Quantum mechanics of radiation damping

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(Received 3 May 1990)

We develop a nonrelativistic formulation for the quantum dynamics of an electron coupled to its own radiation field. For this purpose, we have applied the Feynman-Vernon approach to the composite system in order to obtain the reduced density operator of the electron. In the classical limit, some well-known results, such as the Abraham-Lorentz equation of motion, are reproduced. We have applied the resulting formalism to the problem of interference in order to investigate the possible effects of the incoherent modes of the electromagnetic radiation on the interference fringes. The results allow us to conclude that the coupling to the radiation field is not enough for one to observe a strong influence of those modes on the interference phenomenon.

I. INTRODUCTION

The emission of radiation by accelerated charged particles is responsible for the losses suffered by them. This effect is known as radiation damping.

A quantum description of the dynamics of an electron coupled to its own radiation field must naturally describe this damping phenomenon. There are many different ways to deal with dissipative systems in quantum mechanics, but, in this case, it seems natural to use the system-plus-reservoir approach. In this approach, we consider a conservative composite system, and then apply the canonical process of quantization.

It is easy to see the convenience of this method. Radiation damping is a consequence of the coupling between the electron and its own field. Therefore, the latter is the reservoir responsible for the losses suffered by the particle. Furthermore, we can represent the modes of the field by harmonic oscillators and then we have a particle interacting with a bath of harmonic oscillators, as in the model currently used[1] to study quantum Brownian motion. The same model has been applied to some other problems such as dissipative two-level systems[2] and muon–electron-gas interactions.[3] However, if we apply this model to the electron–radiation-field coupling, we need to modify some of the underlying hypothesis.

The first modification has to do with the initial condition for the density operator of the system plus environment. Following Feynman and Vernon,[4] it is generally supposed that the interaction is suddenly switched on at the initial instant. Before this, the system and the reservoir are assumed to be completely decoupled. Now, in the electron–radiation-field case, as the interaction is always present, the initial condition needs a different treatment. The required formalism has recently been developed in different ways by Hakim and Ambegaokar[5] and by Morais Smith and Caldeira.[6]

The second modification is related to the spectral function of the bath which arises from the form of the interaction itself instead of being imposed as a constraint.

The complete system electron plus radiation field is Hamiltonian and its classical equations of motion allow us to verify the compatibility between this approach and the well-known classical results of the radiation damping, such as the Abraham-Lorentz equation of motion.[7] The purpose of this work is to develop a quantum formulation for the dynamics of an electron coupled to its own radiation field in the nonrelativistic case, and then apply it to a problem of electronic interference in order to investigate the influence of the coupling on the interference fringes.

In Sec. II we show how to describe the complete system electron plus radiation field both in Lagrangian and Hamiltonian formalisms and then investigate its classical limit. The quantum dynamics is developed in Sec. III, while its application to a prototype of an experiment of electronic interference is carried on in Sec. IV. Finally, we discuss the results in Sec. V.

II. DESCRIPTION AND CLASSICAL LIMIT

We describe the complete electron–plus–radiation-field system by means of the Lagrangian

$$L = L_0 + L_I + L_{\rm EM} , \tag{1}$$

where

$$L_0 = \tfrac{1}{2} m \ddot{q}^2 - V_0(\mathbf{q}) \quad \text{(electron)} , \tag{2}$$

$$L_I = \int \left[\frac{1}{c} \mathbf{J} \cdot \mathbf{A} - \rho \phi \right] d^3 r \quad \text{(interaction)} , \tag{3}$$

$$L_{\rm EM} = \frac{1}{8\pi} \int \left[\left[\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} + \nabla \phi \right]^2 - (\nabla \times \mathbf{A})^2 \right] d^3 r \quad \text{(field)} . \tag{4}$$

In these equations, $V_o(\mathbf{q})$ is an external potential; ρ and $\mathbf{J}$ are charge and current densities of the electron, respectively; $\mathbf{A}$ and ϕ are the potentials associated to the self-field of the electron.

We use the radiation gauge and choose $\phi=0$. We also transform the interaction term from $\dot{\mathbf{r}}\cdot\mathbf{A}$ to $\mathbf{r}\cdot\dot{\mathbf{A}}$ using

$$\mathbf{r}\cdot\mathbf{A}=\frac{d}{dt}(\mathbf{r}\cdot\mathbf{A})-\mathbf{r}\cdot\dot{\mathbf{A}} \tag{5}$$

and neglecting the total time derivative.

