PHYSICAL REVIEW A VOLUME 43, NUMBER 11 1 JUNE 1991

# Three-level atom in a broadband squeezed vacuum field. II. Applications

Z. Ficek* and P. D. Drummond
*Department of Physics, The University of Queensland, Queensland 4072, Australia*
(Received 5 October 1990)

Using the formalism developed earlier, we treat spontaneous emission from a three-level atom (ladder system) interacting with a broadband squeezed vacuum field. We obtain expressions for the transient and steady-state populations of the atomic levels with the conditions that the atom interacts with either a multimode perfect squeezed vacuum field, or a three-dimensional vacuum field in which the squeezed modes lie within a solid angle over which squeezing is propagated. The results are compared with those obtained for the atom interacting with a thermal field. We show that in the perfect case the first excited state is not populated when the squeezed vacuum field is in a minimum-uncertainty squeezed state. Moreover, the second excited state can have a steady-state population larger than $\frac{1}{2}$. These features are completely absent when the atom interacts with the thermal field. In addition, for a low-intensity squeezed vacuum field the population in the second excited state exhibits a linear rather than quadratic dependence on the intensity of the squeezed vacuum field. In the three-dimensional case the presence of unsqueezed modes considerably reduces the effect of squeezing on spontaneous emission. However, a significant reduction in a population of the first excited state and a population larger than $\frac{1}{2}$ in the second excited state can be achieved provided the squeezing is propagated over a large solid angle. We also discuss the effect of the two-photon detuning between the double carrier frequency of the squeezing and atomic transition frequency $\omega_3$ on the steady-state atomic population.

## I. INTRODUCTION

In the preceding paper[1] (henceforth referred to as I), a master-equation formalism was developed to study the interaction between a three-level atom in the cascade configuration (ladder system) and a broadband squeezed-vacuum field. As an illustration, we now apply this formalism to the calculation of spontaneous emission, especially to obtain information about the population of the atomic levels.

It is reasonable to expect that when an atom is interacting with a squeezed-vacuum field the atomic dynamics will be strongly affected. A particularly interesting effect of squeezed light on atomic dynamics is the proposed inhibition of atomic phase decay. It has been shown by Gardiner[2] that the two quadratures of the atomic polarization may be damped at different rates, one exhibiting an enhanced decay rate and the other a reduced decay rate compared with the normal radiative decay of the atom. This modification opens the possibility of obtaining subnatural linewidths in resonance fluorescence[3] and in the weak-field atomic absorption spectrum.[4] For experimental observation of the inhibition of atomic phase decay Parkins and Gardiner[5] have proposed a three-dimensional model of interaction between a two-level atom and the squeezed input light. In this model the two-level atom is located in a microscopic plane-mirror Fabry-Pérot cavity, and interacts with a squeezed input field, incident over some finite solid angle. All other modes are in an ordinary vacuum state. With this modification they have demonstrated that a significant reduction in fluctuations experienced by the atom can be achieved in one quadrature when the atom is located at the point at which the input squeezed field is focused. The reduction in fluctuations depends also on the solid angle $\Omega$ over which squeezing is propagated, and increases with increasing $\Omega$.

In recent studies of the applications of squeezed light, multiatom effects in a squeezed vacuum field have been studied. These show some very interesting deviations from the ordinary decay and ordinary emission spectra. In particular, the multiatom resonance fluorescence spectrum in a squeezed vacuum demonstrates an asymmetry under off-resonance excitation by an external coherent laser field.[6] Moreover, a system of atoms interacting with a squeezed vacuum field may generate pairwise atomic squeezed states,[7–9] and a new class of states, which satisfy the equality sign in the Heisenberg uncertainty relation for angular momentum operators, called intelligent spin coherent states.[10] Further work has also been done to incorporate the effect of pumping a single-mode multiatom laser system[11] with squeezed light. These investigations have shown that the anisotropy in the noise of the squeezed field leads to a locking of the laser's phase. Haake, Walls, and Collett[12] have shown how this squeezed-pump-laser model may be realized in practice, with an example of three-level atoms inside an optical cavity pumped by two independent squeezed fields.

In the present paper we demonstrate the effect of squeezed-vacuum field on the dynamics of a three-level atom. In Sec. II, using the master equations derived in paper I, we discuss the equation of motion for the population of the atomic levels when the atom interacts with either a multimode one-dimensional squeezed vacuum field, or a three-dimensional vacuum field in which the squeezed modes lie within a solid angle over which squeezing is propagated. In Sec. III we develop complete dynamical solutions of the equations when the atom interacts with the resonant perfect squeezed vacuum field and find that the system evolves into a steady-state condition that exhibits a number of novel features. We discuss the dependence of the steady-state population distribution upon the intensities of the incident squeezed light, the ratio between the decay constants for both excited states, and the two-photon detuning between the double frequency of the incident squeezed light and transition frequency from the ground to the second excited state. In Sec. IV we obtain solutions for the steady-state population of the atomic levels when the atom interacts with the imperfect squeezed-vacuum field. We discuss the dependence of the steady-state population distribution upon the angle $\theta$ over which squeezing is propagated. In Sec. V we discuss the effect of off-resonance excitation on the steady-state population of the atomic levels. In Sec. VI we summarize our results. In Appendix A we give an alternative derivation, based on the second-order perturbation theory, of the intensity dependence of the population in the second excited state. The results are in good agreement with the overall master-equation approach.

## II. ATOMIC-LEVEL POPULATIONS IN A SQUEEZED VACUUM

In this section, we apply the master equation (I-20), where I refers to the preceding paper, to obtain equations for the populations of the atomic levels. These equations are fundamental to our theory of a three-level atom interacting with a squeezed-vacuum field. We note that equation (I-20) is applicable, with appropriate interpretation, to either wave guided or three-dimensional squeezed fields. The only differences are in the decay rates, and in the squeezing coefficients $M$ and $N$.

Consider an appropriate representation for the density operator as

$$\rho=\sum_{i,j=1}^{3}\rho_{ij}|i\rangle\langle j|, \tag{1}$$

where $\rho_{ij}$ are the matrix elements of the reduced atomic density operator in terms of the atomic states. The master equation (I-20) for the matrix elements $\rho_{ii}$ (the occupation probabilities) leads to the following closed set of four equations of motion, which can be written in matrix form

$$\frac{d\mathbf{X}}{d\tau}=\underline{A}\mathbf{X}+\boldsymbol{\beta}, \tag{2}$$

where $\underline{A}$ is the real $4\times4$ matrix,

$$\underline{A}=\begin{bmatrix} -\alpha(N_2+1) & \alpha N_2 & -\frac{1}{2}\sqrt{\alpha}|M| & \frac{1}{2}|M|\delta_{21} \\ -[N_1-\alpha(N_2+1)] & -(\alpha N_2+2N_1+1) & \sqrt{\alpha}|M| & -\frac{1}{2}|M|(\delta_{12}+\delta_{21}) \\ 0 & 3\sqrt{\alpha}|M| & -\frac{1}{2}(\alpha+\alpha N_2+N_1) & -\frac{1}{2}(\delta_1+\delta_1^L-\Delta) \\ |M|(\delta_{12}-\delta_{21}) & -|M|(\delta_{12}+2\delta_{21}) & \frac{1}{2}(\delta_1+\delta_1^L-\Delta) & -\frac{1}{2}(\alpha+\alpha N_2+N_1) \end{bmatrix}. \tag{3}$$

The vector $\mathbf{X}$ has the following components:

$$\begin{aligned} &X_1=\rho_{33},\quad X_2=\rho_{22},\\ &X_3=(\rho_{13}+\rho_{31})\cos(\varphi_v-\tfrac{1}{2}\Delta\tau)\\ &\qquad+i(\rho_{31}-\rho_{13})\sin(\varphi_v-\tfrac{1}{2}\Delta\tau),\\ &X_4=i(\rho_{13}-\rho_{31})\cos(\varphi_v-\tfrac{1}{2}\Delta\tau)\\ &\qquad+(\rho_{31}+\rho_{13})\sin(\varphi_v-\tfrac{1}{2}\Delta\tau), \end{aligned} \tag{4}$$

while the vector $\boldsymbol{\beta}$ has the components

$$\beta_1=0,\quad \beta_2=N_1,\quad \beta_3=-\sqrt{\alpha}|M|,\quad \beta_4=|M|\delta_{21}. \tag{5}$$

For simplicity, In Eqs. (2)–(5) we have introduced the notation

$$\begin{aligned} &\tau=2\gamma_1 t,\quad \alpha=\gamma_2/\gamma_1,\quad \Delta=\frac{(\omega_1+\omega_2-2\omega)}{\gamma_1},\\ &\delta_1^L=\frac{\Delta\omega_1^0}{\gamma_1},\quad \delta_1=\frac{\Delta\omega_1}{\gamma_1},\delta_{ij}=\frac{\Delta\omega_{ij}}{\gamma_1}, \end{aligned} \tag{6}$$

where $\gamma_i=\gamma_{ii}$, $\Delta\omega_1^0$, $\Delta\omega_1$, and $\Delta\omega_{ij}$ are the damping constants and frequency shifts defined by (I-21), and $\omega$ is the carrier frequency of the squeezed-vacuum field whose double frequency $2\omega$ is tuned close to the atomic transition frequency $\omega_3=\omega_1+\omega_2$.

