Destruction of localization due to coupling to a bath in the kicked-rotator problem

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The quantum kicked rotator is linearly coupled to a heat bath via its momentum coordinate. A momentum-space formulation of the Feynman-Vernon formalism is introduced to find the reduced propagator. The Wigner picture of the dynamics is adopted. It is found that it is a good approximation to ignore the "friction" effect and to replace the bath by a correlated noise source. For certain values of the friction coefficient, this replacement is exact. Destruction of coherence at zero temperature is qualitatively different for an undriven rotator.

I. INTRODUCTION

The kicked rotator constitutes a prototype system for the investigation of Hamiltonian chaos.[1] Quantum mechanically, the chaotic nature of the dynamics is suppressed due to localization.[2,3] Ott, Antonsen, and Hanson[4] found that uncorrelated white noise destroys coherence, and hence localization. However, if noise arises from the coupling to a heat bath, then a more detailed treatment is desired. Such treatment should take into account two effects. One is noise correlations that are expected at low temperatures. The other is "friction" that *may* result in dissipation of energy. While noise results in recovery of diffusion, dissipation of energy tends to balance it, and a steady state is reached. Dittrich and Graham[5] introduced a model for the investigation of this combined effect of noise and dissipation. The master-equation approach that involves the Markovian approximation was used. Thus the model ignored noise correlations and dependence on temperature.

In this work we shall try to analyze a simpler model. The quantum kicked rotator will be assumed to be coupled linearly via its *momentum* coordinate to a heat bath. One may easily convince oneself that no dissipation can arise in the case of such coupling. A first approximation for the effect of the bath is therefore to replace the bath by a noise source. This simplified problem has been studied in the classical limit by Rechester *et al.*[6] Recently, the quantum model was considered,[7] and the effect of noise correlations was explored.

However, it is not self-evident that replacement of the bath by a noise source is indeed justified. In order to investigate this point we follow Caldeira and Leggett[8,9] and use the Feynman-Vernon (FV) formalism[10] to analyze the model. The formalism has been successfully applied to investigate the relaxation problem of a damped harmonic oscillator,[8] the diffusion of a Brownian particle,[11] and the tunneling problem.[9] The formalism has also been applied to time-dependent Hamiltonians[12] and in particular to the investigation of the kicked *particle* problem.[13]

The outline of the paper is as follows. In Sec. II the general formalism is presented. An explicit expression for the propagator is obtained in Sec. III, and transformed to the Wigner representation. Finally, in Sec. IV the Wigner map is analyzed. The conclusions are summarized in Sec. V.

II. MOMENTUM-SPACE FORMULATION OF FEYNMAN-VERNON FORMALISM FOR A ROTATOR

Consider a rotator whose unperturbed Hamiltonian is time dependent

$$\hat{\mathcal{H}}_0=\mathcal{H}_0(\hat{x},\hat{p};\tau),\quad 0\le\tau\le t \tag{2.1}$$

where $\hat{x}$ and $\hat{p}$ are conjugate coordinates $[\hat{x},\hat{p}]=i$ and periodic boundary conditions are imposed over $[0,2\pi]$. The bath is defined by the Hamiltonian

$$\hat{\mathcal{H}}_{\text{bath}}=\sum_{\alpha}\frac{\hat{p}_{\alpha}^{2}}{2m_{\alpha}}+\tfrac{1}{2}m_{\alpha}\omega_{\alpha}^{2}\hat{q}_{\alpha}^{2}\,, \tag{2.2}$$

with $[\hat{q}_\alpha,\hat{p}_\alpha]=i$. The coupling of the system to the bath is assumed to be linear in the momentum coordinate, namely,

$$\hat{\mathcal{H}}_{\text{int}}=\hat{p}\sum_{\alpha}C_{\alpha}\hat{q}_{\alpha} \tag{2.3}$$

where C_α are coupling constants. The total Hamiltonian that describes the system and the bath is

$$\hat{\mathcal{H}}=\hat{\mathcal{H}}_0+\hat{\mathcal{H}}_{\text{int}}+\hat{\mathcal{H}}_{\text{bath}}\,. \tag{2.4}$$

For the sake of later use we define also the Hamiltonian

$$\hat{\mathcal{H}}_F\equiv\hat{\mathcal{H}}_0+\hat{p}F(\tau) \tag{2.5}$$

with $F(\tau)$ some real function of time.

The state of the system whose unperturbed Hamiltonian is (2.1) will be described by the reduced probability matrix

$$\rho(p,P)\equiv\rho(p'',p')\equiv\langle p''|\rho|p'\rangle \tag{2.6}$$

where we introduced the variables

$$P=\tfrac{1}{2}(p''+p')\,, \tag{2.7}$$

$$p=p''-p'\,. \tag{2.8}$$

Note that p'' and p' are integers. Various functions will be denoted by ρ but they will be distinguished by their arguments.