We can change to the Hamiltonian formalism by introducing $\mathbf{p}$ and $\mathbf{\Pi}$, the momenta conjugated to $\mathbf{q}$ and $\mathbf{A}$, respectively. We find

$$H=H_0+H_I+H_{\rm EM}+\cdots \tag{6}$$

with

$$H_0=\frac{p^2}{2m}+V_0(\mathbf{q})\ \ \text{(electron)}\ , \tag{7}$$

$$H_I=4\pi ec\,\mathbf{q}\cdot\mathbf{\Pi}(q,t)\ \ \text{(interaction)}\ , \tag{8}$$

$$H_{\rm EM}=\int\left[2\pi c^2\Pi^2+\frac{1}{8\pi}(\nabla\times\mathbf{A})^2\right]d^3r\ \ \ldots\text{(field)}\ , \tag{9}$$

and where the ellipsis, representing the counterterm, equals

$$2\pi\int\rho^2r^2d^3r\ . \tag{10}$$

This counterterm is not imposed as in the formulation of the quantum Brownian motion but is provided by the transformation (5) and will cancel an additional contribution to the external potential that arises later on in the calculation.

Now we can expand $\mathbf{A}$ and $\mathbf{\Pi}$ in plane waves conveniently normalized in a large box of volume V and write

$$\mathbf{A}(\mathbf{r},t)=\sum_{\mathbf{k},\lambda}q_{\mathbf{k}\lambda}(t)\mathbf{u}_{\mathbf{k}\lambda}(\mathbf{r})\ , \tag{11}$$

$$\mathbf{\Pi}(\mathbf{r},t)=\frac{1}{4\pi c^2}\sum_{\mathbf{k},\lambda}p_{\mathbf{k}\lambda}(t)\mathbf{u}_{\mathbf{k}\lambda}(\mathbf{r})\ , \tag{12}$$

$$\mathbf{u}_{\mathbf{k}\lambda}(\mathbf{r})=\left[\frac{2\pi c^2}{V}\right]^{1/2}\hat{\boldsymbol{\epsilon}}_{\mathbf{k}\lambda}\exp(i\mathbf{k}\cdot\mathbf{r})\ , \tag{13}$$

where $\hat{\boldsymbol{\epsilon}}_{\mathbf{k}\lambda}$ are unit vectors along the polarization directions ($\lambda=1,2$) ($\hat{\boldsymbol{\epsilon}}_{\mathbf{k}1}$, $\hat{\boldsymbol{\epsilon}}_{\mathbf{k}2}$ and $\mathbf{k}$ are mutually orthogonal). We also assume that the polarization directions are equivalent.

This expansion converts the field Hamiltonian into a Hamiltonian of a set of independent harmonic oscillators. The interaction Hamiltonian becomes

$$H_I=\frac{e}{c}\sum_{\mathbf{k},\lambda}p_{\mathbf{k}\lambda}[\mathbf{q}\cdot\mathbf{u}_{\mathbf{k}\lambda}(\mathbf{q})]\ . \tag{14}$$

We will evaluate this term in a certain limit related to the length scale in which we are interested. This length scale is much larger than the characteristic electronic dimension r_0. In the classical limit, r_0 is the classical electron radius, and in the quantum limit, it is the de Broglie wavelength of the electron. In terms of frequency this means that we have a cutoff frequency Ω such that

$$\Omega=\frac{2\pi c}{r_0} \tag{15}$$

and we will study only the low-frequency regime where $\omega\ll\Omega$.

Therefore, in this low-frequency approximation, the exponential term can be treated in the electric dipole approximation. We can further convert the sum into an integral over $\mathbf{k}$. We find

$$H_I=-\frac{e}{4}\int_0^\Omega p_k q k^2 dk\ . \tag{16}$$

Making a canonical transformation on the Hamiltonian in order to exchange q_k and p_k, we modify the interaction term which becomes

$$H_I=-\frac{ec}{4}\int_0^\Omega q_k q k^3 dk\ . \tag{17}$$

It is interesting to put this term in the form

$$H_I=\sum_k a_k q_k q\ . \tag{18}$$

For this purpose we introduce the spectral density function of the reservoir

$$J(\omega)=\frac{\pi}{2}\sum_k\frac{a_k^2}{m_k\omega_k}\delta(\omega-\omega_k)\ , \tag{19}$$

