\alpha_0=\alpha_m$ for masers and $\alpha_0=\alpha_l$ for lasers. $\bar{n}$ denotes the mean number of photons. Its actual value, which also depends on the strength of the injected field, can only be determined from a nonlinear theory. Obviously, $\alpha_0/4\bar{n}$ ($\alpha_0/4$) is of the same form as the phase (amplitude) diffusion coefficient without the injected external field. Thus, the main modification brought about by the injected field can be characterized by two parameters: $2\alpha_1/\alpha_0$ and $|\alpha_3|/\alpha_1$. In Figs. 1–4 we show these quantities versus the dimensionless intensity parameter of the injected field x_m (x_l) for the case of a maser (laser), respectively. Here x_m (x_l) is the Rabi frequency of the injected field times the interaction time τ_0 for the maser (lifetime Γ^{-1} for the laser).

These figures also exhibit the effect of pump statistics on the noise properties of masers and lasers. Figure 1 shows the case of a maser with regular pump ($p=1$). The oscillatory character is associated with a finite number of Rabi oscillations during the transit time of the atoms through the maser cavity. Figure 2 shows the case of a laser with a regular pump ($p=1$). We obtain a smooth behavior, since the Rabi oscillations are averaged out. Note that for the regular pump both the maser and laser models exhibit $|\alpha_3|/\alpha_1>1$, i.e., squeezing of the amplitude fluctuations, for certain values of the external field. Figure 3 shows the case of a maser with a stochastic pump ($p=0$), whereas Fig. 4 corresponds to a laser with a stochastic pump. Note the decrease of $|\alpha_3|/\alpha_1$. For the maser there is still a small part of the full intensity range where this quantity is larger than one and hence squeezing is possible. For the laser, however, this quantity is always less than one and, hence, the stochastically pumped laser does not exhibit squeezing.

From the table the explicit expressions of the phase and amplitude diffusion coefficients for the various schemes can be obtained. In particular

$$D_{\phi\phi}^{(m)}=(\alpha_m/4\tilde{n})[\tfrac{1}{2}+2\sin^2(x_m/2)/x_m^2] =\begin{cases}\alpha_m/4\tilde{n} & \text{if } x_m\to 0,\\ \alpha_m/8\tilde{n} & \text{if } x_m\gg 1,\end{cases} \quad (8)$$

and

$$D_{\rm II}^{(m)}=(\alpha_m/4)[\sin x_m/x_m-2p\sin^4(x_m/2)/x_m^2] =\begin{cases}\alpha_m/4 & \text{if } x_m\to 0,\\ 0 & \text{if } x_m\gg 1,\end{cases} \quad (9)$$

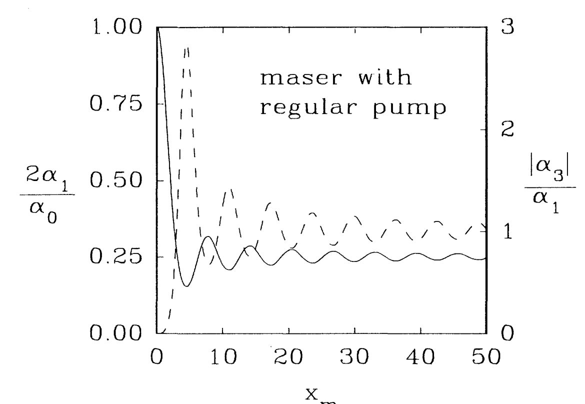

FIG. 1. The parameters $2\alpha_1/\alpha_0$ (solid line) and $|\alpha_3|/\alpha_1$ (dashed line) as a function of the dimensionless intensity parameter $x_m=|\Omega|\tau_0$ for the case of a maser with regular injection ($p=1$). Here $|\Omega|$ is the Rabi frequency of the injected field and τ_0 is the transit time. The scale on the left-hand side (right-hand side) is for the solid line (dashed line).