It is assumed that initially (at $t=0$) the probability matrix of the system and bath is factorized, namely,

$$\rho_{t=0}(p'',q'';p',q')=\rho_{t=0}(p'',p')\rho_{\rm eq}(q'',q') , \qquad (2.9)$$

where $\rho_{t=0}(p'',p')$ represents arbitrary preparation of the system and

$$\rho_{\rm eq}(q'',q')=\langle q''|e^{-\beta\mathcal{H}_{\rm bath}}|q'\rangle/{\rm tr}(e^{-\beta\mathcal{H}_{\rm bath}}) \qquad (2.10)$$

represents the bath in canonical thermal equilibrium. The reciprocal temperature of the bath is β. Units of $\hbar=1$ are used in this section.

Using the formalism of Feynman and Vernon,[10] the propagator $K(pP|p_0P_0)$ of the reduced probability matrix is computed. Thus the evolution of the system is given by

$$\rho_t(p,P)=\sum_{p_0,P_0}K(p,P|p_0,P_0)\rho_{t=0}(p_0,P_0) . \qquad (2.11)$$

In what follows the recipe for the calculation of the propagator is outlined. It is based on FV (Ref. 10) with the appropriate modifications that are required in our case. Note that we follow closely the notations of Ref. 12. The expression for the (reduced) propagator is

$$K(p,P|p_0,P_0)=\sum_{p''}\sum_{p'}U_0[p'']U_0^*[p']e^{i\Delta S_{\rm eff}-S_N} . \qquad (2.12)$$

The sum extends over all paths $\{p'(\tau)|p'(\tau)\in\mathbb{Z}, 0<\tau<t, p'(0)=p_0', p'(t)=p'\}$ (where $\mathbb{Z}$ represents the integer numbers) and similarly for $p''(\tau)$. As usual for path integrals[14] a fine discretization of the time interval $[0,t]$ to small segments of length $\Delta\tau$, such that $t/\Delta\tau$ is a very large integer is implicit. However, we follow the convention to suppress notationally this feature whenever possible.[14] The evolution functional $U_0[p']$ is

$$U_0[p']=\prod_\tau\langle p'(\tau+\Delta\tau)|e^{-i\hat{\mathcal{H}}_0(\tau)\Delta\tau}|p'(\tau)\rangle \qquad (2.13)$$

and $U_0^*[p'']$ is similarly defined. The other functionals appearing in (2.12) have very simple expressions if we use the definitions (2.7) and (2.8) of the variables P and p:

$$\Delta S_{\rm eff}[p,P]\equiv\int_0^t\int_0^\tau d\tau\, d\tau' 2\alpha(\tau-\tau')p(\tau)P(\tau') \qquad (2.14)$$

and

$$S_N[p,p]\equiv\tfrac{1}{2}\int_0^t\int_0^t d\tau\, d\tau'\phi(\tau-\tau')p(\tau)p(\tau') \qquad (2.15)$$

where

$$\alpha(\tau-\tau')=\int_0^\infty\frac{d\omega}{\pi}J(\omega)\sin[\omega(\tau-\tau')] \qquad (2.16)$$

and

$$\phi(\tau-\tau')=\int_0^\infty\frac{d\omega}{\pi}J(\omega)\coth(\tfrac{1}{2}\beta\omega)\cos[\omega(\tau-\tau')] . \qquad (2.17)$$

The spectral distribution of the bath oscillators is

$$J(\omega)\equiv\frac{\pi}{2}\sum_\alpha\frac{C_\alpha^2}{m_\alpha\omega_\alpha}\delta(\omega-\omega_\alpha) . \qquad (2.18)$$

In what follows we assume $J(\omega)$ to be a smooth function with some high-frequency cutoff ω_c. In particular the Caldeira-Leggett choice[8,9] of "Ohmic bath" is

$$J(\omega)=\eta\omega e^{-\omega/\omega_c} . \qquad (2.19)$$

It is useful to define a "noiseless" propagator K_F via

$$K_F\equiv\sum_{p''}\sum_{p'}U_F[p'']U_F^*[p']e^{i\Delta S_{\rm eff}} . \qquad (2.20)$$

In this definition F denotes a given real function of time and U_F is the same as U_0 but with H_0 replaced by the Hamiltonian H_F of (2.5), namely,

$$U_F[p']=U_0[p']\exp\left[i\int F(\tau)p'(\tau)d\tau\right] . \qquad (2.21)$$

We then may obtain the "true" propagator K by averaging K_F over realizations of F taken out of a Gaussian ensemble such that

$$\overline{F(\tau)}=0 , \qquad (2.22)$$

$$\overline{F(\tau)F(\tau')}=\phi(\tau-\tau') , \qquad (2.23)$$

and consequently

$$\overline{\exp\left[-i\int F(\tau)p(\tau)\right]} =\exp\left[-\int\int d\tau\, d\tau'\phi(\tau-\tau')p(\tau)p(\tau')\right] . \qquad (2.24)$$

We turn now to apply this formalism to the case of a periodically kicked rotator.