With the full, three-dimensional master equation the matrix elements $\rho_{ii}$ fulfill an equation of motion identical to Eq. (2), except with the parameters $N_i$ and $|M|$ replaced by the parameters (I-36), which vary with the angle that is squeezed. Moreover, in three dimensions $\gamma_i=\gamma_{ii}$, $\Delta\omega_1^0$, $\Delta\omega_1$, and $\Delta\omega_{ij}$ are defined by (I-37).

It is evident from Eqs. (2)–(4) that the squeezed vacuum field, which is characterized by the parameter $M$, introduces a coupling of the atomic-state populations with the two-photon coherences $\rho_{13}$ and $\rho_{31}$. This coupling of the atomic populations with the two-photon coherences is due to the fact that the squeezed-vacuum field contains strong internal two-photon correlations that are transferred to the atomic system. Moreover, this coupling is dependent on the generalized decay constant

$\eta_{12}=\sqrt{\alpha}\gamma_1$. Clearly, $\eta_{12}$ plays an important role in the interaction of a three-level atom with a squeezed vacuum field.

Since $v(\theta)$ only alters the extent of squeezing, from this point on we will use the parameters $M$ and $N_i$ only. Whether the system is of one or more dimensions, it is the effective squeezing parameter $M$ and effective mean photon numbers $N_i$ that determine the resulting populations and time evolution.

To simplify Eq. (2), we will ignore the Lamb shift $\delta_1^L$ and the shifts $\delta_1$ and $\delta_{ij}$. In Appendix B we show that these parameters are negligibly small when the carrier frequency $\omega$ of the squeezed field is tuned close to the atomic frequencies $\omega_1$ and $\omega_2$, given a typical model of the squeezed radiation produced by a finite-bandwidth nondegenerate parametric amplifier.

## III. RESONANT EXCITATION: PERFECT SQUEEZING

To obtain information about the time evolution of the atomic-level populations we have to solve the set of equations (2). Equation (2) can be easily solved by Laplace transform techniques. We will first consider the case in which the double carrier frequency of the squeezing is resonant with the atomic transition frequency $\omega_3$, i.e., $\Delta=0$. Moreover, we shall assume that the atom interacts with a perfect squeezed vacuum field. This holds when the atom interacts with a multimode one-dimensional squeezed vacuum field or when a multimode three-dimensional squeezed field propagates over the angle $\theta=\pi$, which corresponds to a perfect electric-dipole wave. With these assumptions the time evolution of the populations in the excited atomic levels, given an initial ground state, is

$$\begin{aligned}\rho_{33}(\tau)=&\frac{N_1N_2w-|M|^2(w-m-2\alpha)}{w(3N_1N_2+2N_1+N_2+1-3|M|^2)}+\frac{2\alpha|M|^2(3w-2m-4\alpha)}{w[(m+w)^2-u^2]}\exp(-\tfrac{1}{2}w\tau)\\&+\frac{\alpha[2N_1N_2(u-m-w)+|M|^2(u+m+4\alpha-4w)]}{u(u-m-w)(u-m-2w)}\exp[-\tfrac{1}{2}(m+2w-u)\tau]\\&+\frac{\alpha[2N_1N_2(u+m+w)+|M|^2(u-m-4\alpha+4w)]}{u(u+m+w)(u+m+2w)}\exp[-\tfrac{1}{2}(m+2w+u)\tau]\end{aligned}\tag{7}$$

and

$$\begin{aligned}\rho_{22}(\tau)=&\frac{[N_1(1+N_2)-|M|^2]}{(3N_1N_2+2N_1+N_2+1-3|M|^2)}+\frac{N_1(u-m-2N_1)-2\alpha|M|^2}{u(u-m-2w)}\exp[-\tfrac{1}{2}(m+2w-u)\tau]\\&-\frac{N_1(u+m+2N_1)+2\alpha|M|^2}{u(u+m+2w)}\exp[-\tfrac{1}{2}(m+2w+u)\tau],\end{aligned}\tag{8}$$

where, for simplicity, we have introduced the notation

$$\begin{aligned}&m=(1-\alpha),\quad w=\alpha+\alpha N_2+N_1,\\&u=[(m+2w)^2-4\alpha(3N_1N_2+2N_1+N_2+1-3|M|^2)]^{1/2}.\end{aligned}\tag{9}$$

In Fig. 1 the time evolution of the atomic population is shown for $N_1=N_2=0.2$, $\alpha=1$, and different degrees of squeezing. These graphs show that the atomic population is strongly dependent on the strength of the squeezing. As the degree of squeezing increases, the population of the first excited state $|2\rangle$ decreases [see Fig. 1(a)]. For minimum-uncertainty squeezed states, i.e., $|M|^2=N(N+1)$, this state is populated only in the transient regime. The population of the second excited state $|3\rangle$ increases [see Fig. 1(b)] with an increasing degree of squeezing and is highest for minimum-uncertainty squeezed states. The squeezed vacuum field also introduces a dependence of the atomic population on the ratio $\alpha=\gamma_2/\gamma_1$.

Figure 2 shows the atomic populations $\rho_{22}(\tau)$ and $\rho_{33}(\tau)$ for $N=0.2$, with minimum-uncertainty squeezed states and different values of $\alpha$. It is obvious that the atomic-state populations increase as the ratio $\alpha$ decreases. In addition, for very small $\alpha$ ($\alpha\ll1$) the steady-state population of the second excited state $|3\rangle$ is greater than $\frac{1}{2}$ [see Fig. 2(b)]. For $N_1=N_2$ the steady-state population of the first excited state $|2\rangle$ is equal to zero, independent of $\alpha$. These features are completely absent when the atom interacts with a thermal field. In order to show this more quantitatively, we consider the steady-state populations of the excited atomic levels. From Eqs. (7) and (8) for the steady state ($\tau\to\infty$), we have

$$\rho_{22}=\frac{N_1(1+N_2)-|M|^2}{(3N_1N_2+2N_1+N_2+1-3|M|^2)}\tag{10}$$

and

$$\rho_{33}=\frac{N_1N_2w-|M|^2(w-m-2\alpha)}{w(3N_1N_2+2N_1+N_2+1-3|M|^2)},\tag{11}$$

where $w$, $m$, and $\alpha$ are defined in Eq. (9).

The above steady-state solutions, apart from the parameters $N_1$ and $N_2$, also include the absolute value of the parameter $M$, which means that the squeezed vacuum

