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Modulation instability in the region of minimum group-velocity dispersion of single-mode optical fibers via an extended nonlinear Schrödinger equation

Solange B. Cavalcanti, José C. Cressoni, Heber R. da Cruz, and Artur S. Gouveia-Neto
Departamento de Física, Universidade Federal de Alagoas, Maceió, 57061, Alagoas, Brazil
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Modulation instability in the region of the minimum group-velocity dispersion is analyzed by means of an extended nonlinear Schrödinger equation. It is shown that the critical modulation frequency saturates at a value determined by the fourth-order dispersion. Experimental results demonstrate the viability of generating a train of femtosecond pulses with repetition rates of a few terahertz in reasonable agreement with the theory.

I. INTRODUCTION

Recent investigations on the propagation of light pulses through optical fibers[1] have aroused considerable interest in the study of nonlinear effects described by the nonlinear Schrödinger equation[2] (NLSE). One of these effects, known as modulation instability (MI), first proposed by Hasegawa,[3] is the generation of a train of ultrashort pulses from a perturbed cw radiation. It happens when a cw perturbed radiation experiences an instability that leads to an exponential growth of its amplitude on propagation in optical fibers due to an interplay between the nonlinearity and anomalous dispersion. This phenomenon has been first verified experimentally by Tai, Hasegawa, and Tomita.[4] MI of short pulses has also been considered by Shukla and Rasmussen.[5] The mathematical problem consists in finding periodic solutions of the NLSE and exact analytic expressions for it have been obtained by Akhmedieva, Eleonskii, and Kulagin.[6] Also, using an extended NLSE, Potasek[7] and Vysloukh and Sukhotskova[8] have investigated the influence of third-order dispersion on MI. Recently, there has been an analysis of MI through a phase-matched four-photon-mixing approach which takes into account dispersion effects up to fourth order.[9] This paper is devoted to the investigation of MI, in the region of minimum group-velocity dispersion, through an extended NLSE, taking into account fourth-order dispersive effects. We will demonstrate that this term dominates the critical modulation-instability frequency when second-order dispersion approaches its minimum value at the so-called zero-dispersion wavelength λ_d. Furthermore, experimental results will be presented, demonstrating the generation of a train of femtosecond pulses with repetition rates given by the modulation frequency. This paper is organized as follows. In Sec. II we solve the NLSE for a perturbed cw pulse and investigate its behavior in the anomalous dispersive regime, in Sec. III we show experimental results, and finally Sec. IV is devoted to the conclusions and a discussion of the results.

II. MODULATION INSTABILITY

Consider a linearly polarized light propagating through a dispersive nonlinear medium, such as an ideal single-mode fiber. The electrical field in the slowly varying envelope approximation, may be written as[1]

$$\mathbf{E}(\mathbf{r},t)=\{U(\rho)A(z,t)\exp[-i(\omega_0 t-\beta_0 z)]+\text{c.c.}\}\hat{\mathbf{x}}\ , \tag{2.1}$$

where $\hat{\mathbf{x}}$ is the unitary vector of the polarization direction, $U(\rho)$ is the field distribution, β_0 is the propagation constant at the central frequency of the pulse ω_0, and c.c. stands for complex conjugate. $A(z,t)$ is the amplitude of the envelope which is a slowly varying function of time compared to the optical period. In the case considered here, the refractive index of the fiber is given by

$$n(\omega,I)=n_0(\omega)+n_2 I\ , \tag{2.2}$$

with $I=|A|^2$, and n_0 is the linear refractive index whereas n_2 is the nonlinear Kerr coefficient. The propagation constant that describes chromatic dispersion is expanded in the Taylor series about ω_0 and here we shall retain terms up to fourth order, that is,

$$\begin{aligned}\beta(\omega)&=n(\omega)(\omega/c)\\&=\beta_0+\beta_1(\omega-\omega_0)+\tfrac{1}{2}\beta_2(\omega-\omega_0)^2\\&\quad+(1/3!)\beta_3(\omega-\omega_0)^3+\tfrac{1}{4}\beta_4(\omega-\omega_0)^4\ ,\end{aligned} \tag{2.3}$$

where

$$\beta_n=\left.\frac{d^n\beta}{d\omega^n}\right|_{\omega_0},\quad n=0,1,2,\ldots\ . \tag{2.4}$$

It is convenient to introduce the retarded time

$$T=t-\beta_1 z\ . \tag{2.5}$$

Substituting Eqs. (2.1)–(2.5) in the wave equation, we obtain the following propagation equation for the envelope amplitude:

$$i\frac{\partial A(z,t)}{\partial z}=\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2}+i\frac{\beta_3}{6}\frac{\partial^3 A}{\partial T^3}-\frac{\beta_4}{24}\frac{\partial^4 A}{\partial T^4}-\gamma|A|^2|A|\ . \tag{2.6}$$