where a_k is the coupling coefficient in (18), m_k are the masses, and ω_k are the frequencies of the oscillators of the bath. Transforming the sum over k in (18) into an integral over ω we recover the form (17) if the spectral density behaves like

$$J(\omega)=\begin{cases}\lambda\omega^3, & \omega\le\Omega,\\ 0, & \omega>\Omega,\end{cases} \tag{20}$$

where $\lambda=(2e^2/3c^3)$. We shall also use the notation $\lambda=m\tau$. For electrons, $\tau\cong10^{-24}$ s.

This spectral density needs some remarks. First, it differs from the spectral density used in the study of Brownian motion which behaves linearly below the cutoff frequency. Second, we did not impose its form in order to find some desired behavior in the classical limit, but it is a consequence of the known form of the coupling between the electron and the electromagnetic field in the low-frequency range. Finally, depending on the model, it shows the same behavior as the spectral function for acoustic phonons in a 3D solid (see, for example, Ref. 2).

It is interesting to compare the study of quantum Brownian motion to the one of radiation damping. In the former, one starts with a Hamiltonian like (6) with coordinate-coordinate interaction and a counterterm. Then, one chooses the spectral density in order to reproduce the correct classical limit, namely, the Langevin equation.[1] In the case of radiation damping, the Hamiltonian (6) is obtained from first principles, we have an actual counterterm and the interaction was reduced to the coordinate-coordinate form. We did not impose any particular form for the spectral density, but this was naturally provided by the interaction. Then we have to verify if we find the correct equation of motion in the classical

limit.

To do this we take the Hamilton equations of motion from (6) and employ Laplace transforms to eliminate the field variables. The details of the calculation are in the Appendix and the resulting equation for the electron is

$$m^*\ddot{q}-\lambda\dddot{q}+V_0'(q)=F(t)\ , \tag{21}$$

where m^* is the electron mass, renormalized by the self-interaction, and $F(t)$ is a fluctuating force, satisfying the fluctuation-dissipation theorem. Equation (21) is the so-called Abraham-Lorentz (AL) equation, which gives the approximated classical description of the electron dynamics taking radiation damping into account. It should be noticed that the AL equation itself presents many deficiences, such as runaway solutions, pre-acceleration, etc. We refer the reader to careful discussions on this issue in Ref. 7. We can also argue that there is a correspondence between the approximated description of radiation damping by the third time derivative of position in (21) and the low-frequency limit of the spectral function we have obtained here.

It is shown in the Appendix that $m^*\cong m$ in the nonrelativistic limit studied in this paper.

Now, the quantum dynamics follows from the Feynman-Vernon scheme, with the coupled initial condition (see below) and the particular spectral density (19) and (20).

III. QUANTUM DYNAMICS

We are dealing with a composite system with many degrees of freedom, but the only variables of interest are the ones related to the electron. Thus, if we start with the density operator $\hat{\rho}(t)$ of the whole system, we have to eliminate the field variables performing the trace of $\hat{\rho}(t)$ over them. This procedure defines the reduced density operator of the electron as

$$\tilde{\rho}(t)=\mathrm{tr}_{\mathrm{EM}}\hat{\rho}(t)\ . \tag{22}$$

The time evolution of the total density operator is given by

$$\hat{\rho}(t)=\exp(-iHt/\hbar)\hat{\rho}(0)\exp(iHt/\hbar)\ . \tag{23}$$

So, we have to specify the initial condition $\hat{\rho}(0)$ in order to proceed with the evaluation of $\hat{\rho}(t)$. Since it does not make sense to try to decouple the electron from its own radiation field it would be totally unrealistic to use the factorizable initial condition as proposed by Feynman and Vernon.[4] Instead, we could prepare the initial state of the system by performing a measurement of position on the electron at $t=0$ once it is in equilibrium with the electromagnetic radiation. The corresponding initial condition can be written as

$$\hat{\rho}(\text{entire system},0)=\hat{\rho}(\text{electron},0)\hat{\rho}(\text{entire system in equilibrium})\ . \tag{24}$$

From now on we will represent the electron coordinates by a single variable, say x, and the field variables by the vector

$$\mathbf{R}=(R_1,R_2,\ldots,R_N)\ . \tag{25}$$

In this notation the above initial condition is written as

$$\hat{\rho}(x'\mathbf{R}',y'\mathbf{Q}',0)=\hat{\rho}_0(x',y',0)\hat{\rho}_{\mathrm{eq}}(x'\mathbf{R}',y'\mathbf{Q}')\ , \tag{26}$$

where the primes stand for the values of the variables at $t=0$.

