# Response of a two-level atom to a classical field and a quantized cavity field of different frequencies

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We consider a two-level atom interacting simultaneously with a classically described cw laser field and a quantized cavity field. The classical field is assumed strong, and the cavity field is regarded as a quantum probe of its dressed states. The two fields have different frequencies and amplitudes, but we show that an effective time-independent Hamiltonian can be obtained via two rotating-frame transformations. There is a different effective Hamiltonian in each of an infinite sequence of "probe resonance zones" associated with the subharmonic Rabi sidebands of the dressed states. In the $m$th zone, the Hamiltonian has the same form as the $m$-photon Jaynes-Cummings Hamiltonian. We consider both time-dependent and time-averaged atomic response and make comparisons with recent findings in the case that both fields are fully classical.

## I. INTRODUCTION

In this paper, we extend earlier studies[1,2] of the transient dynamics of a two-level atom interacting simultaneously with two laser fields with different amplitudes and frequencies.[3] Here we investigate the situation in which one of the fields has a fixed amplitude and is described classically but the other is taken to be a quantized cavity field which is allowed to evolve dynamically. The cavity field can be regarded as a quantum probe of the dressed states defined by the classical laser field. Many interesting effects associated with similar systems of fields and two-level atoms have been predicted and observed recently, for example, two-photon gain,[4] cavity-perturbed resonance fluorescence spectra,[5] and atomic squeezing in the cavity.[6]

In our previous studies[1,2] we concentrated on loss-less evolution in the classical transient pump-probe domain. We showed that the short-time response of the atom averaged over many Rabi cycles exhibited new features. The inversion line shape in the neighborhood of every subharmonic resonance showed a sensitive dependence on both the initial Bloch vector orientation and on the initial phase difference between the two fields. In particular, the dependence on the initial phase difference survives time averaging and is not related in a simple way to the transverse phase of the initial Bloch vector. Here, we investigate how these results are changed by the quantization of the probe field.

The approach we use is closely parallel with that used in Refs. 1 and 2. First we transform the system to a suitable rotated basis, and follow by making a second rotating-wave approximation. An effective Hamiltonian is then established in Sec. II. The time evolution of the system may be found analytically as a sum over Fock state occupation numbers. In the limit of small vacuum Rabi frequency the evolution is identical with that obtained from a multiphoton Jaynes-Cummings interaction. In Sec. III we derive an expression for the time evolution operator for the system. Both the time-dependent atomic inversion and the atomic averaged inversion are obtained in Secs. IV and V. Some of their features are discussed. Section VI is devoted to a summary of the differences between the quantum and semiclassical predictions.

## II. EFFECTIVE HAMILTONIAN

The Hamiltonian of our system, in the usual rotating-wave approximation, is given by

$$H=\omega_0 S_z+\omega_c a^\dagger a+\tfrac{1}{2}S_+[re^{-i(\omega_L t+\psi_L)}+\xi a] + \tfrac{1}{2}S_-[re^{i(\omega_L t+\psi_L)}+\xi a^\dagger] , \qquad (2.1)$$

where we have taken $\hbar=1$ for convenience, and $a$ and $a^\dagger$ are creation and annihilation operators for the cavity (probe) field. The so-called vacuum Rabi frequency[7] associated with the cavity field is written $\xi$ and $r$ is the Rabi frequency associated with the classically described laser field. Both are assumed real. The atomic transition frequency is denoted $\omega_0$, the frequencies of the cavity and laser fields are denoted $\omega_c$ and $\omega_L$, and the laser field is assigned the phase $\psi_L$. The $S$'s are the coherence operators for the atom, satisfying angular momentum (not Pauli matrix) commutation relations among themselves. Figure 1 shows the atomic energy levels and the radiative interactions schematically.

In order to eliminate the explicit time dependence of the Hamiltonian we transform to a rotating frame of reference via the unitary operator

$$U(t)=e^{-i(\omega_L t+\psi_L)(a^\dagger a+S_z)} . \qquad (2.2)$$

The new Hamiltonian is defined by

$$H'=U^\dagger(t)HU(t)-iU^\dagger(t)\frac{dU(t)}{dt} \qquad (2.3)$$

and it is easy to show that $H'$ takes the usual time-independent form:

$$H'=\Delta_c a^\dagger a+\Delta_L S_z+\tfrac{1}{2}(r+\xi a^\dagger)S_-+\tfrac{1}{2}S_+(r+\xi a) , \qquad (2.4)$$

 

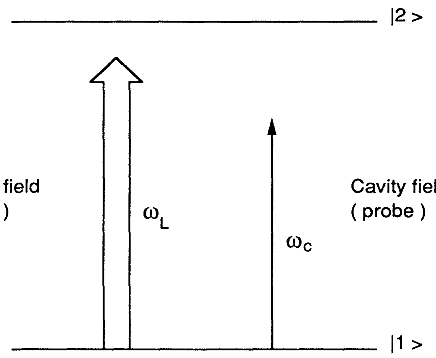


FIG. 1. The two-level atom interacting with the driving and the cavity field simultaneously.

where the detunings are defined to be $\Delta_c=\omega_c-\omega_L$ and $\Delta_0=\omega_0-\omega_L$.