Here, $\gamma=n_2\omega_0/cS_{\text{eff}}$ with S_{eff} as the effective core area. The steady-state solution of (2.6), corresponding to a cw signal, is given by

$$A^{\mathrm{cw}}=A_0\exp(i\phi_{\mathrm{nl}}),\quad A_0=\sqrt{P_0} \tag{2.7}$$

where P_0 is the incident power at $z=0$. Substituting (2.7) in (2.6) we find that (2.7) is a solution when

$$\phi_{\mathrm{nl}}=\gamma|A|^2 z\ . \tag{2.8}$$

Physically, Eqs. (2.7) and (2.8) show that the cw radiation experiences only a power-dependent phase shift after propagation through the fiber. Now we wish to test the stability of such a steady-state solution. To this end we add a small perturbation $a(z,T)$ to the cw light and examine its evolution. Therefore, we substitute the following expression in (2.6):

$$A=(A_0+a)\exp(i\phi_{\mathrm{nl}}),\quad |a|^2\ll|A_0|^2 \tag{2.9}$$

and linearize it in a to find the propagation equation for the perturbation

$$i\frac{\partial a}{\partial z}=\frac{\beta_2}{2}\frac{\partial^2 a}{\partial T^2}+i\frac{\beta_3}{6}\frac{\partial^3 a}{\partial T^3}-\frac{\beta_4}{24}\frac{\partial^4 a}{\partial T^4}-\gamma P_0(a+a^*)\ . \tag{2.10}$$

We now have a linear equation which is readily solved,

$$a(z,T)=a_1\cos(Kz-\Omega T)+ia_2\sin(K_z-\Omega T)\ , \tag{2.11}$$

with K and Ω obeying the following dispersion relation:

$$K=\frac{\beta_3\Omega^3}{6}\pm|\Omega|\left[\beta_2 P_0\gamma+\frac{\Omega^2}{4}\left[\beta_2^2+\frac{\beta_4 P_0\gamma}{3}\right]+\frac{\beta_2\beta_4\Omega^4}{24}+\frac{\beta_4^2\Omega^6}{(24)^2}\right]^{1/2}. \tag{2.12}$$

We are looking for modulation frequencies Ω less than a critical frequency Ω_c corresponding to imaginary wave number K, so that the modulation signal undergoes an exponential growth in amplitude. The first thing we notice from (2.12) is that the cubic dispersion factor β_3 has no influence on the modulation frequency, as pointed out previously in Ref. 7. Also, it is clear that in the normal regime of group-velocity dispersion, K is real for all values of Ω so that the steady state is stable against perturbations. However, in the anomalous regime, K becomes complex for frequencies $\Omega<\Omega_c$, which according to (2.12) is defined by

$$\Omega_c^6+\frac{24\beta_2\Omega_c^4}{\beta_4}+\left[\frac{48P_0\gamma}{\beta_4}+\left[\frac{12\beta_2}{\beta_4}\right]^2\right]\Omega_c^2+(24)^2\frac{\beta_2}{\beta_4^2}P_0\gamma=0\ . \tag{2.13}$$

To illustrate the effect of higher-order dispersion, we plot in Fig. 1 the modulation frequency $\Omega_c/2\pi$ at three regimes obtained from Eq. (2.13). For fiber parameters readily found in practice ($\lambda_0=1.319\ \mu$m, $\lambda_d\sim1.32\ \mu$m, $\gamma=3.05\ \mathrm{W}^{-1}\,\mathrm{km}^{-1}$), it is clear that for $|\beta_2|>1\ \mathrm{ps}^2/\mathrm{km}$ the behavior of Ω_c is dominated by the group delay dispersion. However, for $|\beta_2|<10^{-3}\ \mathrm{ps}^2/\mathrm{km}$ the fourth-order dispersion factor dominates and Ω_c saturates as group delay dispersion tends to zero. Thus, by neglecting β_4 in Eq. (2.12) we obtain the critical modulation frequency as in Ref. 3. Near λ_d we neglect β_2 ending up with the following dispersion relation:

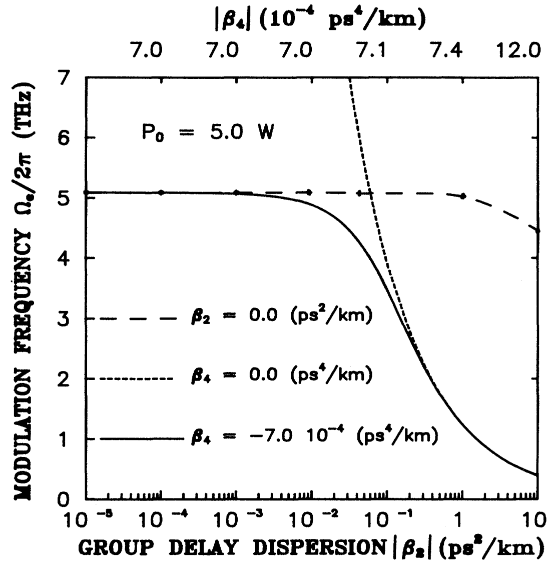

FIG. 1. Critical modulation frequency as a function of the second-order dispersion parameter. The upper axis is associated with the dashed line ($|\beta_2|=0$).

$$K=\frac{\beta_3\Omega^3}{6}\pm\frac{\Omega^2|\beta_4|}{24}[\Omega^4+\mathrm{sgn}(\beta_4)\Omega_c^4]^{1/2}\ . \tag{2.14}$$

It is now clear that for $\beta_2=0$, the critical modulation frequency saturates at a value Ω_c given by

$$\Omega_c=\left[\frac{48P_0\gamma}{|\beta_4|}\right]^{1/4} \tag{2.15}$$

as is depicted in Fig. 2, where we have plotted the modu-

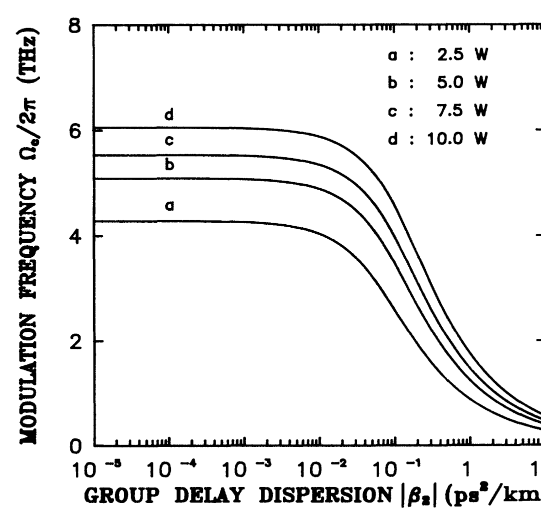

FIG. 2. Critical modulation frequency vs group delay dispersion for various incident powers P_0 with $\gamma=3.05\ \mathrm{km}^{-1}$.

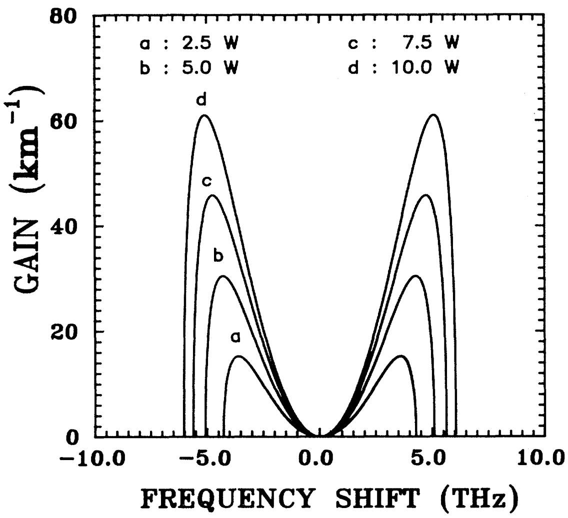

FIG. 3. Gain spectrum of the modulation instability at four power levels. Fiber parameters $|\beta_2| \approx 0$, $|\beta_4| = 7\times 10^{-4}$ ps^4/km, $\gamma = 3.05$ km^{-1} W^{-1}.

lation frequency versus group delay dispersion for various incident powers.

Let us now evaluate the gain spectrum $g(\Omega)$, which is obtained by

$$g(\Omega) = 2\,\mathrm{Im}(K) = \frac{\Omega^2|\beta_4|}{12}(\Omega_c^4 - \Omega^4)^{1/2} , \quad (2.16)$$

where Ω is the shift in frequency from the central initial one ω_0. The maximum gain g_m is then produced at a shifted frequency Ω_m,

$$\Omega_m = \pm \frac{\Omega_c}{2^{1/4}} . \quad (2.17)$$

In this way maximum gain g_m is given by

$$g_m = 2P_0\gamma . \quad (2.18)$$

In Fig. 3, the gain spectrum is shown for various input powers. Comparing this result with the gain spectrum evaluated in the region far from λ_d, where β_2 dominates, we find that although maximum gain is the same in both cases, here the maximum frequency $\Omega_m(\beta_4)$ (where maximum gain is achieved) is larger than the maximum frequency $\Omega_m(\beta_2)$ there by a factor of $2^{1/4}$, as expected, once β_4 is associated with high frequencies.