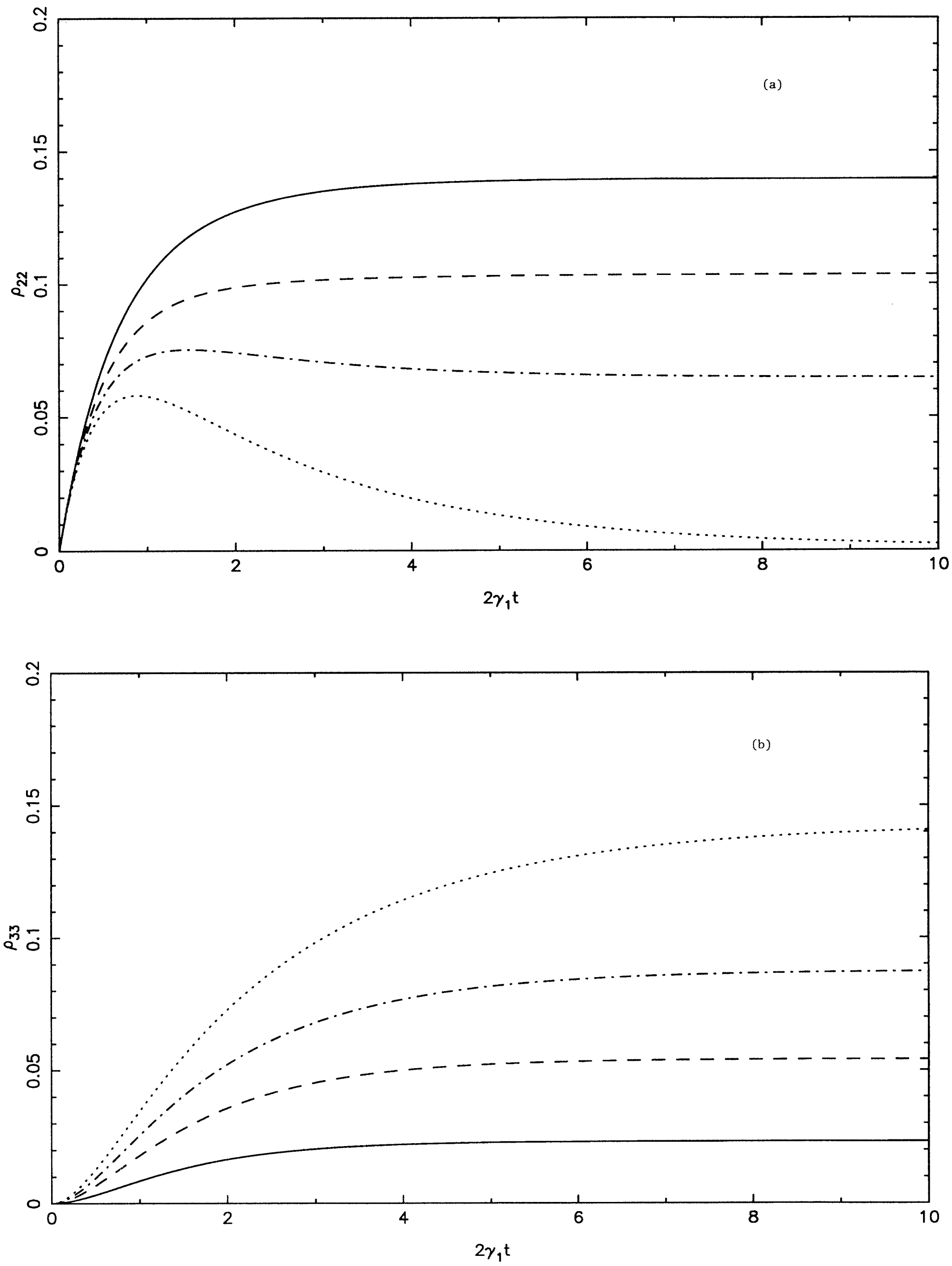


FIG. 1. Time evolution of the population in the state (a) $|2\rangle$ and (b) $|3\rangle$ for $N_1=N_2=0.2$, $\gamma_1=\gamma_2$, and different degrees of squeezing: $|M|=0$(——), $|M|=0.3$(— — —), $|M|=0.4$(—·—·—·), $|M|=[N(N+1)]^{1/2}$(· · · ·).

field changes the steady-state atomic population. When the squeezed vacuum field is replaced by a thermal field at a temperature $T$, we find that $N_1, N_2 \neq 0$, $|M|=0$, and the steady-state atomic populations are

$$\rho_{22}=\frac{N_1(1+N_2)}{(3N_1N_2+2N_1+N_2+1)} \tag{12}$$

and

$$\rho_{33}=\frac{N_1N_2}{(3N_1N_2+2N_1+N_2+1)} . \tag{13}$$

In this case the steady-state populations $\rho_{22}$ and $\rho_{33}$ depend on the relation between $N_1$ and $N_2$ and are independent of the ratio $\alpha$. For $N_1 \gg 1$ and $N_2 \ll 1$ one finds $\rho_{22} \approx 0.5$ and $\rho_{33} \approx \frac{1}{2}N_2$. In this case the first excited state $|2\rangle$ is highly populated, whereas the second excited state

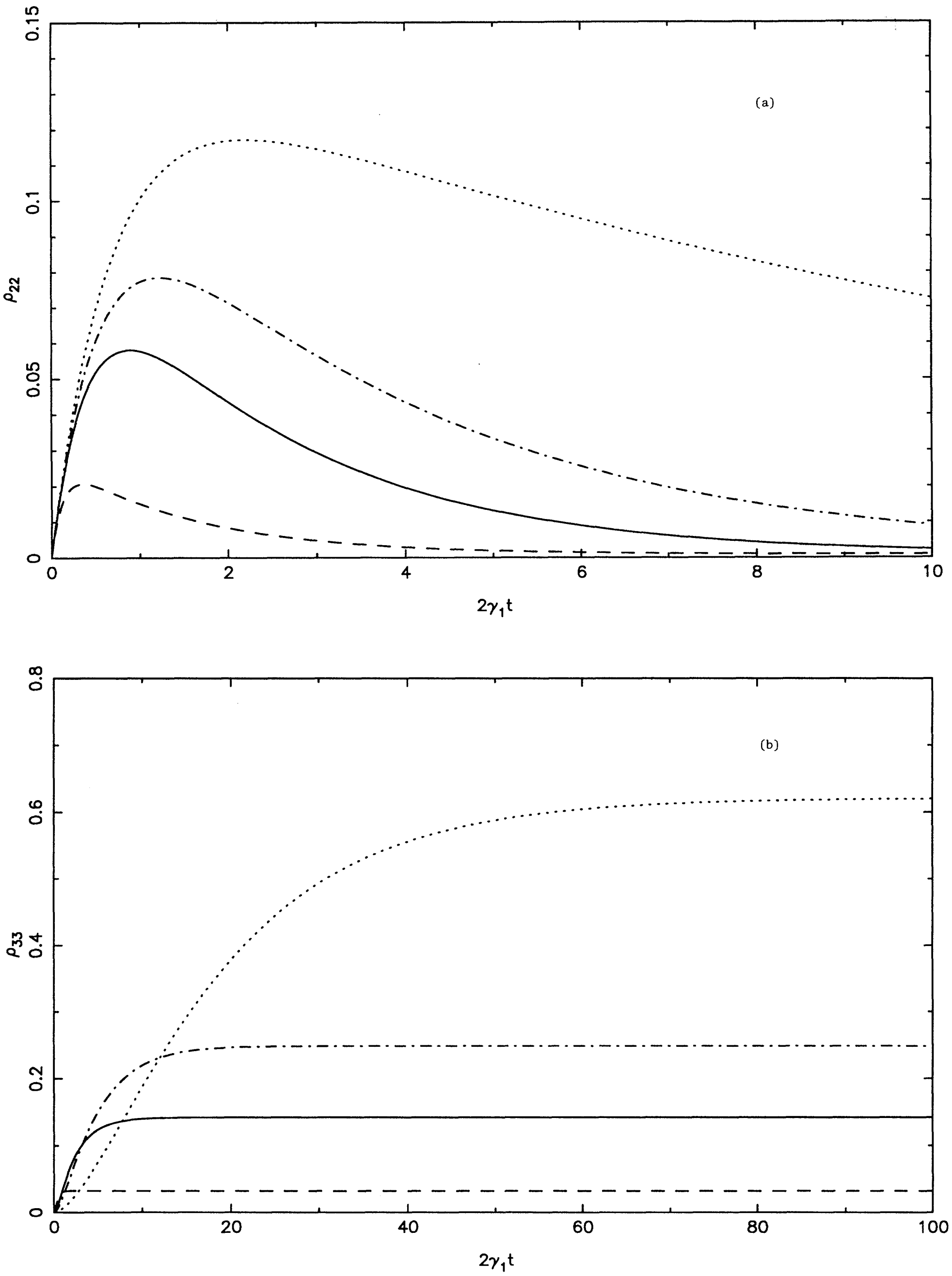


FIG. 2. Time evolution of the population in the state (a) $|2\rangle$ and (b) $|3\rangle$ for $N_1=N_2=0.2$, minimum-uncertainty squeezed states and different $\alpha=\gamma_2/\gamma_1$ : $\alpha=1.0$( —— ), $\alpha=5.0$( — — — ); $\alpha=0.5$ ( —·—·—· ) $\alpha=0.1$ (· · · · ·).

$|3\rangle$ has a very small population. For $N_1 \approx N_2 \gg 1$ the populations $\rho_{22}$ and $\rho_{33}$ asymptotically approach the limit $\frac{1}{3}$. In this case all atomic states are populated with the same probability.

If the atom interacts with a squeezed vacuum field, we have $N_1$, $N_2 \neq 0$ and $|M| \neq 0$. The steady-state atomic population differs substantially from that for the thermal field. For minimum-uncertainty squeezed states we also have $|M|^2=N_2(1+N_1)$ if $N_1>N_2$, and $|M|^2=N_1(1+N_2)$ if $N_2>N_1$. In the first case of $N_1>N_2$, this gives

$$\rho_{22}=\frac{(N_1-N_1)}{1+2(N_1-N_2)} \tag{14}$$

and

$$\rho_{33}=\frac{N_2[1+\alpha(N_1-N_2)]}{(\alpha+\alpha N_2+N_1)[1+2(N_1-N_2)]} \ . \tag{15}$$

Here the population of the state $|2\rangle$ depends on the difference $(N_1-N_2)$ of the intensities of the modes at the frequencies $\omega_1$ and $\omega_2$, and is close to zero when $(N_1-N_2)$ is not too large. For $N_1 \gg N_2 \gg 1$ the population $\rho_{22}$ asymptotically approaches $\frac{1}{2}$. On the other hand, the population of the state $|3\rangle$ is proportional to $N_2$ and, unlike the population in a thermal field, now depends on the ratio $\alpha=\gamma_2/\gamma_1$. This dependence gives the possibility of obtaining the population in the state $|3\rangle$ as greater than $\frac{1}{2}$. For example, when $N_1 \approx N_2$ and $\alpha$ is very small $(\alpha \ll 1)$, then $\rho_{33} \approx N_2/N_1$. This can be close to 1 for $N_2 \approx N_1$. In this case we can obtain an inversion of the atomic population, which cannot be obtained in a thermal field excitation.