In the usual way[1] the reduced density operator assumes the form

$$\tilde{\rho}(x,y,t)=\int\int dx'dy'J(x,y,t;x',y',0)\hat{\rho}_0(x',y',0)\ , \tag{27}$$

where

$$J(x,y,t;x',y',0)=\int_{x'}^{x}\int_{y'}^{y}\mathcal{D}x(t)\mathcal{D}y(t)\exp\frac{i}{\hbar}\{S_0[x(t)]-S_0[y(t)]\}\mathcal{F}([x(t)],[y(t)],x',y') \tag{28}$$

and $\mathcal{F}([x(t)],[y(t)],x',y')$ is the so-called influence functional

$$\mathcal{F}([x],[y],x',y')=\int\int\int d\mathbf{R}'d\mathbf{Q}'d\mathbf{R}\hat{\rho}_{\mathrm{eq}}(x'\mathbf{R}',y'\mathbf{Q}',0)\int_{\mathbf{R}'}^{\mathbf{R}}\int_{\mathbf{Q}'}^{\mathbf{R}}\mathcal{D}\mathbf{R}\,\mathcal{D}\mathbf{Q}$$
$$\times\exp\frac{i}{\hbar}\{S_I[x(t),R(t)]-S_I[y(t),Q(t)]+S_{\mathrm{EM}}[R(t)]-S_{\mathrm{EM}}[Q(t)]\}\ . \tag{29}$$

In the case where the initial condition (26) is valid, the influence functional is a functional of the paths $x(t)$, $y(t)$, and a function of the initial values of the variables of the electron, x',y'.[6] In the equations above, S_0, S_I, and S_{EM} are the actions of the electron, the interaction, and the field, respectively.

For the sake of simplicity we will perform the calculations for

$$V_0(q)=0\ . \tag{30}$$

The result will be convenient for our application to the interference process.

The first thing to do is to calculate the influence functional taking into account the initial condition (26). It can be shown[1,5] that its imaginary part is

$$\mathcal{F}_i=\exp\left[\frac{-i}{\hbar}\int_0^t\int_0^u[\xi(u)]\alpha_i(u-s)[2q(s)]du\,ds\right]\ , \tag{31}$$

where

$$q=\tfrac{1}{2}(x+y)\ , \tag{32}$$

$$\xi=x-y\ , \tag{33}$$

and

$$\alpha_i(u-s)=-\sum_k \frac{a_k^2}{2m_k\omega_k}\sin\omega_k(u-s)\ . \tag{34}$$

With the aid of the spectral density (19) and (20) we easily evaluate Eq. (31) to obtain

$$\mathcal{F}_i=\exp\left[\frac{2i\lambda\Omega^3}{3\hbar\pi}\int_0^t q(u)\xi(u)du+\frac{i}{\hbar}\lambda\int_0^t\xi(u)\ddot{q}(u)du - \frac{2i\lambda\Omega}{\hbar\pi}\int_0^t\xi(u)\ddot{q}(u)du\right]\ . \tag{35}$$

The first term in the above equation has the only effect of renormalizing the external potential (here, $V_0=0$), but we do not need to worry, because the counterterm (10) exactly cancels it. This cancellation can be seen by assuming, for instance, a simple spherical model for the electron's charge distribution.

In the same way, the third term provides an electromagnetic contribution to the mass, leading to a renormalized electron mass.

Thus, the imaginary part of the influence functional reduces to the second term in Eq. (35).