To consider the effect of fiber loss[10] on the critical frequency Ω_c one must replace it by $\Omega_c \exp(-\alpha z/4)$, where α is the attenuation factor. Comparing this expression with the critical frequency evaluated far away from the minimum dispersion wavelength, where β_2 dominates, i.e., $\Omega_c \exp(-\alpha z/2)$, we find that the latter falls off exponentially at twice the rate at which the former decays.

III. EXPERIMENTAL RESULTS

To show the viability of generating a train of femtosecond pulses with repetition rates in the terahertz regime, by operating in the region of the minimum second-order dispersion, we have carried out an experiment as follows. Rather than using a cw source modulated by a small perturbation, we have injected into the fiber a long pulse ($\approx$100 ps) compared with the expected modulation period ($\approx$500 fs), so that we have operated in a quasi-cw situation and looked at stimulated modulation instability.

The light source was a cw mode-locked Nd:YAG laser (YAG denotes yttrium aluminum garnet) operated at 1.32 μm, generating approximately 100 ps at a 100-MHz repetition rate and an average output power of approximately 1.6 W. The fiber used was 500 m long, single mode at 1.32 μm, 7-μm core diameter, and had second- and fourth-order dispersion parameters $\beta_2 \approx -0.1$ ps^2/km and $\beta_4 \approx -7.0\times 10^{-4}$ ps^4/km at 1.32 μm, respectively. The second-order nonlinear parameter was $\gamma = 3.05$ km^{-1} W^{-1}. Temporal and spectral features were measured by using a second-order autocorrelation technique and a 0.5-m scanning spectrograph, respectively.

Figure 4 shows the second-order autocorrelation trace (a) and the corresponding power spectrum (b) for a 6.0-W peak power coupled into the fiber. The frequency of the sidebands ($\approx$2.0 THz) agrees reasonably well with the measured period of the modulation ($\approx$500 fs) inferred from the autocorrelation trace. By raising the pump peak power into the fiber to 8.0 W, a reduction of the modulation period to 470 fs and a corresponding increase in the sidebands frequency to 2.14 THz was observed as shown in Figs. 4(c) and 4(d), respectively. This behavior presented by the results agrees quite well with the theory developed in Sec. II. For higher pump peak power coupled into the fiber the Stokes sidebands undergo Raman preferential gain and the modulation instability evolves into a solitary wave as described in Ref. 11.

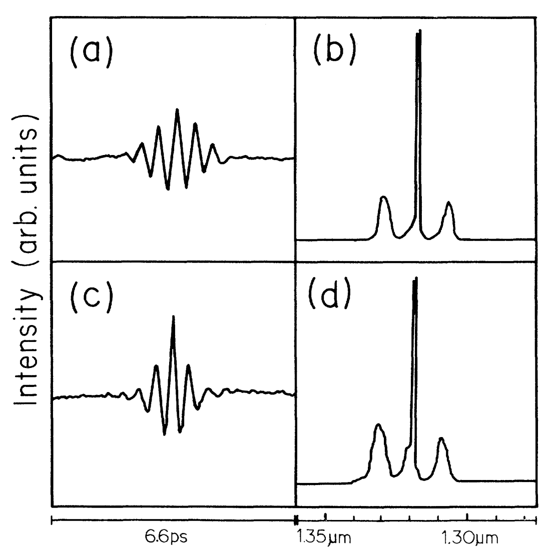

FIG. 4. Autocorrelation trace and corresponding spectra (a) and (b) at 6.0-W and (c) and (d) at 8.0-W pump peak power.

IV. CONCLUSIONS

We have investigated MI of single-mode ideal optical fibers in the region of minimum group-velocity dispersion through an extended NLSE that takes into account dispersion up to fourth order. We have shown that fourth-order dispersion plays a major role for the critical MI frequency in the region around λ_d, thus confirming the result obtained by Kitayama, Okamoto, and Yoshinaga in Ref. 9 from a different route. Also, we have demonstrated that the maximum gain obtained in the present regime is the same as in the region far out from the minimum dispersion regime although at a higher frequency. Furthermore, by including loss effects we find that the critical modulation frequency falls off exponentially at half the rate than in the region where β_2 dominates. Finally, we have shown the generation of a train of femtosecond pulses with repetition rates in the terahertz regime for a situation as close as possible to the theory described in Sec. III.

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