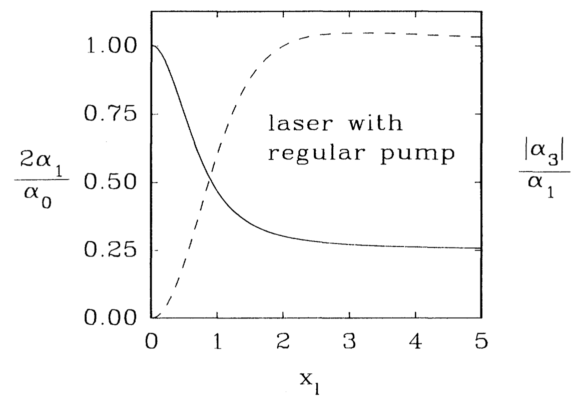

FIG. 2. Same parameters as in Fig. 1 as a function of the dimensionless intensity parameter $x_l=|\Omega|/\Gamma$ for the case of a laser with regular injection ($p=1$). Here Γ is the atomic-decay constant (for simplicity assumed to be the same for both levels). Note that the scale in the laser case is the same for both curves.

for the maser and

$$D_{\phi\phi}^{(l)}=(\alpha_l/4\tilde{n})(1+x_l^2/2)/(1+x_l^2) =\begin{cases}\alpha_l/4\tilde{n} & \text{if } x_l\to 0,\\ \alpha_l/8\tilde{n} & \text{if } x_l\gg 1,\end{cases} \quad (10)$$

and

$$D_{\rm II}^{(l)}=(\alpha_l/4)(1-px_l^2/4)/(1+x_l^2)^2 =\begin{cases}\alpha_l/4 & \text{if } x_l\to 0,\\ 0 & \text{if } x_l\gg 1,\end{cases} \quad (11)$$

for the laser. These expressions summarize our findings. For vanishing intensity of the injected field they reduce to the corresponding expressions for lasers and masers without an injected field. For very high intensities they reproduce the very far-above-threshold behavior of lasers and masers (asymptotical vanishing of the amplitude

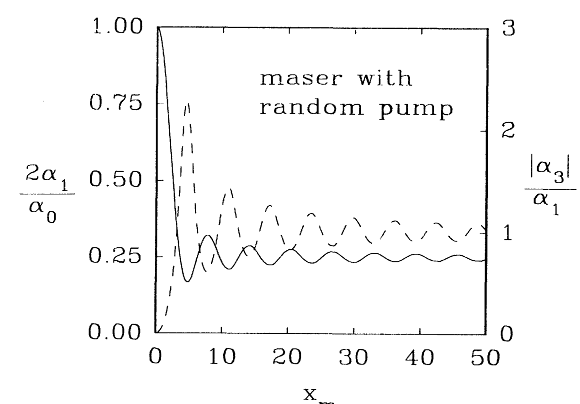

FIG. 3. Same as Fig. 1 for the case of a maser with stochastic pump ($p=0$).

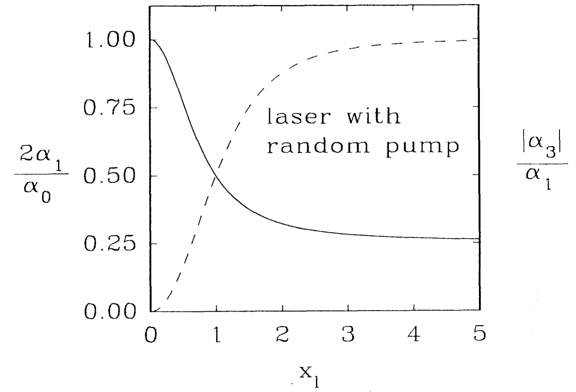

FIG. 4. Same as Fig. 2 for the case of a laser with stochastic pump ($p=0$).

noise and a significant reduction of the phase noise). These expressions also indicate that pump statistics only influence the amplitude noise, since the phase diffusion is independent of p both for the laser and the maser.