III. THE QUANTUM KICKED ROTATOR COUPLED TO A BATH

We consider a particle that is free to move in a ring. The particle is kicked periodically by a cosine potential. The Hamiltonian is

$$\hat{\mathcal{H}}_0=\frac{1}{2M}\hat{p}^2+K\cos\left[2\pi\frac{\hat{x}}{L}\right]\Delta_T(\tau) \qquad (3.1)$$

where M is the mass of the particle, K is the kicking strength parameter, and T is the period of the kicking. We write

$$\Delta_T(\tau)=\sum_{n=-\infty}^{\infty}T\delta(t-nT) . \qquad (3.2)$$

Periodic boundary conditions are imposed on $[0,L]$ and $[\hat{x},\hat{p}]=i\hbar$. We shall choose to work in this section with units $T=1$, $L=2\pi$, $\hbar=1$ so that K and $M\equiv1/\gamma$ are the dimensionless parameters of the problem.

For the Hamiltonian (3.1) the contributing paths in the path-integral expression (2.20) for which $U_F[p']$ does not vanish are those piecewise constant in time, namely, $p'(t'-1<\tau<t')={\rm const}\equiv p'_{t'-1}$ with $t'=1, 2, \ldots, t$. For such paths

$$U_F[p'] = \left[\prod_{t'=1}^{t} \langle p'_{t'} | e^{-iK\cos\hat{x}} | p'_{t'-1} \rangle \right] \times \exp\left[-i \sum_{t'=1}^{t} \left(\tfrac{1}{2}\gamma p'^2_{t'-1} + f_{t'-1} p'_{t'-1} \right) \right] \quad (3.3)$$

where

$$f_{t'-1} \equiv \int_{t'-1}^{t'} F(\tau) d\tau \; . \quad (3.4)$$

Assuming $J(\omega)$ to have the Caldeira-Leggett form (2.19) with cutoff frequency $1 \ll \omega_c$ we obtain (Appendix)

$$\Delta S_{\text{eff}} = \sum_{t'=1}^{t} \frac{1}{\pi} \eta \omega_c P_{t'-1} p_{t'-1} - \eta (P_{t'} - P_{t'-1}) p_{t'} \; . \quad (3.5)$$

The first term, once introduced in the path-integral expression (2.20), results in mass renormalization $\gamma \to \gamma - (1/\pi)\eta\omega_c$. We therefore assume it to be absorbed in γ. The second term will be termed the "friction" term and its effect will be discussed in what follows.

Substitution of (3.3) and (3.5) into the path-integral expression (2.20) yields after regrouping of the terms in the multiplication

$$K_F = \sum_{p''} \sum_{p'} \prod_{t'=1}^{t} \Big(\langle p''_{t'} | e^{-iK\cos\hat{x}} | p''_{t'-1} \rangle \times \langle p'_{t'} | e^{+iK\cos\hat{x}} | p'_{t'-1} \rangle e^{-i\eta(P_{t'} - P_{t'-1}) p_{t'}} \times e^{-i(\gamma P_{t'-1} p_{t'-1} + f_{t'-1} p_{t'-1})} \Big) \; . \quad (3.6)$$

It is natural to introduce K_F as the convolution $(*)$ of the kernels, namely,

$$K_F = K^{\text{kick}} (*) K^{\text{free}} \cdots (*) K^{\text{kick}} (*) K^{\text{free}} \quad (3.7)$$

where

$$K^{\text{free}}(p_{t'}, P_{t'} | p_{t'-1}, P_{t'-1}) = \delta(P_{t'} - P_{t'-1}) \delta(p_t - p_{t'-1}) e^{-i(\gamma P_{t'-1} p_{t'-1} + f_{t'-1} p_{t'-1})} \quad (3.8)$$

and

$$K^{\text{kick}}(p_{t'}, P_{t'} | p_{t'-1}, P_{t'-1}) = e^{-i\eta(P_{t'} - P_{t'-1}) p_{t'}} \langle p''_{t'} | e^{-iK\cos\hat{x}} | p''_{t'-1} \rangle \langle p'_{t'} | e^{+iK\cos\hat{x}} | p'_{t'-1} \rangle \; . \quad (3.9)$$

The simplified indexation of the dummy variables of K^{free} and K^{kick} should not cause any confusion. The δ functions should be interpreted as the Kronecker and Dirac δ functions according to their type of arguments. The function J_ν is the ordinary Bessel function of (integer) order ν.

It is useful to use the Wigner picture of the dynamics. The Wigner representation of the probability matrix is

$$\rho(X,P) = \frac{1}{2\pi} \sum_p e^{ipX} \rho(p,P) \; . \quad (3.10)$$

It is a real function on $[0,2\pi] \times \frac{1}{2}\mathbb{Z}$, where $\mathbb{Z}$ represents integer numbers, and the normalization is

$$\sum_{P \in (1/2)\mathbb{Z}} \int_0^\pi dX \, \rho(X,P) = 1 \; . \quad (3.11)$$

The evolution of the system is the Wigner picture is given by

$$\rho_t(X,P) = \sum_{P_0} \int dX_0 \, K(X,P|X_0,P_0) \rho_{t=0}(X_0,P_0) \quad (3.12)$$

which is the transformed version of Eq. (2.11). The kernel is given via

$$K(X,P|X_0,P_0) = \frac{1}{2\pi} \sum_{p,p_0} e^{ipX} K(p,P|p_0,P_0) e^{-ip_0 X_0} \quad (3.13)$$

which yield for the propagators K^{free} and K^{kick} the following simple expressions:

$$K^{\text{free}}(X,P|X_0,P_0) = \delta(P - P_0) \delta(X - (X_0 + \gamma P + f_t)) \; , \quad (3.14)$$

$$K^{\text{kick}}(X,P|X_0,P_0) = \delta((X - X_0) - \eta(P - P_0)) \times J_{2(P-P_0)}(2K \cos X_0) \; . \quad (3.15)$$