The real part is evaluated in a similar way. Following Ref. 5 we have ($\beta=1/k_BT$)

$$\mathcal{F}_r=\exp\left[-\tfrac{1}{2}\sum_\alpha\coth\left[\frac{\beta E_\alpha}{2}\right]\left|\sum_\gamma\left[\frac{\mu_\gamma\omega_\gamma}{2}\right]^{1/2}(A_\alpha^\gamma I_\gamma-B_\alpha^\gamma I_\gamma^*)\right|^2\right]\ , \tag{36}$$

with the definitions

$$I_\gamma=i(y'-x')+\omega_\gamma\int_0^t[x(u)-y(u)]\exp(i\omega_\gamma u)du\ , \tag{37}$$

$$A_\alpha^\gamma=-\frac{1}{2}\left[\frac{\mu_\gamma\omega_\gamma}{E_\alpha}\right]^{1/2}\frac{1}{E_\alpha+\omega_\gamma}\left[\sum_\beta\frac{\mu_\beta\omega_\beta^2}{(E_\alpha^2-\omega_\beta^2)^2}\right]^{-1/2}\ , \tag{38}$$

$$B_\alpha^\gamma=\frac{1}{2}\left[\frac{\mu_\gamma\omega_\gamma}{E_\alpha}\right]^{1/2}\frac{1}{E_\alpha-\omega_\gamma}\left[\sum_\beta\frac{\mu_\beta\omega_\beta^2}{(E_\alpha^2-\omega_\beta^2)^2}\right]^{-1/2}\ , \tag{39}$$

and

$$\mu_\gamma=\frac{a_\gamma^2}{m_\gamma\omega_\gamma^4}\ , \tag{40}$$

where the energies E_α are the positive solutions of

$$1=\sum_\gamma\frac{\mu_\gamma\omega_\gamma^2}{m(E^2-\omega_\gamma^2)}\ . \tag{41}$$

We find

$$\mathcal{F}_r=\exp S_2\ , \tag{42}$$

where

$$S_2=\left[\frac{-\lambda}{\hbar\pi}\int_0^t f_1(\omega)d\omega\int_0^t\int_0^t\xi(u)\xi(v)\cos\omega(u-v)du\,dv\right]\left[\frac{-\lambda}{\hbar\pi}\int_0^\Omega f_z(\omega)d\omega\left[\xi'^2+\xi'\omega\int_0^t\xi(u)g(u)du\right]\right] \tag{43}$$

with

$$f_1(\omega)=\omega^3\coth\left[\frac{\beta\hbar\omega}{2}\right]\ , \tag{44}$$

$$f_2(\omega)=\frac{\omega\coth\left[\frac{\beta\hbar\omega}{2}\right]}{1+\omega^2\tau^2}\ , \tag{45}$$

and

$$g(u)=2u\omega\cos\omega u-2\sin\omega u\ . \tag{46}$$

The next point is to derive the expression for the propagator of the reduced density operator, which is given by

$$J(q,\xi,t;q',\xi',0)=\int\int\mathcal{D}q(t)\mathcal{D}\xi(t)\exp(iS_1+S_2)\ , \tag{47}$$

where

$$S_1=\frac{m}{\hbar}\int_0^t[\tau\xi(u)\ddot{q}(u)+\dot{\xi}(u)\dot{q}(u)]du\ . \tag{48}$$

To do this we have to evaluate the double path integral. This is easily done by expanding the integrand around the classical paths of the action S_1 and following the same steps of a similar calculation in Ref. 5. This gives

$$J(q,\xi,t;q',\xi',0)=[f(t)]^{-1}\exp(iS_{1\mathrm{cl}}+S_{2\mathrm{cl}})\ , \tag{49}$$

where the actions were calculated along the classical paths of S_1.

Now, the reduced density operator can be written as

$$\tilde{\rho}(q,\xi,t)=\frac{1}{f(t)}\int\int dq'd\xi'\exp(iS_{1\mathrm{cl}}+S_{2\mathrm{cl}})\hat{\rho}_0(q',\xi',0)\ , \tag{50}$$

and all we need is to calculate the function $f(t)$. This is done by normalizing $\tilde{\rho}(q,\xi,t)$ and assuming that $\hat{\rho}_0(q',\xi',0)$ is itself normalized. So, the reduced density operator takes its final form

$$\tilde{\rho}(q,\xi,t)=\frac{m}{\hbar t}\int\int dq'd\xi'\exp(iS_{1\mathrm{cl}}+S_{2\mathrm{cl}})\hat{\rho}_0(q',\xi',0)\ . \tag{51}$$

The above equation is our desired result. This gives the quantum dynamics of the electron taking into account the radiation damping. In this framework we can find the time evolution for any particular $\hat{\rho}_0(q',\xi',0)$. This allows us to investigate the effects of the coupling to the radiation field on the quantum behavior of the electron. We will do this in a particular case, namely, the time evolution of $\tilde{\rho}$ in a prototype interference experiment.