It is interesting to compare our results to previous works. Recently, we have found amplitude squeezing in the injection model of a maser and quenching of the amplitude noise, but no squeezing in the injection model of the laser when initial atomic coherence was present.[6,7,12] The present scheme appears to be more efficient since it predicts amplitude squeezing in the laser. The injected atomic coherence decays with time and has only a reduced effect on the noise performance. The coherence due to the driving field, however, is always restored to its steady-state value and, thus, its effect is enhanced.

In addition to antibunching (amplitude squeezing) the injected field provides phase locking and the reduction of the phase-diffusion noise by 50%. These effects, with the exception of locking, remained unnoticed in previous treatments of lasers with injected signal[16] because they were restricted to a linearized theory in the external field, whereas here we keep the external field to all orders. Finally we note that recently what is believed to be the first experimental observation of CEL operation was reported,[17] in agreement with the predictions of Refs. 1–5 and adding further to the feasibility of the present scheme.

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[1]M. O. Scully, Phys. Rev. Lett. **55**, 2802 (1985); W. Schleich and M. O. Scully, Phys. Rev. A **37**, 1261 (1988).

[2]M. O. Scully and M. S. Zubairy, Phys. Rev. A **35**, 752 (1987); J. Bergou, M. Orszag, and M. O. Scully, *ibid.* **38**, 754 (1988); J. Bergou and M. Orszag, *ibid.* **38**, 763 (1988).

[3]J. Krause and M. O. Scully, Phys. Rev. A **36**, 1771 (1987).

[4]J. Bergou, M. Orszag, and M. O. Scully, Phys. Rev. A **38**, 768 (1988).

[5]M. O. Scully, K. Wódkiewicz, M. S. Zubairy, J. Bergou, N. Lu, and J. Meyer ter Vehn, Phys. Rev. Lett. **60**, 1832 (1988).

[6]N. Lu and J. Bergou, Phys. Rev. A **40**, 237 (1989).

[7]C. Benkert, M. O. Scully, J. Bergou, and M. Orszag, Phys. Rev. A **41**, 4062 (1990).

[8]Y. Yamamoto, S. Machida, and O. Nilsson, Phys. Rev. A **34**, 4025 (1986).

[9]P. Filipowicz, J. Javanainen, and P. Meystre, Phys. Rev. A **34**, 3077 (1986).

[10]J. Bergou, L. Davidovich, M. Orszag, C. Benkert, M. Hillery, and M. O. Scully, Opt. Commun. **72**, 82 (1989); J. Bergou, L. Davidovich, M. Orszag, C. Benkert, M. Hillery, and M. O. Scully, Phys. Rev. A **40**, 5073 (1989); C. Benkert, M. O. Scully, J. Bergou, L. Davidovich, M. Hillery, and M. Orszag, *ibid.* **41**, 2756 (1990).

[11]T. A. B. Kennedy and D. F. Walls, Phys. Rev. A **40**, 6366 (1989); F. Haake, S. M. Tan, and D. F. Walls, *ibid.* **40**, 7121 (1989); F. Haake, S. M. Tan, and D. F. Walls, *ibid.* **41**, 2808 (1990).

[12]C. Benkert and M. O. Scully, Phys. Rev. A **42**, 5544 (1990).

[13]M. Sargent III, M. O. Scully, and W. E. Lamb, Jr., *Laser Physics* (Addison-Wesley, Reading, MA, 1974).

[14]G. S. Agarwal, Phys. Rev. A **34**, 4055 (1986).

[15]J. Bergou, M. Orszag, M. O. Scully, and K. Wódkiewicz, Phys. Rev. A **39**, 5136 (1989); N. Lu, S.-Y. Zhu, and G. S. Agarwal, *ibid.* **40**, 258 (1989).

[16]W. W. Chow, M. O. Scully, and E. W. van Stryland, Opt. Commun. **15**, 6 (1975).

[17]M. P. Winters, J. L. Hall, and P. E. Toschek, Phys. Rev. Lett. **65**, 3116 (1990).