Significant further simplification occurs if either the atomic transition or the cavity field is exactly resonant with the laser field.[8] We are most interested in the cavity field as a variable-frequency probe and retain $\omega_c\neq\omega_L$ but consider the case of resonant pumping $\Delta_0=0$. By rearranging terms the Hamiltonian can then be written

$$H'=\Delta_c a^\dagger a+rS_z'+\frac{\xi}{2}(a+a^\dagger)S_z'+\frac{\xi}{4}(a-a^\dagger)(S'_+-S'_-)\ , \tag{2.5}$$

where we have relabeled the coherence operators for later convenience as follows:

$$S_z'=S_x\ , \tag{2.6a}$$

$$S'_+=S_z+iS_y\ , \tag{2.6b}$$

$$S'_-=S_z-iS_y\ . \tag{2.6c}$$

It is easy to check that the new set also obeys angular momentum commutation relations among themselves. The eigenvectors of $S_z'$ are the dressed states of the atom in the laser field alone and the operators $S'_+$ and $S'_-$ are the corresponding raising and lowering operators.

Next we make another unitary transformation

$$\tilde{H}=T^\dagger H'T\ , \tag{2.7}$$

where $T$ is an atomic-state-dependent displacement operator of the cavity field state:

$$T=\exp\left[\frac{\xi}{2\Delta_c}(a-a^\dagger)S_z'\right]. \tag{2.8}$$

Then the new Hamiltonian reads

$$\tilde{H}=\Delta_c a^\dagger a+rS_z'+\frac{\xi}{4}(S'_+V-S'_-V^\dagger)(a-a^\dagger)+\frac{\xi^2}{16\Delta_c}\ , \tag{2.9}$$

where the operator $V$ is an ordinary field displacement operator, defined by

$$V=\exp\left[\frac{\xi}{2\Delta_c}(a^\dagger-a)\right] \tag{2.10}$$

$$=\exp\left[\frac{-\xi^2}{8\Delta_c^2}\right]\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\left[\frac{\xi}{2\Delta_c}\right]^{m+n}\frac{(-1)^n}{n!m!}a^{\dagger m}a^n\ . \tag{2.11}$$

The Hamiltonian $H$ describes a two-level system with its energy levels separated by $r$, coupled to a single quantized mode of radiation of frequency $\Delta_c$, and undergoing a complicated multiphoton interaction of infinte order.

There is an infinite sequency of "probe resonance zones" in the neighborhood of the Rabi subharmonic resonances at $\Delta_c=r/m$ $(m=\pm1,\pm2,\dots)$, where $H$ can be further simplified by means of another rotating-wave approximation (RWA) as discussed in detail in Refs. 1 and 2. The conditions for the second RWA are (i),

$$|r-m\Delta_c|\ll r,\quad m=\pm1,\pm2,\dots$$

and (ii),

$$|m|\xi\alpha\ll r\ ,$$

where $\alpha$ is the square root of the average cavity photon number. Condition (i) restricts $\Delta_c$ within the neighborhood of the $m$th subharmonic resonance, and condition (ii) ensures that the cavity intensity is weak enough to be considered a probe. In the spirit of the RWA, the slowly varying terms are identified and retained. In the $m$th resonance zone $S_+$ has the zeroth-order time dependence $\exp(+irt)$ that is approximately canceled by the zeroth-order time dependence $\exp[-i(\Delta_c/m)t]$ of all field operators of the form $a^{\dagger n}a^{n+m}$, for any $n$. Thus the effective RWA Hamiltonian for the $m$th resonance zone is

$$H_{\text{eff}}=\Delta_c a^\dagger a+rS_z'+\frac{\xi}{4}(S'_+B_m+B_m^\dagger S'_-)\ , \tag{2.12}$$

where we have dropped the constant term in (2.9) because it plays no role in the dynamics of the system. The operator $B_m$ is given for $m>0$ by

$$B_m=\exp\left[\frac{-\xi^2}{8\Delta_c^2}\right]\sum_{n=0}^{\infty}\frac{(-1)^n}{n!(n+m)!}\left[\frac{\xi}{2\Delta_c}\right]^{2n+m-1}\times\left[2n+m-\frac{\xi^2}{4\Delta_c^2}\right]a^{\dagger n}a^{n+m} \tag{2.13}$$

and the negative-$m$ operators are obtained from the relation

$$B_{-m}=(-1)^m B_m^\dagger\ . \tag{2.14}$$

Concerning the effective RWA $m$th-zone Hamiltonian given in (2.12), two remarks may be made. First, the physical picture is relatively simple, because the basic role of $B_m$ is to destroy $m$ photons of frequency $\Delta_c$. This feature will enable us to find analytic expressions for the time dependence of the dynamical variables of interest.