Now consider the minimum-uncertainty situation when $N_2 \geq N_1$. From Eq. (I-7) we have that, for the minimum-uncertainty squeezed states, $|M|^2=N_1(1+N_2)$. Thus Eqs. (10) and (11) lead to

$$\rho_{22}=0 \tag{16}$$

and

$$\rho_{33}=\frac{N_1}{(\alpha+\alpha N_2+N_1)} \; . \tag{17}$$

Here, the population of the first excited state $|2\rangle$ is always equal to zero, independent of the relation between $N_1$ and $N_2$, whereas the population of the second excited state $|3\rangle$ can be greater than $\frac{1}{2}$. In fact, if $\alpha$ is small then $\rho_{33} \approx 1$. Just as in the case $N_1 > N_2$ we can obtain a total population inversion in a three-level atom interacting with the squeezed-vacuum field. These new effects, which do not appear in a normal vacuum as well as in a thermal field, are due to the fact that the squeezed vacuum field contains strong internal two-photon correlations that are transferred to the atomic system. These correlations generate two-photon transitions in the three-level atom, which directly transfer atomic population from the ground state $|1\rangle$ to the second excited state $|3\rangle$ without population of the state $|2\rangle$. These lead to high population of the state $|3\rangle$.

It is interesting to note from Eq. (7) that in a low-intensity incident field $(N_1,N_2 \ll 1)$ the time evolution of the population of the $|3\rangle$ state in a thermal field $(|M|=0)$ has a quadratic dependence on $N$. For $N_1=N_2=N$ this has the form

$$\rho_{33}(\tau) \approx \frac{N^2}{(1-\alpha)}[(1-\alpha)-e^{-\alpha\tau}+\alpha e^{-\tau}] \; . \tag{18}$$

In a squeezed-vacuum field, however, the time evolution of the population $\rho_{33}(\tau)$ depends *linearly* on $N$, and for the minimum-uncertainty squeezed states has the form

$$\rho_{33}(\tau) \approx \frac{N}{\alpha}(1-2e^{-\alpha\tau/2}+e^{-\alpha\tau}) \; . \tag{19}$$

This unexpected linear dependence on $N$ of the population $\rho_{33}(\tau)$ may be explained heuristically as follows. In a thermal field there are no internal two-photon correlations and the transition $|1\rangle \rightarrow |3\rangle$ appears as a two step transition $|1\rangle \rightarrow |2\rangle \rightarrow |3\rangle$. At low intensity these two transitions are both proportional to intensity $N$, resulting in a quadratic dependence on $N$ of the two-photon transition $|1\rangle \rightarrow |3\rangle$. In the squeezed-vacuum field, however, there exists highly correlated photon pairs that can effectively transfer the atomic population from $|1\rangle$ to $|3\rangle$ in a single step proportional to $N$. In Appendix A we give another derivation of the linear dependence of the population $\rho_{33}(\tau)$ on the intensity of the squeezed field using second-order perturbation theory. In this derivation it is clear that there are two ways to populate state $|3\rangle$, either through intensity terms like $N^2$, or through coherence terms like $|M|^2$. It is the $|M|^2$ terms that dominate at low intensity, and since $|M|^2 \approx N$ at low intensities, there is a linear intensity dependence in a minimum-uncertainty squeezed state.

Recently, Gea-Banacloche[13] and Javanainen and Gould[14] reported that the low-intensity two-photon transition rate in a three-level atom interacting with a squeezed-vacuum field is linearly dependent on the intensity of squeezing, giving results similar to ours.

## IV. EFFECTS OF IMPERFECT SQUEEZING

In this section, we investigate in more detail the effects of imperfect squeezing, or "nonideal" coupling between the three-level atom and the squeezed vacuum field. In paper I we have derived the master equation for the atomic density operator $\rho$, assuming that the atom interacts with a three-dimensional vacuum field in which squeezed modes have propagation vectors inside a solid angle $\Omega$. This "nonideal" coupling between the atom and the squeezed vacuum field means that the atom interacts not only with the squeezed modes but also with ordinary (unsqueezed) vacuum modes. As stated earlier, the master equation (I-20) for the occupation probabilities $\rho_{ii}$ of the atomic levels leads to a closed system of equation of motion identical to Eq. (2), but with $N_i$ and $M$ replaced by the parameters (I-36). In this case the squeezing parameters $N_i$ and $M$ are dependent on the angle $\theta$ over which squeezing is propagated. When the angle $\theta$ is smaller than $\pi$, then $v(\theta)$ is smaller than 1, and the effect of squeezing on the atomic population distribution can be reduced. In order to show this, consider the steady-state populations of the atomic levels $|2\rangle$ and $|3\rangle$.

Replacing $N_i$ and $M$ by the input field parameters (I-36) and for minimum-uncertainty squeezed states, Eqs. (10) and (11), with $N_1 > N_2$ leads to

$$\rho_{22}=\frac{v(\theta)[N_1'-v(\theta)N_2']}{\{v(\theta)[2N_1'+N_2'-3v(\theta)N_2']+1\}} \tag{20}$$

and

$$\rho_{33}=\frac{v^2(\theta)N_2'\{1-\alpha v(\theta)N_2'+[1+\alpha-v(\theta)]N_1'\}}{[\alpha+\alpha v(\theta)N_2'+v(\theta)N_1']\{v(\theta)[2N_1'+N_2'-3v(\theta)N_2']+1\}} \; , \tag{21}$$

while for $N_2 \geq N_1$,

$$\rho_{22} = \frac{v(\theta)N_1'[1-v(\theta)]}{\{v(\theta)[2N_1'+N_2'-3v(\theta)N_1']+1\}} \tag{22}$$

and

$$\rho_{33} = \frac{v^2(\theta)N_1'\{1-v(\theta)N_1'+[1+\alpha-\alpha v(\theta)]N_2'\}}{[\alpha+\alpha v(\theta)N_2'+v(\theta)N_1']\{v(\theta)[2N_1'+N_2'-3v(\theta)N_1']+1\}} . \tag{23}$$

By comparing Eq. (22) with Eq. (16) we see that for $N_2 \geq N_1$ in the three-dimensional case the population $\rho_{22}$ is different from zero. In contrast, in the one-dimensional case for perfect squeezing the population $\rho_{22}$ is equal to zero. This difference is due to the fact that in the one-dimensional case we have assumed that all modes interacting with the atom are squeezed, while in the three-dimensional case only those modes are squeezed whose propagation vectors lie inside the angle $\theta$ over which squeezing is propagated. The value of the population in the state $|2\rangle$ is now dependent on the angle $\theta$, and can be significantly reduced when $\theta$ is large. This is shown in Fig. 3, where the steady-state population $\rho_{22}$ given by Eq. (10) is plotted versus the angle $\theta$ for the thermal field and for the minimum-uncertainty squeezed field. It is evident that a pronounced reduction of the population in the state $|2\rangle$ can be achieved for large values of $\theta$. For $\theta=\pi$, which is the case when all modes of the three-dimensional field are squeezed, the population of the state $|2\rangle$ is equal to zero, just as for the one-dimensional squeezed field.

Figure 4 illustrates the effect of the ratio $\alpha=\gamma_2/\gamma_1$ on the stationary population of the state $|3\rangle$. We plot $\rho_{33}$, given by Eq. (23), as a function of the angle $\theta$ for $N_1'=N_2'=1$, and different values of $\alpha$, and compare them with the value of population $\rho_{33}$ for the thermal field. It is seen that a large difference appears between the populations $\rho_{33}$ in the thermal field and the squeezed-vacuum field. This difference increases when $\alpha$ decreases and is due to the fact that the strong two-photon correlations contained in the squeezed field involve the two-photon transition $|1\rangle \to |3\rangle$ leading to an increase of the population in the level $|3\rangle$. In addition, for small $\alpha$ ($\alpha \ll 1$) the transition rate $|3\rangle \to |2\rangle$ is small, which enlarges the population in the level $|3\rangle$.

Figure 5 shows the stationary populations $\rho_{33}$ as a function of $N'$ ($N'=N_1'=N_2'$) when the squeezed-vacuum field is incident over the angle $\theta=120°$, in comparison with a thermal field. For an incident thermal field, the population $\rho_{33}$ increases with increasing $N'$. The maximum value is $\frac{1}{3}$ for $N' \gg 1$. In the squeezed vacuum field, however, the population $\rho_{33}$ increases with increasing $N$, attains its maximum value for moderate $N'$, and then decreases for large $N'$. The position of the maximum value $\rho_{33}$ depends on the ratio $\alpha$ and shifts towards

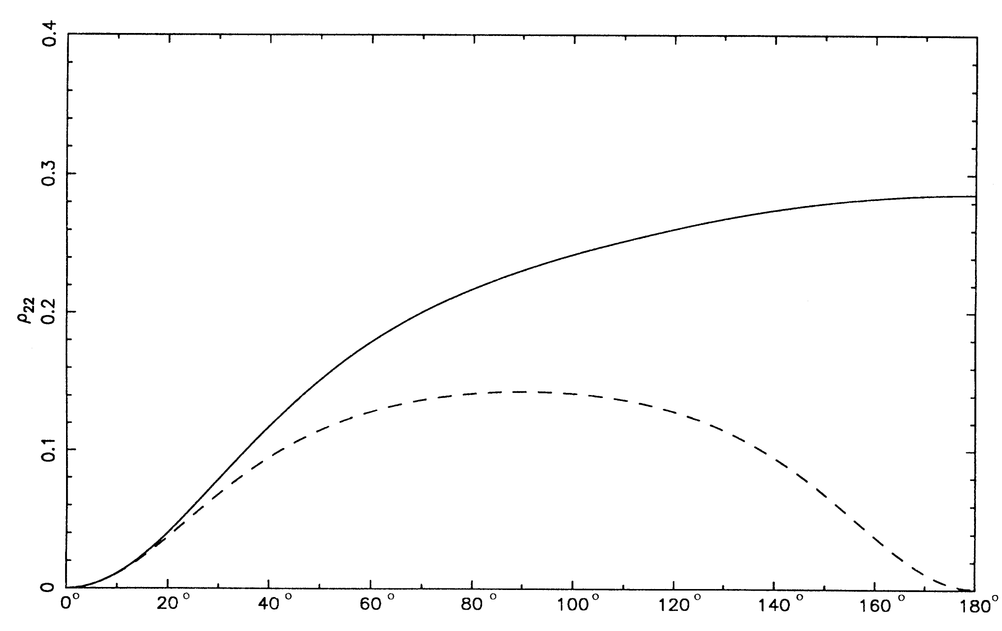


FIG. 3. Stationary population of the state $|2\rangle$ vs the angle $\theta$ over which squeezing is propagated for $N_1'=N_2'=1.0$, $\alpha=1.0$, and thermal field (——), and minimum-uncertainty squeezed field (— — —).