In order to obtain the true propagator we should average K_F over realizations of F satisfying (2.22) and (2.23). Instead, following definition (3.4), we may average over realization of f such that

$$\overline{f_{t'}} = 0 \quad (3.16)$$

and

$$\overline{f_{t'} f_{t''}} = \phi_{t',t''} \equiv \int_{t'-1}^{t'} \int_{t''-1}^{t''} d\tau' d\tau'' \phi(\tau' - \tau'') \; . \quad (3.17)$$

For the Caldeira-Leggett choice (2.19) one obtains (A9) at the limit of high temperature ($\beta \ll 1$)

$$\phi_{t',t''} \equiv \phi_\sigma(t' - t'') \equiv \sigma \delta_{t',t''} \quad (3.18)$$

with $\sigma = 2\eta/\beta$, while in the limit of low temperature ($t \ll \beta$)

$$\phi_{t',t''} = \phi_\eta(t' - t'') + \phi_\lambda(t' - t'') \quad (3.19)$$

where

$$\phi_\eta(t' - t'') = \frac{2\pi}{6} \eta \delta_{t',t''} - \frac{\eta}{\pi} \frac{1}{(t' - t'')^2} (1 - \delta_{t',t''}) \quad (3.20)$$

and

$$\phi_\lambda(t' - t'') = 2\lambda \delta_{t',t''} - \lambda \delta_{|t' - t''|,1} \quad (3.21)$$

with $\lambda = (\eta/\pi) \ln\omega_c - (2\pi/6)\eta$.

IV. ANALYSIS OF THE WIGNER MAP

In order to make the following discussion more transparent it is convenient to choose the units $M=1$, $L=2\pi$, $T=1$ so that $\hbar$ and K are the dimensionless parameters of (3.1). The propagators (3.14) and (3.15) in these units are

$$K^{\mathrm{free}}(X,P|X_0,P_0)=\delta(P-P_0)\delta(X-(X_0+P_0+f_t)) \tag{4.1a}$$

and

$$K^{\mathrm{kick}}(X,P|X_0,P_0)=\delta((X-X_0)-\eta(P-P_0)) \times J_{2(P-P_0)/\hbar}\left[2\frac{K}{\hbar}\cos X_0\right] . \tag{4.1b}$$

It should be stressed that Wigner function $\rho(X,P)$ is defined now on the domain $[0,2\pi]\times(\hbar/2)\mathbb{Z}$ where $\mathbb{Z}$ are the integer numbers. Using well-known formulas for $J_\nu(x)$ in the limit $\nu \ll x$ and $x \ll \nu$, it is easily verified that in the limit $\hbar \to 0$ the map (4.1) corresponds to the classical map

$$X'=X_{t-1}+P_{t-1}+f_t , \tag{4.2a}$$

$$P_t=P_{t-1}+K\sin X' , \tag{4.2b}$$

$$X_t=X'+\eta(P_t-P_{t-1}) . \tag{4.2c}$$

This map is of Langevin type and includes the *retarded* response of the bath to the change of the momentum. We could have guessed this classical map in advance using the analogy to the case of a damped particle where the coupling is via the x coordinate and consequently the friction is proportional to the velocity $\dot{x}$. Here the coupling is via the p coordinate and we expect the classical equations of motion to be

$$\dot{P}=-\frac{\partial \mathcal{H}_0(X,P)}{\partial X} , \tag{4.3a}$$

$$\dot{X}=\frac{\partial \mathcal{H}_0(X,P)}{\partial P}+\eta\dot{P}|_{\mathrm{retarded}}+F(t) . \tag{4.3b}$$

Integrating these equations over one step (free propagation plus a kick) we indeed obtain the map (4.2).

In the absence of noise and "friction" ($\eta=0$), as the value of K is increased, the *classical* dynamics follows the Kolmogorov Arnold Moser (KAM) scenario.[1] For $K=0$, free rotator, the dynamics is integrable while for $K_c \ll K$ ($K_c \approx 0.9716$), the last KAM trajectories are already destroyed, and diffusion in momentum with coefficient $D=D_{\mathrm{cl}}(K)\approx\frac{1}{2}K^2$ takes place. Quantum mechanically, for $K=0$ the propagator (4.1) is identical to the classical Liouville propagator of phase-space distributions. For $K_c \ll K$ the (pure) eigenstates of the propagator are localized in momentum[3] with localization length $\hbar\xi$, which is given in the semiclassical limit ($\hbar \ll 1$) by $\xi=\frac{1}{2}[D_{\mathrm{cl}}(K)/\hbar^2]$. As a result classical diffusion is suppressed[2] after time $t^* \approx \xi$. An example for numerical simulation is given in Fig. 1. In what follows we shall discuss the effects of noise and friction on the dynamical behavior of the rotator both in the integrable case ($K=0$) and in the chaotic regime ($K_c \ll K$).