Second, by combining conditions (i) and (ii) we obtain $\xi\alpha \ll \Delta_c$, which suggests keeping only the lowest-order term $n=0$ in $B_m$. The effective $m$th-zone Hamiltonian then takes its simplest form

$$H_{\text{eff}} \approx \Delta_c a^\dagger a + rS'_z + \frac{\xi}{4(m-1)!}\left[\frac{\xi}{2\Delta_c}\right]^{m-1} \times (S'_+ a^m + \text{H.c.}) , \tag{2.15}$$

which we recognize as the $m$th-order multiphoton Jaynes-Cummings Hamiltonian.[9]

## III. TIME EVOLUTION OPERATOR

With the help of the effective Hamiltonian (see Fig. 2), one can obtain an explicit expression for the time evolution operator, and the time dependence of any dynamical variable may then be obtained. Here, we consider both the time-dependent and time-averaged atomic inversion, which are the simplest quantities to describe the response of the atom.

The time evolution operator is given by

$$S_m(t) = U(t) T \exp(-iH_{\text{eff}} t) T^\dagger U^\dagger(0) \tag{3.1}$$

Any state vector $|\Psi(t)\rangle$ at time $t$ is related to its initial state by

$$|\Psi(t)\rangle = S_m(t)|\Psi(0)\rangle . \tag{3.2}$$

It is well known that $\exp(-iH_{\text{eff}}t)$ is expressible as a $2\times2$ matrix,[10] using the two $S_z$ eigenvectors as basis. After doing some lengthy algebra, $S_m(t)$ reads

$$S_m(t) = \exp(-i\theta a^\dagger a)\begin{bmatrix} C_{11} & C_{21} \\ C_{21} & C_{22}\end{bmatrix}\exp(i\psi_L a^\dagger a) , \tag{3.3}$$

where we have defined operators $C_{ij}$ by

$$C_{11} = e^{i(\psi_L - \theta)/2}[(GD_{11} + G^\dagger D_{21})G^\dagger + (GD_{12} + G^\dagger D_{22})G]/2 , \tag{3.4a}$$

$$C_{21} = e^{i(\psi_L + \theta)/2}[(GD_{11} - G^\dagger D_{21})G^\dagger + (GD_{12} - G^\dagger D_{22})G]/2 , \tag{3.4b}$$

$$C_{12} = e^{-i(\psi_L + \theta)/2}[(GD_{11} + G^\dagger D_{21})G^\dagger - (GD_{12} + G^\dagger D_{22})G]/2 , \tag{3.4c}$$

$$C_{22} = e^{-i(\psi_L - \theta)/2}[(GD_{11} - G^\dagger D_{21})G^\dagger - (GD_{12} - G^\dagger D_{22})G]/2 , \tag{3.4d}$$

with $\theta = \omega_L t + \psi_L$, and $G = V^2$. The $D_{ij}$'s take the form

$$D_{11} = e^{-i\Delta_c t(a^\dagger a + m/2)}[\cos(d_m t) - i\delta \sin(d_m t)/d_m] , \tag{3.5a}$$

$$D_{21} = -i\xi e^{-i\Delta_c t(a^\dagger a + m/2)}[B_m^\dagger \sin(d_m t)/d_m]/4 , \tag{3.5b}$$

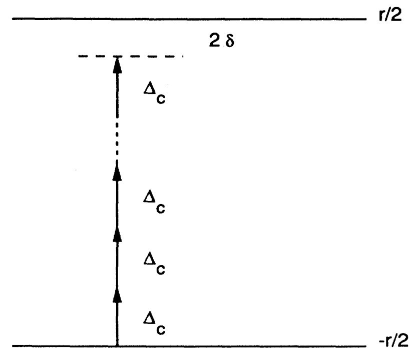


FIG. 2. Schematic diagram showing the system described by the effective Hamiltonian.

$$D_{12} = -i\xi e^{-i\Delta_c t(a^\dagger a - m/2)}[B_m \sin(d'_m t)/d'_m]/4 , \tag{3.5c}$$

$$D_{22} = e^{-i\Delta_c t(a^\dagger a - m/2)}[\cos(d'_m t) + i\delta \sin(d'_m t)/d'_m] , \tag{3.5d}$$

where we have used the abbreviations

$$d_m = (16\delta^2 + \xi^2 B_m B_m^\dagger)^{1/2}/4 , \quad d'_m = (16\delta^2 + \xi^2 B_m^\dagger B_m)^{1/2}/4 , \tag{3.6}$$

and the quantity

$$\delta = (r - m\Delta_c)/2 \tag{3.7}$$

measures the detuning from the $m$th resonance. In this representation, any initial state vector $|\Psi(0)\rangle$ of the system takes the form

$$|\Psi(0)\rangle = \begin{bmatrix} p_2|\Phi_f\rangle_2 \\ p_1|\Phi_f\rangle_1 \end{bmatrix} , \tag{3.8}$$

where the $p_i$'s ($i=1,2$) are the probability amplitudes for the atom, and the $|\Phi_f\rangle_i$'s are the field states connected with the level $i$.