IV. APPLICATION TO THE INTERFERENCE PROBLEM

We consider a simple one-dimensional problem which contains the essential physical information we need in order to analyze more complex situations.

Suppose we have, at $t=0$, two wave packets of width σ, centered at $x=a$ and $x=-a$, and propagating against each other with average momenta $-\hbar k$ and $\hbar k$, respectively. We represent them by

$$\psi_1(x)=\exp(ikx)\exp\left[-\frac{(x+a)^2}{2\sigma^2}\right]\ , \tag{52}$$

$$\psi_2(x)=\exp(-ikx)\exp\left[-\frac{(x-a)^2}{2\sigma^2}\right]\ , \tag{53}$$

and the initial state of the system by

$$\psi(x)=\psi_1(x)+\psi_2(x)\ . \tag{54}$$

Thus the initial density operator for this system takes the form

$$\hat{\rho}_0(x,y,0)=\hat{\rho}_1(x,y,0)+\hat{\rho}_2(x,y,0)+\hat{\rho}_{\mathrm{int}}(x,y,0)\ , \tag{55}$$

where the first and the second terms describe the independent propagation of each wave packet, while the last one describes the interference due to the initial overlap of the packets. Since we are interested in the interference process, we retain only this last term.

In the case where the electron is not coupled to any bath we can predict that the two wave packets will coincide at the origin and then they will move far apart with their widths spreading. The interference term is small in the beginning, reaches its maximum when the centers coincide, and becomes small again after the crossing.

In order to know what happens when we take into account the coupling to the radiation field we have to insert $\hat{\rho}(0)$ into the reduced density operator in Eq. (51). We assume that $a\gg\sigma$ in order to neglect the initial overlap of the wave packets. The normalized initial reduced density operator is then

$$\hat{\rho}_{\mathrm{int}}(q,\xi,0)=C\exp(2ikq)\exp\left[-\frac{1}{\sigma^2}(q^2+\xi^2/4+a\xi+a^2)\right]+C\exp(-2ikq)\exp\left[-\frac{1}{\sigma^2}(q^2+\xi^2/4-a\xi+a^2)\right]\ , \tag{56}$$

with the normalization constant

$$C=\frac{1}{2\sigma\sqrt{\pi}}\ . \tag{57}$$

The relevant information in $\tilde{\rho}_{\mathrm{int}}(t)$ is in its diagonal terms ($\xi=0$). Performing all the integrations we find

$$\hat{\rho}_{\mathrm{int}}(q,\xi=0,t)=\hat{\rho}_{\mathrm{int}}(q,t)=\frac{2\sqrt{\pi}}{\sigma(t)}\exp\left[-\frac{[q+a(t)]^2}{2\sigma^2(t)}\right]\exp\left[-\frac{[q-a(t)]^2}{2\sigma^2(t)}\right]$$
$$\times\cos\left[\frac{mqB(t)}{2\hbar t\alpha(t)}\right]\exp\left[-\frac{\lambda H(t)}{\hbar\pi\alpha(t)}\left[\sigma^2k^2+\frac{a^2}{\sigma^2}\right]\right]\ , \tag{58}$$

where

$$a(t)=a-\frac{\hbar k}{m}t\ , \tag{59}$$

$$\sigma^2(t)=\sigma^2+\frac{\hbar^2t^2}{m^2\sigma^2}+\frac{4\hbar t^2\tau}{\pi m}H(t)\ , \tag{60}$$

$$\alpha(t)=\frac{1}{4\sigma^2}+\frac{\sigma^2m^2}{4\hbar^2t^2}+\frac{m\tau}{\hbar\pi}H(t)\ , \tag{61}$$

$$H(t)=\int_0^\Omega\frac{f_i(\omega)}{\omega^2}d\omega\left[2\cos\omega t-\frac{6\sin\omega t}{\omega t}+3+\frac{2}{\omega^2t^2}(1-\cos\omega t)\right]+\int_0^\Omega f_2(\omega)d\omega\left[1+\frac{2\tau}{t}(1-\cos\omega t)+2\left[\frac{\sin\omega t}{\omega t}-1\right]\right]\ , \tag{62}$$

and

$$B(t)=\frac{\sigma^2 mk}{\hbar t}+\frac{a}{\sigma^2}\ . \tag{63}$$

When $\lambda=0$, expression (58) has the general form of an interference term, namely,

$$\hat{\rho}_{\rm int}(q,t)=2\sqrt{\hat{\rho}_1(q,t)}\sqrt{\hat{\rho}_2(q,t)}\cos f(q,t)\ , \tag{64}$$

where the function $\cos f(q,t)$ describes the interference fringes.