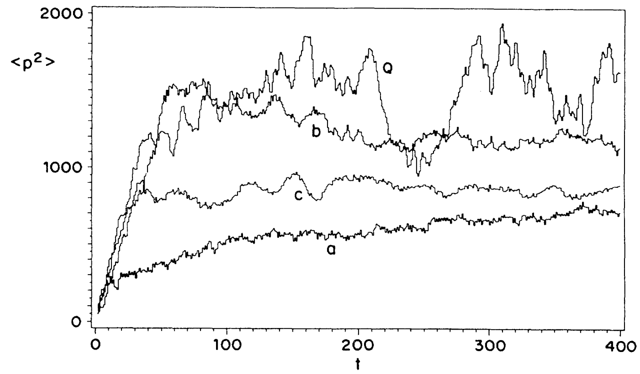

FIG. 1. The kinetic energy $\langle p^2 \rangle$ of a rotator as a function of time (iterations) in the absence of noise. Curve Q—quantum-mechanical behavior, the parameters are $K=10$ and $\hbar=2\pi(0.3/\sqrt{5}-1)$. Initially the rotator was prepared with zero momentum. Curves a, b, and c—the semiclassical map (4.5). The parameters K and $\hbar$ are the same while $\eta=0$, $\eta=0.21$, and $\eta=0.25$, respectively. Note that the qualitative dynamical behavior is the same.

The effect of the bath on a free rotator ($K=0$) is, both classically and quantum mechanically, a stochastic process. The motion for a wave packet can be described as a free motion plus spatial diffusion superimposed. The reduced propagator is simply

$$K(X,P|X_0,P_0)=\frac{1}{\sqrt{2\pi}\delta x(t)} \times\exp\left[-\frac{1}{2}\left[\frac{X-X_0}{\delta x(t)}\right]^2\right]\delta(P-P_0) \tag{4.4}$$

where the spreading is given by

$$\delta x^2(t)=\int_{\tau'=1}^{t}\int_{\tau''=1}^{t}d\tau' d\tau''\phi(\tau'-\tau'') . \tag{4.5}$$

At high temperatures $\delta x^2(t)=2(\eta/\beta)t$ while at the limit of zero temperature the diffusion is logarithmic in time, namely, $\delta x^2(t)\sim(2/\pi)\hbar\eta\ln(\omega_c t)$. Let us now use these results to find the time scale for destruction of quantum coherence if $K=0$.

Consider a rotator which is prepared initially in a pure state that constitutes a superposition $(1/\sqrt{2})(|p_1\rangle+|p_2\rangle)$. The Wigner function is

$$\rho_{t=0}(X,P)=\tfrac{1}{2}\rho^{(1)}+\tfrac{1}{2}\rho^{(2)}+\rho^{\mathrm{int}} \tag{4.6}$$

where

$$\rho^{(i)}_{t=0}(X,P)=\frac{1}{2\pi}\delta_{P,p_i}, \quad i=1,2 \tag{4.7}$$

and

$$\rho^{\mathrm{int}}(X,P)=\frac{1}{2\pi}\delta_{P,(p_2+p_2)/2}\cos\left[\frac{X}{\delta x_c}\right] \tag{4.8}$$

with $\delta x_c = \hbar/|p_2 - p_1|$. One easily finds using (4.4) that the superposition (4.6) will turn gradually into a mixture. Namely,

$$\rho_t^{\text{int}}(X,P) = e^{-(1/2)[\delta x(t)/\delta x_c]^2} \rho_{t=0}^{\text{int}}(X - Pt, P) \tag{4.9}$$

while $\rho_t^{(i)}(X,P) = \rho_{t=0}^{(i)}(X - Pt, P)$. One may define a coherence time t_c via $\delta x(t) = \delta x_c$. More generally, a superposition of momentum eigenstates which is localized within some range $\Delta p = \hbar\xi$ results in details on spatial scale for $\delta x_c = 1/\xi$. The coherence time t_c is determined then via $\delta x^2(t) = 1/\xi$, since on this time scale the interference is smeared due to a diffusion process. At high temperatures the coherence time is

$$t_c^{\text{free rotator}} = \left[\frac{1}{\xi}\right]^2 \frac{1}{\sigma} \tag{4.10}$$

with $\sigma = 2\eta/\beta$, while at the limit of zero temperature

$$t_c \sim \frac{1}{\omega_c} \exp\left[\frac{\pi}{2}\left[\frac{1}{\xi}\right]^2 \frac{1}{\hbar\eta}\right],$$

which is an exponentially long time.

In what follows we assume $K_c < K$. Classically, the effect of noise is to destroy dynamical correlations.[6] When the variance of the noise is sufficiently large, namely, $1 < \phi(0)$, the diffusion coefficient approaches the random-phase value $D = \frac{1}{2}K^2$ irrespective of the value of η. Noise correlations are expected to be of little importance due to the exponential instability of the phase-space trajectories.