Of special interest for study is the case in which the initial cavity field is in a coherent state $|\alpha\rangle$:

$$|\Phi_f\rangle_1 = |\Phi_f\rangle_2 = |\alpha\rangle , \tag{3.9}$$

where $\alpha = |\alpha| e^{i\psi_c}$. The coherent state is well suited for making comparison with semiclassical calculations. In the coherent-state case we will consider both time-dependent and time-averaged inversions.

## IV. TIME-DEPENDENT ATOMIC INVERSION

The atomic inversion is defined as $w(t) = \langle S_z \rangle$. In the $m$th resonance zone we find

$$w(t) = 1 - 2|\langle\alpha| e^{-i(\psi_L a^\dagger a)}(p_1^* C_{21}^\dagger + p_2^* C_{22}^\dagger)(p_1 C_{21} + p_2 C_{22}) e^{i(\psi_L a^\dagger a)}|\alpha\rangle|^2 . \tag{4.1}$$

By expanding $|\alpha\rangle$ in the Fock basis and making use of the fact that the operators $D_{ij}$'s are functions of operators $a^\dagger$ and $a$, Eq. (4.1) may be rewritten as

$$w(t)=1-\tfrac{1}{2}\sum_{s=0}^{\infty}|R_s(t)|^2 , \tag{4.2}$$

where $R_s(t)$ is the sum of the series,

$$R_s(t)=\sum_{n=0}^{\infty}\{(p_1+p_2)[G_{s,n}F_{11}(n)-G_{s,n}F_{21}(n)] +(p_1-p_2)[G_{s,n}F_{12}(n)+G_{n,s}F_{22}(n)]\} , \tag{4.3}$$

where $G_{s,n}$ are the matrix elements of the operator $G$ represented in the Fock space, and the definitions of $F_{ij}(n)$ are given by the matrix elements of $D_{ij}$ between Fock states and appropriate coherent states specified as follows:

$$F_{11}(n)=\langle n|D_{11}|\alpha_+\rangle,\quad F_{21}(n)=\langle n|D_{21}|\alpha_+\rangle , \quad F_{12}(n)=\langle n|D_{12}|\alpha_-\rangle,\quad F_{22}(n)=\langle n|D_{22}|\alpha_-\rangle , \tag{4.4}$$

with

$$\alpha_{\pm}=|\alpha|e^{i(\psi_L-\psi_c)}\pm\frac{\xi}{4\Delta_c} . \tag{4.5}$$

In this way the dependence of the initial phase difference between the classical field and the cavity field is included through the variable $\alpha_{\pm}$. The explicit form of $F_{ij}(n)$ can be easily evaluated:

$$\begin{aligned}
F_{11}(n)&=e^{-|\alpha_+|^2/2}\frac{\alpha_+^n}{\sqrt{n!}}e^{-i\Delta_c t(n+m/2)}\{\cos[d_m(n)t]-i\delta\sin[d_m(n)t]/d_m(n)\} ,\\
F_{22}(n)&=e^{-|\alpha_-|^2/2}\frac{\alpha_-^n}{\sqrt{n!}}e^{-i\Delta_c t(n-m/2)}\{\cos[d_m'(n)t]+i\delta\sin[d_m'(n)t]/d_m'(n)\} ,\\
F_{12}(n)&=-i\xi e^{-|\alpha_-|^2/2}\frac{\alpha_-^{n+m}}{\sqrt{(n+m)}}e^{-i\Delta_c t(n+m/2)}\{B_m^{n,n+m}\sin[d_m'(n+m)t]/d_m'(n+m)\}/4 ,\\
F_{21}(n)&=-i\xi e^{-|\alpha_+|^2/2}\frac{\alpha_+^{n-m}}{\sqrt{(n-m)!}}e^{-i\Delta_c t(n-m/2)}\{B_m^{n-m,n}\sin[d_m(n-m)t]/d_m(n-m)\}/4 ,
\end{aligned} \tag{4.6}$$

where the following abbreviations have been used:

$$d_m(n)=\langle n|d_m|n\rangle,\quad d_m'(n)=\langle n|d_m'|n\rangle , \tag{4.7}$$

and

$$B_m^{j,k}=\langle j|B_m|k\rangle . \tag{4.8}$$

Equation (4.2) is quite complicated to analyze. We have performed some numerical calculations on the simplest situation where the cavity is at the principal resonance (i.e., $\Delta_c=r$). In this case, $w(t)$ displays interesting features for different initial conditions. They are discussed as follows.