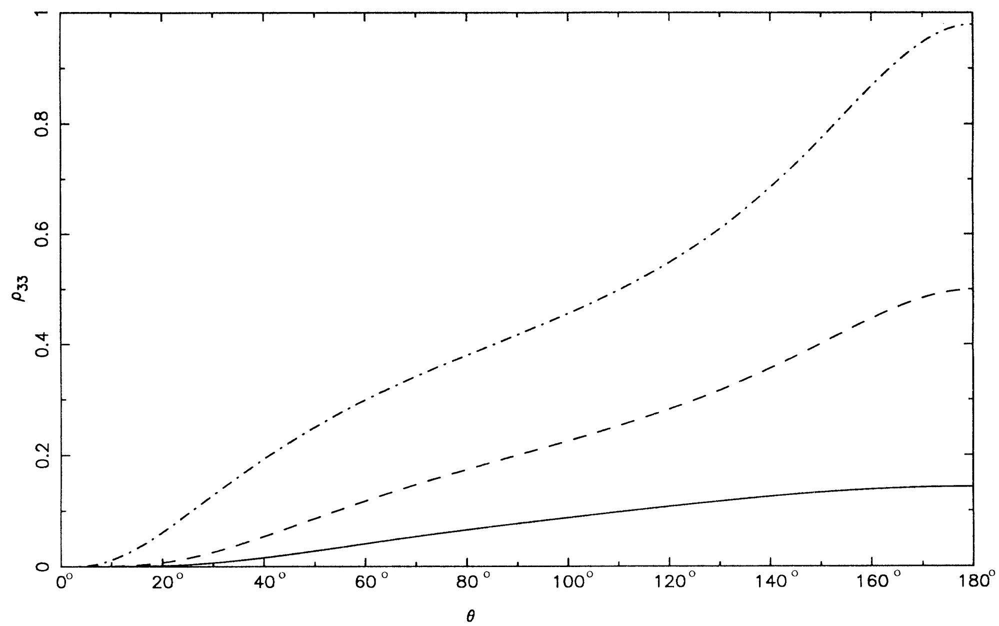


FIG. 4. Stationary population of the state $|3\rangle$ vs the angle $\theta$ over which squeezing is propagated for $N_1'=N_2'=1.0$ and thermal field (——), minimum-uncertainty squeezed field and $\alpha=0.5$(— — —), minimum-uncertainty squeezed field and $\alpha=0.01$(—·—·—·).

smaller $N$ as $\alpha$ decreases. For $\alpha=0.01$ the maximum value of $\rho_{33}$ is greater than $\frac{1}{2}$. In the one-dimensional case the atomic population is inverted, *independent* of $N$ for $\alpha \ll 1$ [see Eq. (17)]. However, in the three-dimensional case this effect appears only for small $N$.

In conclusion, the presence of unsqueezed modes in the interaction between the three-level atom and the electromagnetic field has a destructive effect on the reduction of the population in the state $|2\rangle$ and the inversion of the atomic population. Despite this, the presence of the

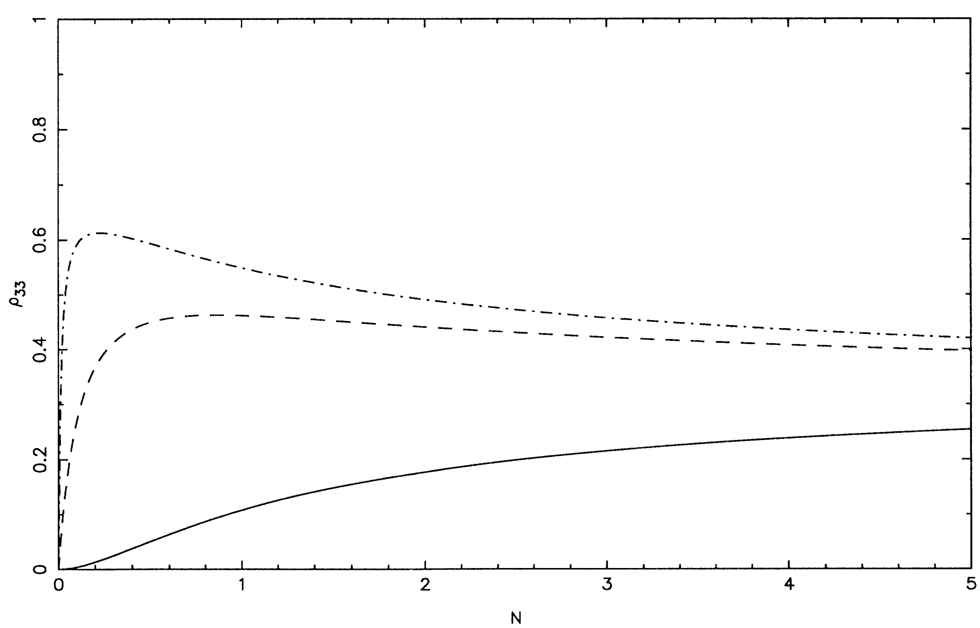


FIG. 5 Stationary population of the state $|3\rangle$ vs the intensity $N'$ of the incident field for $\theta=120°$ and the incident thermal field (——), minimum-uncertainty squeezed field and $\alpha=0.1$ (— — —), and minimum-uncertainty squeezed field and $\alpha=0.01$ (—·—·—·).

unsqueezed modes still does allow a significant reduction of the population in the state $|2\rangle$ and inversion of the atomic population, provided the angle $\theta$ over which squeezing is propagated is sufficiently large. Moreover, from Eq. (23), for a weak intensity of the squeezed vacuum field the population of the level $|3\rangle$ is proportional to $v^2(\theta)N_1$, which means that its linear dependence on the intensity of the squeezed field is not affected by the presence of the unsqueezed modes. We expect that even more dramatic effects would occur for a three-level atom placed inside a three-dimensional microcavity, as this could be used to reduce the relative coupling of unsqueezed modes.

## V. OFF-RESONANCE EXCITATION

Let us now consider the case in which $\Delta \neq 0$. In this case a finite two-photon detuning exists between the atomic transition $\omega_3$ and the double carrier frequency of the squeezing. From Eq. (2) with the perfect squeezing the steady-state solutions for the populations of the excited atomic levels are

$$\rho_{22}=\frac{[N_1(N_2+1)-|M|^2](\alpha+\alpha N_2+N_1)^2+N_1(N_2+1)\Delta^2}{(3N_1N_2+2N_1+N_2+1)[(\alpha+\alpha N_2+N_1)^2+\Delta^2]-3|M|^2(\alpha+\alpha N_2+N_1)^2} \tag{24}$$

and

$$\rho_{33}=\frac{(\alpha+\alpha N_2+N_1)[N_1N_2(\alpha+\alpha N_2+N_1)-|M|^2(\alpha N_2+N_1-1)]+N_1N_2\Delta^2}{(3N_1N_2+2N_1+N_2+1)[(\alpha+\alpha N_2+N_1)^2+\Delta^2]-3|M|^2(\alpha+\alpha N_2+N_1)^2} . \tag{25}$$

For a thermal field, $|M|=0$, and the steady-state populations (24) and (25) are independent of $\Delta$. If the atom interacts with a squeezed-vacuum field, $N_1,N_2 \neq 0$, $|M|\neq 0$, and the stationary population of the atomic excited states depends on the two-photon detuning $\Delta$. For $\Delta\neq 0$ the population of the first excited state $|2\rangle$ depends on the ratio $\alpha=\gamma_2/\gamma_1$. This is in contrast to the exact resonance ($\Delta=0$) where this population is independent of $\alpha$ [see Eq. (10)]. Moreover, for $\Delta\neq 0$ the population of the state $|2\rangle$ is different from zero even for a minimum-uncertainty squeezed state.