According to Eqs. (58) and (59) the wave packets follow the free particle classical paths, with the velocities

$$\mp\frac{\hbar k}{m}\ . \tag{65}$$

Furthermore, there is a spreading of the wave packets as expected.

The new remarkable fact is in the last exponential,

$$D(t)=\exp\left[-\frac{\lambda H(t)}{\hbar\pi\alpha(t)}\left[\sigma^2k^2+\frac{a^2}{\sigma^2}\right]\right]\ , \tag{66}$$

which resembles the form already obtained in the interference term of wave packets in the case of Brownian motion.[8] The quantities σ, k, and a depend only on the preparation procedure and we shall analyze $D(t)$ for realistic values of these parameters in what follows.

Let us consider, for instance, electrons with energies of the order of some electronvolts. In this case the wave vector k is of the order of 10^{10} m^{-1} and the de Broglie wavelength of the particle is about 10^{-1} Å. This leads us to consider the initial width σ as 1 Å and so, the factor $(\sigma k)^2$ is very close to one. On the other hand the initial separation of the wave packets can be taken as the interatomic spacing in a solid which means that $2a\cong$ few angstroms, and that $(a/\sigma)^2$ would be at most of the order of 10^2. We can also estimate the cutoff frequency Ω as 10^{19} sec^{-1}.

Taking all these values into account and remembering that $\tau=6.2\times10^{-24}$ sec we can finally study the behavior of $D(t)$ in the limits of interest. We found that for temperatures below 10^9 K we can safely use the zero temperature limit and $H(t)/\alpha(t)$ tends to a constant for times of the order of 10^{-24} sec. Since the wave packets take much longer than that to overlap we shall only be interested in the long time behavior of $D(t)$.

Carefully analyzing the integrals involved in the exponent of $D(t)$ we found that for long times

$$D(t)\cong\exp-[10^{-2}(a/\sigma)^2] \tag{67}$$

which is not much smaller than one. This only tells us that although there should be a tendency for the interference term to be destroyed by the coupling to the electromagnetic field, the latter is not sufficiently effective in so doing.

Although the specific numerical values of the physical quantities involved in the problem do not allow for the destruction of interference, there still remains the question about the origin of $D(t)$.

The emission of radiation that happens in our prototype experiment takes place only during the initial preparation of the packets and afterward there is no emission of radiation at all. However, the coupling to the field introduces not only dissipation but also a fluctuating force to the electron motion. Here, only the fluctuations can be responsible for the existence of $D(t)$.

A similar effect has also been shown to exist in the example of quantum Brownian motion when $\lambda\to0$ and $T\to\infty$, but in such a way that $\lambda T\to$constant (see Ref. 8 for details). This situation can be shown to be equivalent to an undamped particle acted on by an external stochastic force without temperature.

V. CONCLUSIONS

Using the system-plus-reservoir approach we were able to describe the quantum dynamics of a nonrelativistic charged particle taking into account the coupling to its own radiation field in the low-frequency limit. The approach (within the appropriated limits) provides the expected classical limit of the problem, namely, the Abraham-Lorentz equation of motion. We also showed that this problem is another example that can be treated within the scheme of coordinate-coordinate coupling with the appropriate choice of the spectral function $J(\omega)$. Moreover, the counterterm for the renormalization of the potential spontaneously appears in the present example, whereas the mass renormalization has the usual meaning as in quantum electrodynamics.

The resulting generalized Feynman-Vernon approach can clearly be applied to the charged-particle–radiation-field system in many circumstances. Obviously a very interesting example to be studied is the quantum dynamics of the charged particle in the presence of an external force $f(t)$ such that df/dt is not zero.

In the specific application to the interference problem, we conclude that the incoherent modes of the electromagnetic radiation would tend to destroy the interference fringes depending on the way the two initial wave packets are prepared. However, for specific values of the quantities involved in more realistic problems we found that this possible correction is negligible.