### A. Vacuum cavity field plus atomic ground state ($|\alpha|=0, p_1=1, p_2=0$)

The motion of the atomic inversion is just Rabi oscillations (at frequency $r$) but the amplitude is slowly modulated by a much slower frequency $\xi/4$, where $\xi$ is the vacuum field Rabi frequency $\xi$. The result is illustrated in Fig. 3. Analytically, an approximate expression for $w(t)$ may be derived from (4.2) for this initial condition. We find, by keeping the leading order terms in the series (4.3),

$$w(t)\approx-\cos(\Delta_c t)\cos(\xi t/4) . \tag{4.9}$$

### B. Strong cavity field plus atomic dressed state ($|\alpha|\gg 1, p_1=p_2=1/\sqrt{2}$)

In this case, the expression for $w(t)$ may be approximated as[11]

$$w(t)\approx w_s(t)+w_f(t) , \tag{4.10}$$

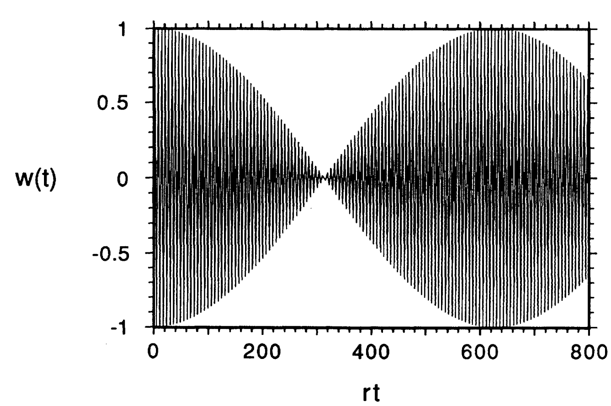


FIG. 3. Time-dependent atomic inversion for the principal resonance. The values of the parameters are $r=1$, $\xi=0.02$ and the initial conditions are vacuum field and atomic ground state.

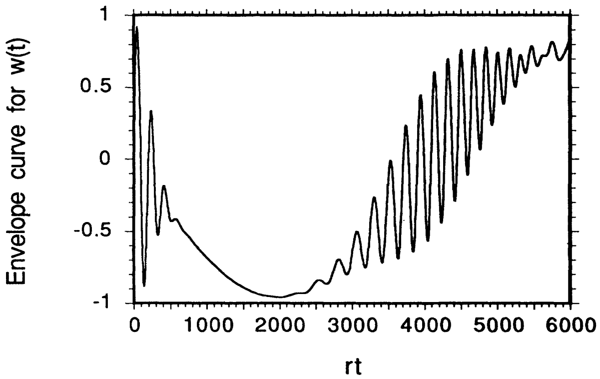


FIG. 4. The envelope curve for the atomic inversion as a function of dimensionless time *rt* for the principal resonance. It is obtained by connecting the $w(t)$ at discrete times $t=(2n+\frac{1}{2})\pi/\Delta_c$. The values of the parameters are $r=1$, $\xi=0.02$, $|\alpha|^2=10$. The initial atomic state is one of the dressed states with $p_1=p_2=1/\sqrt{2}$.

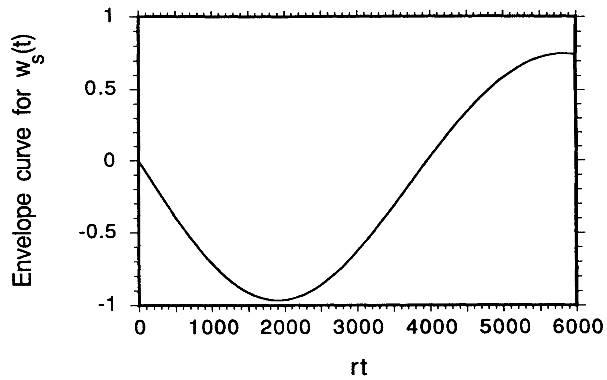


FIG. 6. Time development of the envelope for the slow component $w_s(t)$.

where $w_s(t)$ and $w_f(t)$ are slow and fast components, respectively. They are defined by the following equations:

$$w_s(t)=\sum_{n=0}^{\infty} e^{-|\alpha|^2}\frac{|\alpha|^{2n}}{n!}\sin(\Delta_c t)\sin(\Omega_n^s t) \tag{4.11}$$

and

$$w_f(t)=\sum_{n=0}^{\infty} e^{-|\alpha|^2}\frac{|\alpha|^{2n}}{n!}\sin(\Delta_c t)\sin(\Omega_n^f t) \ , \tag{4.12}$$

where

$$\Omega_n^f=\frac{\xi}{4}(\sqrt{n+1}+\sqrt{n}\,), \quad \Omega_n^s=\frac{\xi}{4}(\sqrt{n+1}-\sqrt{n}\,) \ , \tag{4.13}$$

are the fast and slow frequencies, respectively.