We turn now to discuss the effect of the bath on quantal localization. For $K = 0$ we have observed that friction is effectively absent. By inspection of the quantum propagator (4.1) we may generalize this observation for $K \neq 0$ provided $\eta = (4\pi/\hbar)N_{\text{integer}}$. Thus for either $K = 0$ or $\eta = (4\pi/\hbar)N_{\text{integer}}$ we could have ignored the friction term ΔS_{eff}, and therefore the following two models are equivalent: (a) Quantum kicked rotator coupled to a heat bath as in (2.4); and (b) quantum kicked rotator coupled to a noise source as in (2.5). The latter model is of course *much* more convenient to both analytical and numerical analysis. This latter model was investigated in Ref. 7; the results will now be summarized.

The effect of *noise* is to destroy coherence. If the noise is strong, namely, coherence is destroyed within one kicking period ($t_c^{\text{free rotator}} < 1$), then classical-like diffusion is recovered. Weak noise destroys localization perturbatively. The mechanism for the destruction of quantum coherence depends on the type of noise correlations (Fig. 2). One may define the coherence time to be the inverse of the average decay rate of a quasienergy eigenstate. The coefficient diffusion is given then by the expression[4,7]

$$D = (\hbar\xi)^2 \frac{1}{t_c} . \tag{4.11}$$

A first-order perturbative calculation enables one to obtain a formula for t_c which depends on the noise correlation function. It is found that at high temperatures $t_c = t_c^{\text{free rotator}}$ while at low temperatures $t_c \ll t_c^{\text{free rotator}}$.

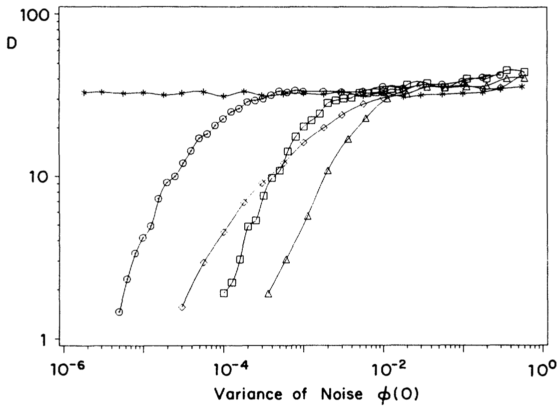

FIG. 2. The diffusion coefficient D as a function of the noise variance $\phi(0)$. The symbol $*$ is for the classical standard map, while $\Diamond$ and $\triangle$ are for the quantal map (4.1) with $\eta = 0$. The correlation functions are (3.18) and (3.21), respectively. The parameters are the same as in Fig. 1. The symbols $\odot$ and $\Box$ are for the semiclassical map with $\eta = 0.25$ and the correlation functions are (3.18) and (3.21) as before.

In the latter case destruction of coherence is dominated by the cutoff-dependent component of the noise, unlike the $K = 0$ case. This effect is due to the singular nature of the kicking driving.

For $\eta \neq (4\pi/\hbar)N_{\text{integer}}$ the replacement of the bath by a c-number noise source is an approximation. However, one expects this approximation to be an excellent one. In order to support this claim one may try first to use some kind of generalization of the standard perturbative treatment. Such a generalization has not been found. However, it arises that at least to the high-temperature white noise and for the dominant λ component of the zero-temperature noise one may obtain the expression for t_c via a heuristic picture.[7] The purpose of the following discussion is to extend this heuristic approach to the general case, namely, $\eta \neq (4\pi/\hbar)N_{\text{integer}}$ using a semiclassical picture of the dynamics. Thus it will be demonstrated that the mechanism for destruction of localization is essentially the same irrespective of the value of η.

Consider the map (4.1). We may assume without loss of generality that $0 < \eta < 4\pi/\hbar$, and consider the effect of the noise separately. It is useful to define a new coordinate $Y \equiv X - \eta P$ and rewrite the Wigner map in the (Y,P) phase-space coordinates

$$K^{\text{free}}(Y,P|Y_0,P^0) = \delta(P - P_0)\delta(Y - (Y_0 + P_0 + f_t)) , \tag{4.12a}$$

$$K^{\text{kick}}(Y,P|Y_0,P_0) = \delta(Y - Y_0) J_{2[(P-P_0)/\hbar]} \times \left[2\frac{K}{\hbar}\cos(Y_0 + \eta P_0)\right] . \tag{4.12b}$$

