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Stability of matter interacting with photons

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We prove the stability of the ground state of an ensemble of charged particles interacting with a quantized electromagnetic field. We show that recently predicted instabilities of such a system do not occur.

The stability of an ensemble of charged particles interacting via Coulomb forces is the most striking feature of atoms and molecules. Although it has been rightly believed that the stability of matter is entirely due to quantum mechanics, it took almost a half of a century, since the discovery of the Schrödinger equation, for the completion of a full proof.[1]

The most powerful results about the stability of the ground state of charged matter have been obtained in cases in which the interactions have been caused by the longitudinal (Coulomb) part of the electric field[1] or when the transverse field has been external and classical.[2]

If charged particles are allowed to interact with the quantized transverse electric field, the problem of stability has to include the question of stability of the interacting photon field. The state of the combined system matter plus radiation involves now the interacting charged particles exchanging real and virtual photons. Such a state belongs to a larger Hilbert space also involving photon states.

To the best of our knowledge, we are unaware of any results about the stability of the ground state of such a system.

In some past and recent publications involving the problem of spontaneous emission of any two-level system interacting with some kind of bosons, it has been suggested that a nondissipative instability of the ground state can occur.[3] Most of these results have been obtained in a framework of some model and simplified, atom-field interactions.

It is the purpose of this Brief Report to prove the stability of the ground state even if the photon field is included. Our proof deals with the following nonrelativistic Hamiltonian for the spinless charged particles:

$$H = H_F + H_A \ , \tag{1}$$

where the field Hamiltonian

$$H_F = \sum_{\lambda} \int d_3k \, \hbar\omega a^{\dagger}_{\lambda\mathbf{k}} a_{\lambda\mathbf{k}} \ , \tag{2}$$

involves all the polarizations λ and all the continuum modes $\mathbf{k}$ of the photon field. The atomic part has the standard minimal coupling form:

$$H_A = \sum_j \frac{(\mathbf{p}_j - e_j \mathbf{A})^2}{2m_j} + V \ , \tag{3}$$

where the potential V includes all possible electric longitudinal interactions (Coulomb interactions) between the charged particles.

We treat neutral atoms or molecules, so the well-known dipole approximation may be introduced. In this approximation the vector potential $\mathbf{A}$ is evaluated at the center of the atom for all its constituents. In this approximation the proof of the stability turns out to be extremely simple. The extension of this proof beyond the dipole approximation is still possible but involves a much more complicated mathematical relation based on the so-called diamagnetic inequalities.[1]

We start the proof of the stability of the ground state, noting that the Hamiltonian H_A given by Eq. (3) is unitarily equivalent to the following atomic Hamiltonian, free from the quantized electromagnetic field:

$$H_0 = \sum_i \frac{p_j^2}{2m_j} + V \ . \tag{4}$$

This equivalence is achieved with the help of the so-called Power-Zienau unitary transformation:[4]

$$U = \exp\left[-\frac{i}{\hbar} \sum_j e_j \mathbf{r}_j \cdot \mathbf{A} \right] \ . \tag{5}$$

Due to this unitary equivalence we conclude that H_A is bounded from below and

$$\langle \psi | H_A | \psi \rangle \geq E_0 \ , \tag{6}$$

where ψ is a state of the combined system particles plus radiation and E_0 is the stable lower bound of H_0 without the transverse quantized electric field. The free energy of the quantized electromagnetic field is a positive quadratic form for which we have

$$\langle \psi | H_F | \psi \rangle \geq 0 \ . \tag{7}$$

Combining the two inequalities (6) and (7) we come to the conclusion that

$$\langle \psi | H | \psi \rangle \geq E_0 \qquad (8)$$

for all vectors $|\psi\rangle$, i.e., the ground state of a system of charged particles is stable if interactions with photons are included.

This remarkably simple proof involves an arbitrary atomic Hamiltonian H_0 [Eq. (4)] for which the stability of the ground state has been shown for static longitudinal electric interactions. Fifteen years ago we had also shown that no field-induced phase transition can occur in atomic systems coupled to a quantized electromagnetic field.[5] The present proof and its generalization beyond the dipole approximation parallels generalizations of Ref. 5 known as "no-go theorems."[6,7]

In summary we conclude that the system of the charged particles interacting with photons is stable, and various claims about the field-induced instabilities that have been published recently occur because of the absence of the A^2 term in the model Hamiltonians.[8]

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[1]See the review articles by E. H. Lieb, Rev. Mod. Phys. **48**, 553 (1976) and **53**, 603 (1981).

[2]This is the Bohr–van Leeuven theorem, according to which the phenomenon of spontaneous magnetization does not arise in classical physics. See, for example, R. K. Pathria, *Statistical Mechanics* (Pergamon, New York, 1972).

[3]B. Fain, Phys. Rev. A **37**, 546 (1988); Phys. Rev. Lett. **61**, 2197 (1988); **63**, 2694C (1989).

[4]See C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Photons and Atoms* (Wiley, New York, 1989), Chap. IV.

[5]K. Rzążewski, K. Wódkiewicz, and W. Zakowicz, Phys. Rev. Lett. **35**, 422 (1975).

[6]I. Bialynicki-Birula and K. Rzążewski, Phys. Rev. A **19**, 301 (1979).

[7]K. Gawędzki and K. Rzążewski, Phys. Rev. A **23**, 2134 (1981).

[8]For spin systems interacting with the magnetic field, the A^2 term leads to the diamagnetic term. The absence of this term in a model Hamiltonian results in the instability of the ground state which agin is an artifact of the model.