Since the factor $\sin(\Delta_c t)$ is a rapidly oscillating function of time, the description of the time development of $w(t)$ is more transparent by looking at it stroboscopically. We take the values of the atomic inversion only when $t=(2n-\frac{1}{2})\pi/\Delta_c$ [i.e., at the instant that $\sin(\Delta_c t)$ reaches its minimum]. The curve connecting these points is identified as the envelope for $w(t)$ because it simply describes the amplitude variation of the inversion. In Fig. 4, the envelope curve for $w(t)$ is plotted. Similarly, we plot the envelope curves for $w_f(t)$ and $w_s(t)$ in Figs. 5 and 6, respectively. The decomposition of $w(t)$ into the fast and slow components introduced in Eq. (4.10) is clear now.

It is interesting to see that the envelope for $w_f(t)$ displays collapses and revivals in the course of time. In fact, the expressions (4.10)–(4.12) share similar structures for the time-dependent solution of the dipole moment in the coherent Jaynes-Cummings (JC) model[12] besides the factor $\sin(\Delta_c t)$ which accounts for the rapid oscillation driven by the laser field. The similarity may be understood by the fact that the operator $S_z$ equals $(S'_+ + S'_-)/2$. Therefore the atomic inversion of the original atom is equivalent to the dipole moment of the dressed atom.

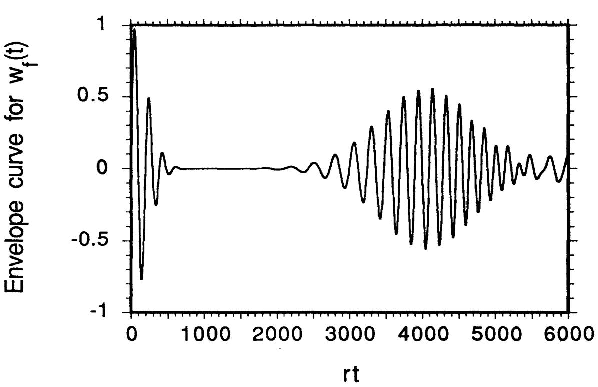


FIG. 5. Time development of the envelope for the fast component $w_f(t)$.

## V. TIME-AVERAGED INVERSION

The time-averaged inversion in the transient regime[1,2] is the simplest quantity to characterize the exchange of the energy between the atom, the pump, and the probe. We carry out the time averaging of $w(t)$ over a period $T$

$$\overline{W}_m=\frac{1}{T}\int_0^T w(t)dt \ , \tag{5.1}$$

where the subscript $m$ denotes the $m$th resonance zone, and the period $T$ is taken to be much shorter than any relaxation time but much larger than the longest period of the atomic oscillations, which is of order $(\Delta_c/\xi)^{m-1}/(m\xi)$. The final result ($m>0$) is

$$\overline{W}_m=(w_0/2)\sum_{n=0}^{\infty}V_{m+n,n}\frac{e^{-(|\alpha_+|^2+|\alpha_-|^2)/2}}{\sqrt{n!(n+m)!}}\frac{\mathrm{Re}(\alpha_-^{n+m}\alpha_+^{*n})}{1+x_n^2}$$
$$+(u_0/2)\sum_{n=0}^{\infty}V_{m+n,n}\left[e^{-|\alpha_+|^2}\frac{|\alpha_+|^{2n}}{n!}+e^{-|\alpha_-|^2}\frac{|\alpha_-|^{2n+2m}}{(n+m)!}\right]\frac{x_n}{1+x_n^2}$$
$$-(v_0/2)\sum_{n=0}^{\infty}V_{m+n,n}\frac{e^{-(|\alpha_+|^2+|\alpha_-|^2)/2}}{\sqrt{n!(n+m)!}}\mathrm{Im}(\alpha_-^{n+m}\alpha_+^{*n})\frac{1}{1+x_n^2}$$
$$+\frac{1}{2}\sum_{n=0}^{\infty}V_{m+n,n}\left[e^{-|\alpha_+|^2}\frac{|\alpha_+|^{2n}}{n!}-e^{-|\alpha_-|^2}\frac{|\alpha_-|^{2n+2m}}{(n+m)!}\right]\frac{x_n}{1+x_n^2} \tag{5.2}$$