These results are shown in Fig. 6(a), where the steady-state population $\rho_{22}$ is plotted as a function $\Delta$ for $\alpha=1$ and different $N_1=N_2=N$. It is evident from Fig. 6(a) that the population of state $|2\rangle$ is zero only for $\Delta=0$ and increases as $|\Delta|$ increases. The population of the second excited state $|3\rangle$ takes a maximum value at $\Delta=0$ and decreases with increasing $|\Delta|$. In Fig. 6(b) the steady-state population $\rho_{33}$ is plotted against the detuning $\Delta$ for $\alpha=1$ and different $N_1=N_2=N$. It should be noted here that for a large detuning $\Delta$ ($|\Delta|\gg 1$) both the steady-state populations (24) and (25) go to their thermal equilibrium values and are independent of the degree of squeezing. Moreover, for large $\Delta$ the population $\rho_{33}$ is proportional to $N^2$ rather than to $N$ as for $\Delta=0$.

For imperfect squeezing the populations of the excited atomic levels are given by equations identical to Eqs. (24) and (25), but with $N_i$ and $M$ replaced by the parameters (I-36). In this case the population of the atomic levels depends on the two-photon detuning as well as on the angle $\theta$ over which squeezing is propagated. Figure 7 shows a dependence of the atomic populations $\rho_{22}$ and $\rho_{33}$ on the two-photon detuning, with minimum-uncertainty squeezed states for $\gamma_1=\gamma_2$, $N_1'=N_2'=1.0$ and different values of $\theta$. It is obvious from Fig. 7 that the atomic-state populations are sensitive to the angle $\theta$ over which squeezing is propagated. For small $\theta$ the populations $\rho_{22}$ and $\rho_{33}$ show a small sensitivity to the two-photon detuning. As the angle $\theta$ increases the population $\rho_{22}$ ($\rho_{33}$) shows a pronounced minimum (maximum) at $\Delta=0$. For $\theta=\pi$, which is the case when all modes of the three-dimensional field are squeezed, the population $\rho_{22}$ is equal to zero as for perfect squeezing, when $\Delta=0$.

The present analysis of dependence of the atomic population on the two-photon detuning can be experimentally verified using, for example, two-photon absorption spectroscopy techniques.[15] Because the fluorescence from the middle and the highest states is proportional to $\gamma_1\rho_{22}$ and $\gamma_2\rho_{33}$, respectively, one can model these experiments by monitoring the intensity of the fluorescence from the middle or highest states as a function of the two-photon detuning. Figure 7 shows the frequency dependence of the intensity of the fluorescence from the middle $|2\rangle$ and the highest $|3\rangle$ states. These features are completely absent when the atom interacts with a thermal field. Our results indicate, for the first time, the possible applications of *wide-band* squeezed vacuum fields in two-photon spectroscopy.

## VI. SUMMARY

Starting from the master equations derived in paper I, we have considered here the effect of the squeezed-vacuum field on the atomic population distribution in a three-level atom. We have shown that when the atom interacts with a multimode broadband vacuum field with perfectly mode-matching squeezing the first excited state $|2\rangle$ of the atom is not populated in the steady state. Moreover, the second excited state $|3\rangle$ can be populated with a probability greater than $\frac{1}{2}$. However, this happens only when the ratio $\alpha=\gamma_2/\gamma_1$ of the spontaneous-emission rates $\gamma_2$ and $\gamma_1$ for the transitions $|3\rangle\rightarrow|2\rangle$ and $|2\rangle\rightarrow|1\rangle$, respectively, is much smaller than 1. Further analysis shows that for a low intensity $N$ of the squeezed-vacuum field, the population in the state $|3\rangle$ is linearly proportional to $N$. This is in contrast to the case when the atom interacts with the thermal field, where the population in the state $|3\rangle$ is proportional to $N^2$. When the double frequency of the squeezed vacuum field is detuned from the frequency $\omega_3$ of the transition $|1\rangle\rightarrow|3\rangle$ the population in the first excited state $|2\rangle$ is different from zero. For large $|\Delta|$ the population in the state $|3\rangle$ is proportional to $N^2$, independent of degree of squeezing, and the populations return to their thermal equilibrium values.

We have also modified our model to treat the case of an atom interacting with a three-dimensional vacuum field in which only those modes are squeezed whose propagation vectors lie inside the solid angle over which squeezing is propagated. In this case the population in the first excited state $|2\rangle$ is different from zero when the angle $\theta$ over which squeezing is propagated is smaller

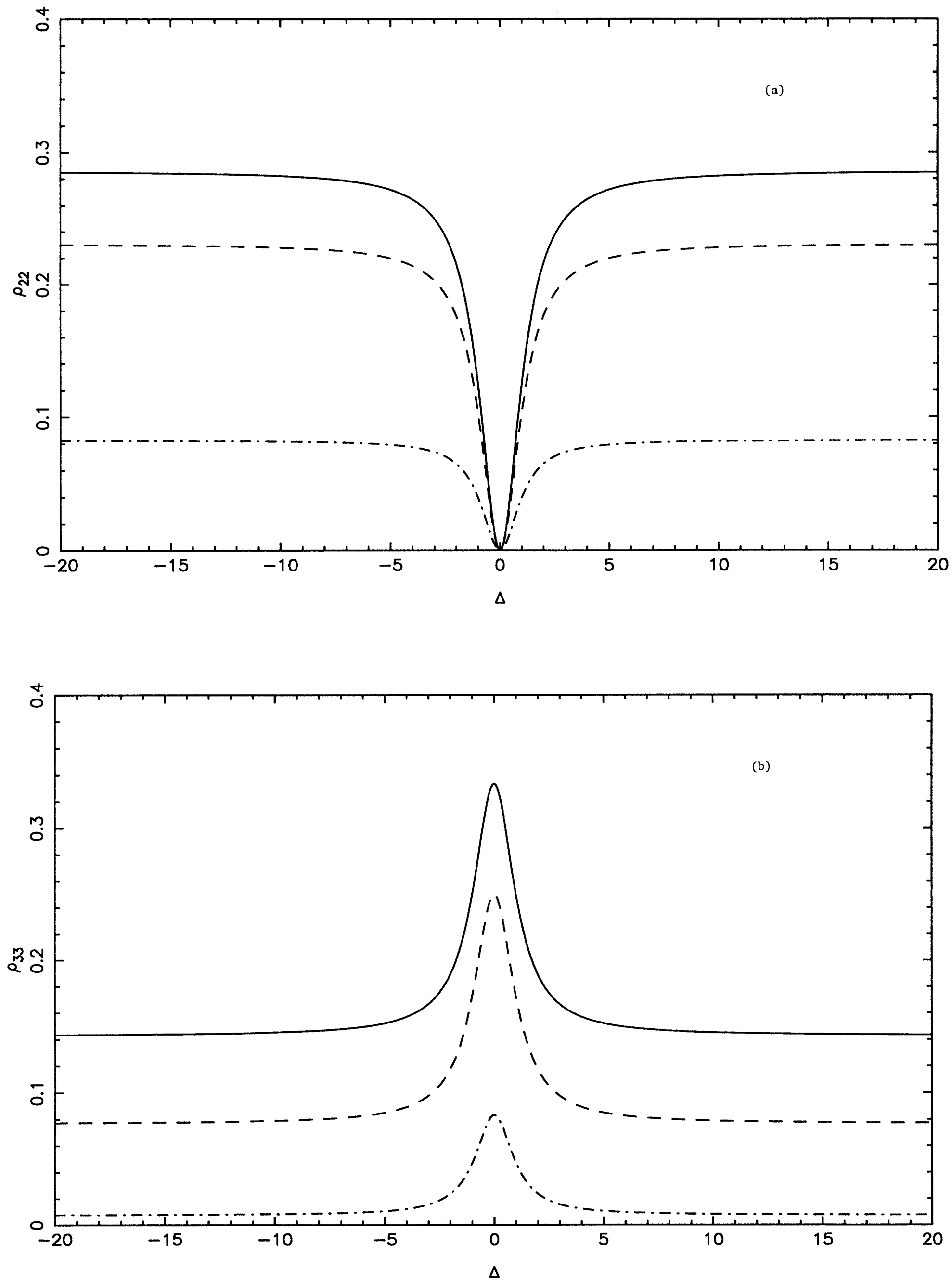


FIG. 6. Steady-state population of the level (a) $|2\rangle$ and (b) $|3\rangle$ vs the detuning for $\Delta$ for $\alpha=1.0$, minimum-uncertainty squeezed states, and different $N=N_1=N_2$: $N=1.0$ (——), $N=0.5$(— — —), and $N=0.1$(—·—·—·).

than $\pi$. This leads also to a lower population in the state $|3\rangle$. Despite this, for a sufficiently large angle $\theta$ it is still possible to obtain a significant reduction of the population in the state $|2\rangle$ and the occupation probability of the state $|3\rangle$ greater than $\frac{1}{2}$.

The present analysis of interaction between the three-level atom and squeezed-vacuum field shows that steady-state atomic populations are sensitive to the squeezed-vacuum–atom coupling. This is distinct from the two-level atom case, where only polarization dynamics are altered. As has been shown in this paper, in the experimentally more realistic three-dimensional case the detection of the effects induced by the squeezed vacuum field is possible, but it is an open question as to whether these effects are accessible to observation. However, it appears likely that direct effects on atomic populations will be more readily detectable than the indirect effects on decay rates that occur in two-level-atom cases.