A semiclassical approximation for this map is given by

$$Y_t = Y_{t-1} + P_{t-1} + f_t , \tag{4.13a}$$

$$P_t = P_{t-1} + \hbar \left[\frac{K}{\hbar} \sin(Y_t + \eta P_{t-1}) \right]_{\mathrm{NI}} \tag{4.13b}$$

where $(\ )_{\mathrm{NI}}$ denotes the nearest-integer part of a real number. This semiclassical map (with $\eta = 0$ and $f = 0$) was phenomenologically introduced in Ref. 15 and later on analyzed by Berman, Kolovsky, and Izrailev.[16] This map takes into account the effect of the discrete nature of phase space but disregards the quasistochastic nature of the Wigner propagator. It was shown that the semiclassical map is capable of giving the right qualitative behavior, namely, resonances and suppression of diffusion, which is characteristic to the quantum map. We therefore make the conjecture that the semiclassical map (4.9) gives the right qualitative behavior also when $\eta \neq 0$ and $f = 0$. [The effect of noise ($f \neq 0$) will be discussed in the next paragraph.] On the basis of the argumentation that is introduced in Ref. 16, we expect a crossover after time $t^* \sim K^2/\hbar^2$ from classical-like diffusion in momentum to saturationlike steady state, irrespective of the value of η. This point was verified numerically (Fig. 1), however, it was found that the time scale t^* is quite sensitive to the value of η in irregular fashion. It should be stressed that a similar sensitivity is found with respect to the value of $\hbar$. The reason is that the map (4.9) overamplifies the effect of quantum resonances.

In order to discuss the effect of noise we should first clarify the significance of the time scale t^*. On time scales larger than t^* the dynamics appear to be quasiperiodic. It is attributed to the resolution of periodic orbits (in the sense of Ref. 16). The longer periodic orbits are of characteristic period t^* and extend over a phase-space portion of dimension $(2\pi)(\hbar t^*)$. Furthermore, while phase space is naturally discretized on momentum scale $\delta p = \hbar$, it is now also effectively discretized on spatial scale $\delta x \sim 2\pi/t^*$. Thus these periodic orbits are the analog of the quasienergy eigenstates whose Wigner function has similar features.[7,13] We now may follow the argumentation of Ref. 7 in order to find the characteristic time scale t_c for the destruction of these periodic orbits by noise. If the variance of the noise is large enough to destroy these periodic orbits within one time step, namely, $(1/t^*)^2 \leq \phi(0)$, we expect the dynamical behavior to become classical and correlations of the noise are expected to be of little importance. However, when the noise is weak the time scale t_c is found to depend on the nature of the noise correlations. One may easily convince oneself that for the uncorrelated white noise (3.18), spatial diffusion causes destruction of details on the spatial scale $2\pi/t^*$ on time scale $t_c \sim (2\pi/t^*)^2(1/\sigma)$. In the case of the short-range correlation (3.21) one may deduce following argumentation of Ref. 7 that $t_c \sim (2\pi/t^*)(1/\lambda)$, which is by factor t^* longer (for the same noise variance), but unlike the integrable case is not exponentially long. Assuming the noise to be weak ($t^* \ll t_c$) we expect perturbative destruction of localization. The diffusion coefficient is given then by (4.11). Substitution of the time scale t_c yields

$$D = C(\hbar t^*)^2(t^*)^2\sigma \tag{4.14}$$

for the uncorrelated white noise (3.18) and

$$D = C'(\hbar t^*)^2(t^*)\lambda \tag{4.15}$$

for the λ-correlated noise (3.21). The prefactors C and C' are of order unity. The first result (4.14) is relevant in the case of high temperature with $\sigma = 2(\eta/\beta)$, while the latter result (4.15) is relevant to the limit of zero temperature with $\lambda = (1/\pi)(\ln\omega_c - 2\pi/6)\hbar\eta$. If the cutoff frequency ω_c is not very high then also the long-range component (3.20) of the zero-temperature noise should be taken into account. The semiclassical pictures cannot give us a quantitative estimate for the contribution of this component.

V. CONCLUSIONS

An exact investigation of the dynamics in the case of a chaotic system which is coupled to a heat bath has been presented. The quantum kicked rotator was coupled linearly to the bath via its momentum coordinate. It was shown that the only effect of the bath is this model is to destroy coherence, while "friction" has an insignificant effect that does not result in dissipation. Thus it is a good approximation to replace the heat bath by a noise source. For certain values of the "friction" parameter this replacement is even exact.

This work constitutes the basis for the investigation of the effect of correlated noise on localization which has been reported in Ref. 7 where the rotator has been coupled to a correlated noise source. Using a semiclassical picture of the dynamics we demonstrated that the result of the latter reference should hold also if the friction effect is taken into account, namely, localization is destroyed also at zero temperature and linear diffusion in energy is recovered. The mechanism for destruction of coherence depends on the type of noise correlations and therefore results in different time scales.

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APPENDIX

In this Appendix explicit expressions for the kernel $\alpha(\tau)$ and the correlation function $\phi(\tau)$ are introduced, and details of the derivation of (3.5) and (3.18)–(3.21) are presented. The Caldeira-Leggett choice (2.19) of the spectral distribution of the bath oscillators is assumed.