and for negative $\Delta_c$,

$$\overline{W}_{-m}=(w_0/2)\sum_{n=0}^{\infty}V_{n,n+m}\frac{e^{-(|\alpha_+|^2+|\alpha_-|^2)/2}}{\sqrt{n!(n+m)!}}\frac{\mathrm{Re}(\alpha_+^{n+m}\alpha_-^{*n})}{1+x_n^2}$$
$$+(-1)^m(u_0/2)\sum_{n=0}^{\infty}V_{n,n+m}\left[e^{-|\alpha_+|^2}\frac{|\alpha_+|^{2n+2m}}{(n+m)!}+e^{-|\alpha_-|^2}\frac{|\alpha_-|^{2n}}{n!}\right]\frac{x_n}{1+x_n^2}$$
$$-(v_0/2)\sum_{n=0}^{\infty}V_{n,n+m}\frac{e^{-(|\alpha_+|^2+|\alpha_-|^2)/2}}{\sqrt{n!(n+m)!}}\mathrm{Im}(\alpha_+^{n+m}\alpha_-^{*n})\frac{1}{1+x_n^2}$$
$$+\frac{(-1)^m}{2}\sum_{n=0}^{\infty}V_{n,n+m}\left[e^{-|\alpha_+|^2}\frac{|\alpha_+|^{2n+2m}}{(n+m)!}-e^{-|\alpha_-|^2}\frac{|\alpha_-|^{2n}}{n!}\right]\frac{x_n}{1+x_n^2}\,, \tag{5.3}$$

where the following abbreviations have been made:

$$x_n=\frac{4\delta}{\xi(\langle n|B_mB_m^\dagger|n\rangle)^{1/2}} \tag{5.4}$$

and

$$V_{n+m,n}=\langle n+m|V|n\rangle$$
$$=\exp\left[\frac{-\xi^2}{8\Delta_c^2}\right]\sum_{j=0}^{n}\left[\frac{\xi}{2\Delta_c}\right]^{m+2j}\times\frac{(-1)^{m+j}\sqrt{n!(n+m)!}}{j!(m+j)!(n-j)!} \tag{5.5}$$

and $u_0, v_0, w_0$ are the initial Bloch vector components:

$$u_0=2\,\mathrm{Re}(p_2^*p_1e^{-i\psi})\,,$$
$$v_0=2\,\mathrm{Im}(p_2^*p_1e^{-i\psi})\,, \tag{5.6}$$
$$w_0=|p_2|^2-|p_1|^2\,.$$

Now we compare our result (5.2) with the previous result of semiclassical calculations[1,2] which reads

$$\overline{W}_m^c\approx\frac{1}{m!}\left[\frac{-\xi|\alpha|}{2\Delta_c}\right]^m\frac{-v_0\sin(m\psi)+w_0\cos(m\psi)+u_0x}{1+x^2}\,, \tag{5.7}$$

where

$$x\approx-\frac{4(m-1)!\delta}{\xi|\alpha|}\left[\frac{-\xi|\alpha|}{2\Delta_c}\right]^{m-1}\,. \tag{5.8}$$

The approximation sign means that only the terms of lowest order in $(\xi|\alpha|/\Delta_c)$ were kept. We notice that expression (5.2) is quite different from (5.7). The discrepancy may be considered as the consequence of the quantum nature of the cavity field. However, one can show that expression (5.7) is just the large photon number limit $(|\alpha|\gg1)$ of (5.2). The key step for proving this is to identify $\alpha_+\approx\alpha_-$. Then the quantity $\exp(-|\alpha|^2)|\alpha|^{2n}/n!$ appearing in the summation is just the Poisson distribution, which has a sharp peak at $n=|\alpha|^2$. We then evalu-

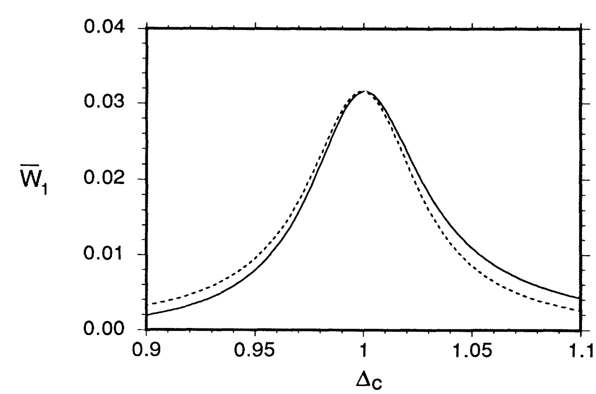


FIG. 7. Time averaged inversion as a function of atom-cavity deturning $\Delta_c$ for the $m=1$ resonance zone. The values of the parameters are $r=1$, $\xi=0.02$, $|\alpha|^2=10$, $\psi=0$, $u_0=v_0=0$, and $w_0=-1$. The solid line is for the quantum results and the dashed line is for the semiclassical result.

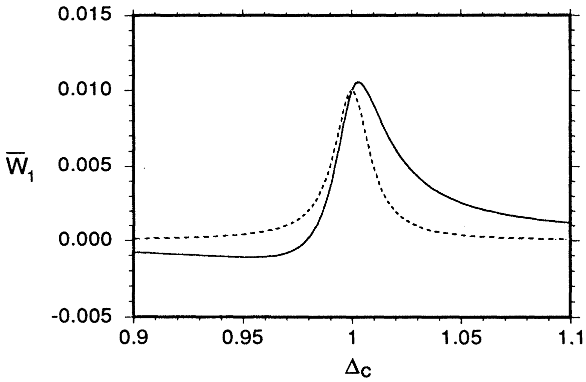


FIG. 8. Same as in Fig. 3 but now for $|\alpha|^2=1$.