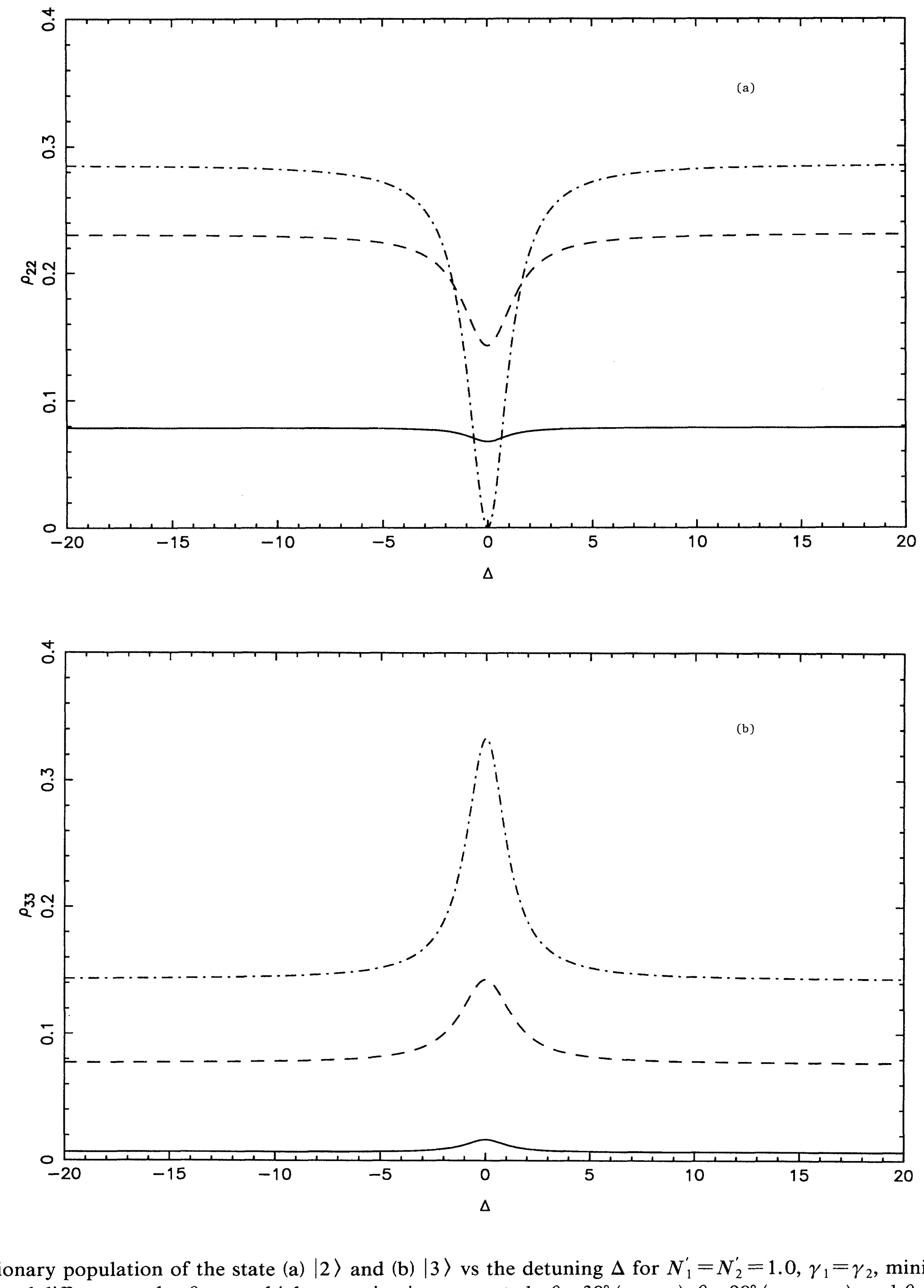


FIG. 7. Stationary population of the state (a) $|2\rangle$ and (b) $|3\rangle$ vs the detuning $\Delta$ for $N_1'=N_2'=1.0$, $\gamma_1=\gamma_2$, minimum-uncertainty squeezed states and different angles $\theta$ over which squeezing is propagated: $\theta=30°$ (——), $\theta=90°$ (— — —), and $\theta=180°$ (—·—·—).

## APPENDIX A

We found in Sec. III that at low intensity $N$ of the incident squeezed-vacuum field the time evolution of the population in the state $|3\rangle$ depends linearly on $N$. By contrast, in the thermal field case the time evolution of the population in the state $|3\rangle$ depends on $N^2$. We have explained this effect as connected with the two-photon transitions induced by the squeezed-vacuum field. We give below a verification of these results using second-order perturbation theory to calculate the population in the state $|3\rangle$.

Assume that our system (three-level atom and squeezed reservoir) is described by the state vector

$$|\Psi(t)\rangle = U(t)|\Psi(0)\rangle, \tag{A1}$$

where $U(t)$ is a unitary operator defined as ($\hbar=1$)

$$U(t)=\exp\left[-\int_0^t dt' H(t')\right]. \tag{A2}$$

In Eq. (A1), $|\Psi(0)\rangle$ is the state of the system at time $t=0$ given by

$$|\Psi(0)\rangle = |1\rangle|r(\omega),\varphi(\omega)\rangle , \tag{A3}$$

where $|1\rangle$ is the ground state of the atom. The state $|r(\omega),\varphi(\omega)\rangle$ is the initial state of the field, which we assume is defined in the squeezed state as[16]

$$|r(\omega),\varphi(\omega)\rangle = S(r(\omega),\varphi(\omega))|0\rangle , \tag{A4}$$

where $S(r(\omega),\varphi(\omega))$ is the multimode squeezing operator given by

$$S(r(\omega),\varphi(\omega)) = \exp\left[\int d\omega_\lambda r(\omega_\lambda) a(\omega_\lambda) a(2\omega-\omega_\lambda) e^{-2i\varphi(\omega_\lambda)} - \text{H.c.}\right]. \tag{A5}$$

In the above equation (A5), $r(\omega_\lambda)$ and $\varphi(\omega_\lambda)$ are, respectively, the degree and the phase of squeezing. One can show that in this squeezed-vacuum state, the quantities $M(\omega_\lambda), N(\omega_\lambda)$ are given by

$$M(\omega_\lambda) = -\sinh[r(\omega_\lambda)]\cosh[r(\omega_\lambda)]e^{2i\varphi(\omega_\lambda)} ,$$
$$N(\omega_\lambda)=\sinh^2[r(\omega_\lambda)] . \tag{A6}$$

The density operator $W(t)$ can be defined from (A1) as

$$W(t)=U(t)W(0)U^\dagger(t) , \tag{A7}$$

where $W(0)=|1\rangle\langle 1|\otimes\rho_R$, and $\rho_R$ is the density operator for the squeezed reservoir. Applying perturbation theory to the unitary operator (A2) and including only the two-photon terms, we have

$$W(t) \approx \tfrac{1}{4} U^{(2)}(t) W(0) U^{(2)\dagger}(t) = \tfrac{1}{4}\int_0^t dt_1 \int_0^{t_1} dt_2 \int_0^t dt_3 \int_0^{t_3} dt_4 H_{\text{in}}(t_1) H_{\text{in}}(t_2) W(0) H_{\text{in}}(t_3) H_{in}(t_4) , \tag{A8}$$

where $W(t)$ is given in the interacting picture and $H_{\text{in}}(t)$ is the interaction Hamiltonian (I-4).

With Eqs. (A8) and (I-4) the population of the second excited atomic state is

$$\rho_{33}(t) = \langle 3|\text{Tr}_b W(t)|3\rangle = \gamma_1\gamma_2 \int_0^t dt_1 \int_0^{t_1} dt_2 \int_0^t dt_3 \int_0^{t_3} dt_4 \langle r(\omega),\varphi(\omega)|\Phi^\dagger(t_3)\Phi^\dagger(t_4)\Phi(t_1)\Phi(t_2)|r(\omega),\varphi(\omega)\rangle e^{i\omega_1(t_2-t_3)} e^{i\omega_2(t_1-t_4)} , \tag{A9}$$

where

$$\Phi(t) = \frac{1}{\sqrt{2\pi}} \int d\omega_\lambda a(\omega_\lambda) e^{i\omega_\lambda t} , \tag{A10}$$

and $\gamma_i$ ($i=1,2$) are the damping constants defined in Eq. (I-21). The second-order correlation function that appears in Eq. (A9) can be evaluated using Eqs. (A4) and (A5). With some algebraic calculation one can show that[17]

$$\langle r(\omega),\varphi(\omega)|\Phi^\dagger(t_3)\Phi^\dagger(t_4)\Phi(t_1)\Phi(t_2)|r(\omega),\varphi(\omega)\rangle = |M|^2 e^{2i\omega(t_1-t_3)}\delta(t_3-t_4)\delta(t_1-t_2) + 2N^2\delta(t_3-t_2)\delta(t_4-t_1) , \tag{A11}$$

where for simplicity we have supposed that $M,N$ are approximately independent of frequency. Substituting (A11) into (A9) we arrive at

$$\rho_{33}(t) = 2\gamma_1\gamma_2\left[N^2t^2 + \frac{|M|^2}{\Delta^2}[1-\cos(\Delta t)]\right] , \tag{A12}$$

where $\Delta=(\omega_3-2\omega)$ is the detuning between the atomic transition frequency $\omega_3$ and the double frequency of the squeezed vacuum field.