Substituting (3.19) into the expression (2.16) one obtains

$$\alpha(\tau) = -\eta \frac{\partial}{\partial \tau} \left[\frac{1}{\pi} \frac{\tau_c}{\tau_c^2 + \tau^2} \right] . \tag{A1}$$

Note that $\alpha(\tau)$ decays on scale $\tau_c \equiv 1/\omega_c$ and has total area $(1/\pi)\eta\omega_c$. In what follows we assume $\tau_c \ll 1$. Operating with the kernel $\alpha(\tau - \tau')$ on a piecewise constant path $P(\tau)$ yields within the segment $t' < \tau < t' + 1$

with $t'=0,1,2,\ldots,t-1$

$$\int_0^\tau 2\alpha(\tau-\tau')P(\tau')d\tau'=\frac{2}{\pi}\eta\omega_c P_{t'} -\frac{2}{\pi}\eta\omega_c\frac{1}{1+(\omega_c\tau)^2}(P_{t'}-P_{t'-1}) . \tag{A2}$$

For $t'=0$ one should make the replacement $P_{t'-1}\to 0$. Thus discontinuities of $P(\tau)$ are suppressed on time scale τ_c. It is now easy to find via substitution in (2.14) that

$$\Delta S_{\text{eff}}=\sum_{t'=1}^{t}\frac{2}{\pi}\eta\omega_c P_{t'-1}p_{t'-1}-\eta P_0 p_0 -\eta\sum_{t'=1}^{t-1}(P_{t'}-P_{t'-1})p_{t'} . \tag{A3}$$

The expression (3.5) differs from this expression. First, the second term is omitted, since it may be factorized out of the propagator and has the effect of operating on the initial state with

$$K_{\text{switching}}(X_0,P_0|X_-,P_-) =\delta(P_0-P_-)\delta(X_0-(X_-+\eta P_-)) . \tag{A4}$$

The second difference stems from the fact that we added a term which represents the effect of the last kick which is observed only at the time $t+\tau_c$.

We turn now to discuss the correlation function $\phi(\tau)$. Substitution of (2.19) into (2.17) yields

$$\phi(\tau)=\frac{\eta}{\pi}\left[\frac{\tau_c^2-\tau^2}{(\tau_c^2+\tau^2)^2}\right] +2\frac{\pi}{\beta}\frac{1}{2\pi\beta}\left[\frac{1}{(\tau/\beta)^2}-\left[\frac{\pi}{\sinh(\pi\tau/\beta)}\right]^2\right] \tag{A5}$$

where $\tau_c=1/\omega_c$. It has two regimes of behavior, the short time where

$$\phi(\tau)=-\frac{\eta}{\pi}\frac{1}{\tau^2},\quad \tau_c\ll\tau\ll\beta \tag{A6}$$

and the long-time regime

$$\phi(\tau)=-2\frac{\eta}{\beta}\frac{2\pi}{\beta}e^{-(2\pi/\beta,\tau)},\quad \beta\ll\tau . \tag{A7}$$

It satisfies the sum rule

$$\int_0^\infty\phi(\tau)d\tau=2\frac{\eta}{\beta} . \tag{A8}$$

Hence, at high temperature, when β is shorter than the period of the kicking ($\beta\ll 1$), the correlation function can be replaced by a δ function, i.e.,

$$\phi(\tau)=2\frac{\eta}{\beta}\delta(\tau) \tag{A9}$$

while at low temperature ($t\ll\beta$) we may use (A6) to obtain (3.18)–(3.21) via (3.17). Details of the explicit calculation may be found in Appendix A of Ref. 12.

[1]A. J. Lichtenberg and M. A. Lieberman, *Regular and Stochastic Motion* (Springer, Berlin, 1983).

[2]G. Casati, B. V. Chirikov, F. M. Izrailev, and J. Ford, in *Stochastic Behavior in Classical and Quantum Hamiltonian Systems,* edited by G. Casati and J. Ford (Springer, New York, 1979), p. 334.

[3]S. Fishman, D. R. Grempel, and R. E. Prange, Phys. Rev. Lett. **49**, 509 (1982); D. R. Grempel, R. E. Prange, and S. Fishman, Phys. Rev. A **29**, 1639 (1984); S. Fishman, R. E. Prange, and M. Griniasty, *ibid.* **39**, 1628 (1989).

[4]E. Ott, T. M. Antonsen, Jr., and J. D. Hanson, Phys. Rev. Lett. **53**, 2187 (1984).

[5]T. Dittrich and R. Graham, Z. Phys. B **62**, 515 (1986); Ann. Phys. (N.Y.) **200**, 363 (1990).

[6]A. B. Rechester, M. N. Rosenbluth, and R. B. White, Phys. Rev. A **23**, 2664 (1981).

[7]D. Cohen (unpublished).

[8]A. O. Caldeira and A. J. Leggett, Physica **121A**, 587 (1983).

[9]A. O. Caldeira and A. J. Leggett, Ann. Phys. (N.Y.) **140**, 374 (1983).

[10]R. P. Feynman and F. L. Vernon, Jr., Ann. Phys. (N.Y.) **24**, 118 (1963).

[11]V. Hakim and V. Ambegaokar, Phys. Rev. A **32**, 423 (1985).

[12]D. Cohen and S. Fishman, Phys. Rev. A **39**, 6478 (1989).

[13]D. Cohen (unpublished).

[14]L. S. Schulman, *Technique and Applications of Path Integration* (Wiley, New York, 1981).

[15]B. V. Chirikov, F. M. Izrailev, and D. L. Shepelyansky, Sov. Rev. **2C**, 209 (1981).

[16]G. P. Berman, A. R. Kolovsky, and F. M. Izrailev, Physica A **152**, 273 (1988).