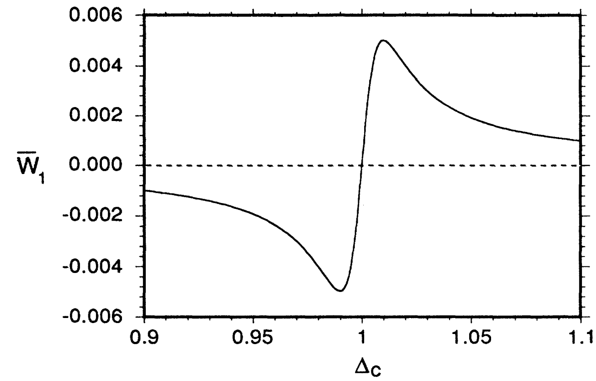


FIG. 9. Same as in Fig. 4 but now for $|\alpha|^2=0$.

ate the summations around the peak, and formula (5.7) follows quite easily.

The features of the average inversion (5.2) change dramatically when $\alpha$ is small. In Fig. 7–9, we present some graphical illustrations of $\overline{W}_m$ as a function of $\Delta_c$ for various values of $|\alpha|$. In order to compare the semiclassical result, we draw $\overline{W}_m^c$ on the same graph with a dashed line. The initial atomic state for these graphs is taken to be the ground state and the Rabi frequency $r$ is normalized to unity.

In Fig. 7, where the value of $|\alpha|$ is comparatively large, the curves of $\overline{W}_m$ and $\overline{W}_m^c$ are almost the same. Both of them show a Lorentizian-like peak at the resonance. Their slight difference is reflected in the width of the peak. The width for $\overline{W}_m$ is larger than that of $\overline{W}_m^c$. This fact may be interpreted as the influence of quantum fluctuations. The same feature appears in Fig., 8, but now $\overline{W}_m$ and $\overline{W}_m^c$ have a larger difference.

When the atom starts in a vacuum cavity field ($\alpha=0$), the atomic inversion is very small. We obtained a "dispersive" shape for $\overline{W}_m$ in Fig. 9. In this case, it is easy to show that $\overline{W}_m$ takes the approximate form

$$\overline{W}_m \approx \tfrac{1}{2}(1+u_0)\left[\frac{-\xi}{2\Delta_c}\right]^m \frac{x_0}{1+x_0^2}\ , \tag{5.9}$$

where only the lowest-order term in $\xi/\Delta_c$ is kept. Expression (5.9) shows no dependence upon the relative phase between the pump and the cavity field. This is expectable because the phase of the vacuum field may be considered to be completely random. Interestingly, the inversion is always zero at the exact resonance and reaches its extremum at

$$\Delta_c = \frac{r}{m} \pm \frac{\xi}{2\sqrt{m!}}\left[\frac{\xi}{2\Delta_c}\right]^{m-1}\ . \tag{5.10}$$

This equation reduces to $\Delta_c = r \pm \xi/2$ for the principal resonance zone. We see that the separation $\xi$ is exactly the vacuum Rabi frequency for the system.

The detailed mechanism of how the "dispersive" shape comes out is still not well understood. One may regard the result (5.9) as a consequence of spontaneous emission into the vacuum, and the emitted photons then interact with the atom again. In this aspect, the fact is related to the vacuum field splitting. However, the dynamics is more complicated because of the presence of the strong driven field. Further investigation on how the vacuum field modifies the atomic dynamics as well as the spectral properties of the cavity field would be necessary.

## VI. SUMMARY

In summary, we have established an effective rotating-frame Hamiltonian for a two-level atom interacting simultaneously with a variable-frequency quantized cavity field mode and with a classically described laser field. The two fields have different frequencies and amplitudes, and the cavity field is regarded as a quantum-mechanical probe.

There is a different Hamiltonian for each probe resonance zone, and for sufficiently large pump intensity the $m$th-zone Hamiltonian was shown to be the $m$-photon Jaynes-Cummings Hamiltonian.

We obtained explicit analytic expressions for the system evolution operator, and evaluated the atomic inversion and its time average. The time-dependent inversion at the principal resonance was discussed for both strong field and vacuum cavity field cases in which the atom is specially prepared in the dressed state and the ground state, respectively. We found that besides the rapid oscillation of frequency $\Delta_c$, the atom-cavity interaction modifies the dynamics of the inversion quite significantly. In particular, the occurrence of the collapse and revival behavior marks the quantum signature of the cavity field. The time average, as a function of cavity frequency, could be compared numerically with previous expressions obtained fully semiclassically. The greatest deviations were found for near-vacuum probe states, as could be expected.

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