At a low intensity of the squeezed-vacuum field ($N \ll 1$), with $|M|^2=N(N+1)$ and $\Delta\to 0$, from Eq. (A12) we have the result

$$\rho_{33}(t) \approx \gamma_1\gamma_2 N t^2 . \tag{A13}$$

In this case the population $\rho_{33}(t)$ linearly depends on $N$, which confirms our result in Sec. III. For a large detuning $\Delta$, from (A12) we get

$$\rho_{33}(t) \approx 2\gamma_1\gamma_2 N^2 t^2 . \tag{A14}$$

In this case the population $\rho_{33}(t)$ shows a quadratic dependence on $N$, just as it does for the interaction with the thermal field.

In all above calculations we have assumed that the atom interacts with a one-dimensional squeezed-vacuum field. It is not difficult to repeat all the above calculations with the Hamiltonian (I-22) for the three-dimensional vacuum field, in which only those modes are squeezed whose wave vectors lie inside a solid angle $\Omega$ over which squeezing is propagated. These calculations lead to a population in level $|3\rangle$ given by

$$\rho_{33}(t)=2\gamma_1\gamma_2 v^2(\theta)\left[N'^2t^2+\frac{|M'|^2}{\Delta^2}[1-\cos(\Delta t)]\right], \tag{A15}$$

where $v(\theta)$ is given by Eq. (I-34).

In this case the population $\rho_{33}(t)$ shows the same behaviour, as a function of $N$, as for the one-dimensional squeezed vacuum field. This confirms our conclusion in Sec. IV that the linear dependence of the population $\rho_{33}$ on the intensity of the squeezed field is not affected by the presence of the unsqueezed modes, provided the intensity is sufficiently low.

## APPENDIX B

In this appendix we estimate the order of magnitude of the $\delta_1$ and $\delta_{12}=-\delta_{21}$ parameters that are due to the interaction with a squeezed-vacuum field, which is assumed to be on two-photon resonance.

For a one-dimensional squeezed vacuum field these parameters are defined in Eq. (I-21) and have the form

$$\delta_1=-\frac{1}{\gamma_1}\int_0^\infty d\omega_\lambda|g_1(\omega_\lambda)|^2N(\omega_\lambda)\times\left[\frac{1}{\omega_1-\omega_\lambda}+\frac{1}{\omega_1+\omega_\lambda}\right], \tag{B1}$$

$$\delta_{12}=\frac{1}{\gamma_1}\int_0^\infty d\omega_\lambda\frac{g_1(\omega_\lambda)g_2(2\omega-\omega_\lambda)}{(\omega_\lambda-\omega_1)}M(\omega_\lambda)\,. \tag{B2}$$

To evaluate these integrals we need to assume some spectral form for the squeezing parameters $N(\omega_\lambda)$ and $M(\omega_\lambda)$. We will use as a typical example the squeezing spectrum at the output of a nondegenerate parametric oscillator[18] tuned to resonance with the atomic transition frequencies $\omega_1$ and $\omega_2$:

$$N(\omega_\lambda)=\sum_{i=1}^{2}\left[\frac{A}{(\omega_\lambda-\omega_i)^2+\Gamma^2}-\frac{A'}{(\omega_\lambda-\omega_i)^2+\Gamma'^2}\right], \tag{B3}$$

$$M(\omega_\lambda)=\sum_{i=1}^{2}\left[\frac{A}{(\omega_\lambda-\omega_i)^2+\Gamma^2}+\frac{A'}{(\omega_\lambda-\omega_i)^2+\Gamma'^2}\right]. \tag{B4}$$

The intensity spectrum $N(\omega_\lambda)$ and squeezing correlation $M(\omega_\lambda)$ associated with the parametric modes at $\omega_1$ and $\omega_2$ equals the difference and sum, respectively, of two Lorentzians centered at $\omega_\lambda=\omega_i$, with widths $\Gamma$ and $\Gamma'$ and weights $A$ and $A'$, giving four Lorentzians in all. We assume a small squeezing bandwidth relative to the carrier frequency $\omega$, which is typical for the output of a nondegenerate parametric oscillator. This assumption is equivalent to $\Gamma/\omega\ll1$ and $\Gamma'/\omega\ll1$. Of course, it is also necessary that $\Gamma/\gamma\gg1$ to obtain Markovian results.

Using Eqs. (B3) and (B4) we obtain

$$\delta_1=\frac{1}{(\omega-\delta)\pi}\int_{-\omega}^{\infty}d\varepsilon\frac{(\varepsilon+\omega)}{(\varepsilon+\delta)}\left[\frac{A}{(\varepsilon-\delta)^2+\Gamma^2}-\frac{A'}{(\varepsilon-\delta)^2+\Gamma'^2}\right]+O\left[\frac{1}{\omega}\right], \tag{B5}$$

and

$$\delta_{12}=\left[\frac{\gamma_2}{\gamma_1}\right]^{1/2}\frac{\pi^{-1}}{(\omega_1\omega_2)^{1/2}}\int_{-\omega}^{\infty}d\varepsilon\frac{(\omega^2-\varepsilon^2)^{1/2}}{(\varepsilon+\delta)}\left[\frac{A}{(\varepsilon-\delta)^2+\Gamma^2}+\frac{A'}{(\varepsilon-\delta)^2+\Gamma'^2}\right]+O\left[\frac{1}{\omega}\right], \tag{B6}$$

where $\varepsilon=\omega_\lambda-\omega$ and $\delta=(\omega-\omega_1)$. Evaluating the integrals we find

$$\delta_1=\left[\frac{A}{\Gamma}\frac{\delta}{(4\delta^2+\Gamma^2)}-\frac{A'}{\Gamma'}\frac{\delta}{(4\delta^2+\Gamma'^2)}\right]+O\left[\frac{1}{\omega}\right], \tag{B7}$$

and

$$\delta_{12}=\left[\frac{\gamma_2}{\gamma_1}\right]^{1/2}\frac{\omega}{(\omega_1\omega_2)^{1/2}}\left[\frac{A}{\Gamma}\frac{\delta}{(4\delta^2+\Gamma^2)}+\frac{A'}{\Gamma'}\frac{\delta}{(4\delta^2+\Gamma'^2)}\right]+O\left[\frac{1}{\omega}\right]. \tag{B8}$$

Since $\omega/\Gamma\gg1$ and $\omega/\Gamma'\gg1$ then $\delta/(4\delta^2+\Gamma^2)\ll1$ and we can neglect the parameters $\delta_1$ and $\delta_{12}$.

*On leave from Institute of Physics, Adam Mickiewicz University, Poznań, Poland.


[1]Z. Ficek and P. D. Drummond, preceding paper, Phys. Rev. A **43**, 6247 (1991).

[2]C. W. Gardiner, Phys. Rev. Lett. **56**, 1917 (1986).

[3]H. J. Carmichael, A. S. Lane, and D. F. Walls, Phys. Rev. Lett. **58**, 2539 (1987); J. Mod. Opt. **34**, 821 (1987).

[4]H. Ritsch and P. Zoller, Phys. Rev. A **38**, 4657 (1988).

[5]A. S. Parkins and C. W. Gardiner, Phys. Rev. A **40**, 3796 (1989); **42**, 5765(E) (1990).

[6]A. S. Shumovsky and T. Quang, J. Phys. B **22**, 131 (1989); M. R. B. Wahiddin, S. S. Hassan, J. Timonen, and R. K. Bullough, in *Coherent and Quantum Optics, VI,* edited by J. H. Eberly, L. Mandel, and E. Wolf (Plenum, New York, 1990), p. 1195; Z. Ficek and B. C. Sanders, Quantum Opt. **2**, 269 (1990).

[7]S. M. Barnett and M. A. Dupertuis, J. Opt. Soc. Am. B **4**, 505 (1987).

[8]G. M. Palma and P. L. Knight, Phys. Rev. A **39**, 1962 (1989).

[9]A. K. Ekert, G. M. Palma, S. M. Barnett, and P. L. Knight, Phys. Rev. A **39**, 6026 (1989).

[10]G. S. Agarwal and R. R. Puri, Opt. Commun. **69**, 267 (1989); Phys. Rev. A **41**, 3782 (1990).

[11]M. A. Marte, H. Ritsch, and D. F. Walls, Phys. Rev. A **38**, 3577 (1988).

[12]F. Haake, D. F. Walls, and M. J. Collett, Phys. Rev. A **39**, 3211 (1989).

[13]J. Gea-Banacloche, Phys. Rev. Lett. **62**, 1603 (1989).

[14]J. Javanainen and P. L. Gould, Phys. Rev. A **41**, 5088 (1990).

[15]M. D. Levenson and S. S. Kano, *Introduction to Nonlinear Laser Spectroscopy* (Academic, Boston, 1988).

[16]C. M. Caves and B. L. Schumaker, Phys. Rev. A **31**, 3068 (1987).

[17]B. L. Schumaker, Phys. Rep. **135**, 318 (1986).

[18]P. D. Drummond and M. D. Reid, Phys. Rev. A **41**, 3930 (1990).