ACKNOWLEDGMENTS

This work has been partially supported by the FAPESP (Fundacão de Amparo à Pesquisa do Estado de São Paulo) and the CNPq (Conselho Nacional de Desenvolvimento Cientfico e Tecnológico).

APPENDIX: THE CLASSICAL EQUATION OF MOTION

The composite system electron-plus-radiation field is described by a Hamiltonian that depends on the variables of the electron and the field. Clearly, the classical behavior is found by writing down the Hamiltonian equations of motion for this Hamiltonian [Eq. (6)]. However, these equations are coupled through the variables of the field and we must eliminate them in order to obtain the classical equation of motion for the electron only.

First, the Hamilton equations of motion are

$$m\ddot{q} = -V_0'(q) - \sum_k a_k q_k \tag{A1}$$

and

$$\ddot{q}_k = -c^2k^2q_k - a_k q \ . \tag{A2}$$

Now performing the Laplace transform of the equations above, we can eliminate the variables q_k and then take the inverse transform of the equation for q. We find

$$m\ddot{q} = -V_0'(q) + \mathcal{L}^{-1}\left[\sum_k \left[a_k \frac{sq_k(0)+\dot{q}_k(0)}{s^2+c^2k^2} + \frac{a_k^2\tilde{q}(s)}{s^2+c^2k^2}\right]\right], \tag{A3}$$

where $\mathcal{L}^{-1}$ means the inverse transform. The first sum inside the square brackets gives

$$F(t) = \sum_k \left[a_k q_k(0)\cos ckt + \frac{a_k\dot{q}_k(0)}{ck}\sin ckt\right] . \tag{A4}$$

In order to calculate average values in an ensemble of identically prepared systems, we assume that the reservoir is in thermodynamical equilibrium at $t=0$ and remember that the theorem of equipartition of energy is valid in the classical limit. Thus, it is straightforward to find that the average of $F(t)$ is

$$\langle F(t)\rangle = 0 \tag{A5}$$

and the correlation is

$$\langle F(t)F(t')\rangle = \frac{2k_BT}{\pi}\int_0^\infty \frac{J(\omega)}{\omega}\cos\omega(t-t')d\omega \ . \tag{A6}$$

This last equation agrees with the expression given by the fluctuation-dissipation theorem,

$$\langle F(t)F(t')\rangle = \frac{\hbar}{\pi}\int_0^\infty J(\omega)\coth\left[\frac{\hbar\omega}{2k_BT}\right]\cos\omega(t-t')d\omega \ , \tag{A7}$$

for high temperatures ($k_BT \gg \hbar\omega$). In the case focused here, $J(\omega)$ is given by (20).

The remaining term in Eq. (A3) is

$$\mathcal{L}^{-1}[\tilde{R}(s)\tilde{q}(s)] \ , \tag{A8}$$

with

$$\tilde{R}(s) = \sum_k \frac{a_k^2}{a^2+c^2k^2} \ . \tag{A9}$$

Employing the spectral function $J(\omega)$, this gives

$$\tilde{R}(s) = \frac{2m\tau\Omega^3}{3\pi} - \frac{2m\tau\Omega}{\pi}s^2 + m\tau s^3 + \frac{2m\tau}{\Omega\tau} + O\left[\frac{1}{\Omega^3}\right] . \tag{A10}$$

for $\Omega \gg 1$, the two last terms are negligible in this equation and the inverse transform then gives

$$\frac{2m\tau\Omega^3}{3\pi}q(t) - \frac{2m\tau\Omega}{\pi}\ddot{q}(t) + m\tau\dddot{q}(t) \ . \tag{A11}$$

The effect of the first term above is to renormalize the external potential $V_0(q)$. However, as the counterterm[1] exactly cancels it, the bare potential is unaffected. If we assume a specific model for the charge distribution of the electron, we can easily see this.

The second term in (A11) is an electromagnetic contribution to the bare electron mass. This gives rise to a renormalized electron mass

$$m^* = m + \Delta m \ , \tag{A12}$$

where

$$\Delta m = \frac{2m\tau\Omega}{\pi} \ . \tag{A13}$$

But, in this case, taking into account the numerical values, we can conclude that $\Delta m \ll 1$ and thus we have essentially $m^* \cong m$.

Finally, the last term in (A11) is the well-known radiation damping and the resulting classical equation of motion is just the Abraham-Lorentz equation (21).

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