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Calculation of electron-pair production by 7.5-MeV photons on Sn and U

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We present in this Brief Report the results of the distorted-wave Born approximation calculations of the electron-pair-creation cross section by 7.5-MeV photons on Sn and U. The results are compared with the cross sections obtained by using semiempirical formulas of Overbo [Phys. Lett. **71B**, 412 (1977)] and of Maximon and Gimm [Natl. Bur. Stand. (U.S.). Internal Report No. 78-1456 (1978)] and the interpolated experimental data.

I. INTRODUCTION

The calculation of the electron-pair-creation cross sections in distorted-wave Born approximation (DWBA) in the Coulomb field of the atomic nucleus by low- and intermediate-energy photons has been performed by many researchers.[1–7] Except for the calculations of Sud and Sharma[6] and Wright, Sud, and Kosik[7] all other DWBA calculations of the pair-production cross sections have been for low-energy photons. Overbo, Mork, and Olsen[2] have computed the electron-pair creation cross sections by photons of energies ranging from threshold to 5.0 MeV in the field of a large number of atomic nuclei. The technique of the Overbo, Mork, and Olsen[2] cannot be used for calculating the pair-production cross sections for photon energies higher than 5.0 MeV as their technique does not permit the evaluation of accurate DWBA radial integrals. For photon energies greater than 50.0 MeV, the Davies-Bethe-Maximon[8] expression, obtained by using the Sommerfeld-Maue wave functions for electron and positron, gives accurate pair-production cross sections. In the energy range 5–50 MeV the pair-production cross sections have been evaluated by using the semiempirical formulas of Overbo[9] and Maximon and Gimm.[10] The semiempirical formulas are functions of atomic number, photon energy, and a number of arbitrary parameters. The parameters of the semiempirical formulas are obtained by fitting them to the low-energy DWBA pair-production cross section computed by Overbo, Mork, and Olsen.[2] The formulas are constructed so that in the high-energy approximation, the semiempirical formulas approach the Davies-Bethe-Maximon expression.[8] The widely used table for the total photon absorption cross sections by Hubbell, Gimm, and Overbo[11] makes use of Overbo's[9] formula to evaluate the contribution of the nuclear-pair-production cross sections in the intermediate energy range. The two empirical formulas are found to differ from each other and give different results in the energy range 5–50 MeV. So there is uncertainty in the available pair-production cross sections. Therefore, there is considerable interest in the DWBA calculations of the pair-production cross section. Wright, Sud, and Kosik[7] used a different technique to evaluate the radial integrals and calculated the DWBA pair-production cross sections by 10.0- and 20.0-MeV photons on uranium. We present in this Brief Report the results of our DWBA pair-production cross section calculations by 7.5-MeV photons on Sn and U by using the technique developed by (Wright, Sud, and Kosik[7]). In this technique the radial integrals for the photon energy w are obtained by integrating a first-order matrix differential equation in the photon energy parameter and using as an initial value the radial integrals evaluated for the photon energy parameter for which the integrals can be evaluated accurately. We refer the reader to see Ref. 7 for details of the technique and for explicit expression, for the DWBA pair-production cross section used in our calculation. We present in Sec. II the results of our calculations and compare them with the Bethe-Heitler[12] calculations, the interpolated results of Overbo,[9] and those of Maximon and Gimm,[10] and the interpolated data at 7.5 MeV photon energy obtained from the available experimental data.

II. RESULTS AND CONCLUSIONS

We present in Tables I and II the results of our calculations of the differential cross sections ($d\sigma/dE_+$) for pair production by 7.5-MeV photons on tin ($Z=50$) and uranium ($Z=92$) for different positron energies. The calculational details for evaluating the DWBA radial integrals and the differential cross sections have been discussed at length by Wright, Sud, and Kosik.[7] The differential cross section for the pair production can be expressed as

$$\frac{d\sigma}{dE_+} \sum_{q=1}^{\infty} T_q ,$$

where $q=|k_+|+|k_-|-1$, k_+ and k_- are lepton partial-wave quantum numbers, and T_q is the partial differential cross section. We have evaluated T_q for $q=1$–40, 60, 80, 100, and 150. The value of T_q for intermediate values of q are obtained by a spline interpolation

TABLE I. Differential pair-production cross sections for $Z=92$ with 7.5-MeV photons for different positron energies; $d\sigma/dE_+$ is the result of present DWBA calculation and $d\sigma_{\mathrm{BH}}/dE_+$ is the plane wave Bethe-Heitler cross section.

E_+ (MeV)	$Z^{-2}(d\sigma/dE_+)$ (mb/MeV)	$Z^{-2}(d\sigma_{\mathrm{BH}}/dE_+)$ (mb/MeV)
0.75	0.0601	0.132 4
1.50	0.2137	0.253 2
2.50	0.2609	0.298 1
3.50	0.2728	0.306 9
4.00	0.2748	0.306 9
5.00	0.2747	0.298 1
5.50	0.2715	0.283 6
6.00	0.2612	0.253 2
6.50	0.2326	0.189 6
6.75	0.2014	0.132 4
6.9549	0.1514	0.047 85

TABLE II. Differential-pair production cross sections for $Z=50$ with 7.5-MeV photons for different positron energies; $d\sigma/dE_+$ is the result of the present DWBA calculation and $d\sigma_{\mathrm{BH}}/dE_+$ is the plane-wave Bethe-Heitler cross section.

E_+ (MeV)	$Z^{-2}(d\sigma/dE_+)$ (mb/MeV)	$Z^{-2}(d\sigma_{\mathrm{BH}}/dE_+)$ (mb/MeV)
0.65	0.050 6	0.099 54
0.75	0.097 4	0.132 4
1.50	0.242 4	0.253 2
2.5	0.287 6	0.298 1
3.5	0.298 2	0.306 9
4.0	0.299 1	0.306 9
5.0	0.293 5	0.298 1
5.5	0.283 3	0.283 6
6.0	0.259 7	0.253 2
6.5	0.201 8	0.189 6
6.75	0.161 6	0.132 4
6.9549	0.035 89	0.047 85

of $\log T_q$. It is observed that $\log T_q$ behaves linearly for large q. We use the slope a of the $\log T_q$ versus q at $q=q_{\max}$ to calculate the contribution of the remaining term (Wright, Sud, and Kosik[7]). The differential cross section can be expressed as

$$\frac{d\sigma}{dE_+}=\sum_{q=1}^{q\max} T_q+R \ ,$$

where the remainder R is given as

$$R=T_{q_{\max}}e^a/(1-e^a) \ .$$

The remainder at $q=q_{\max}$ is less than 1% of the total sum for uneven sharing of energy by positron and electron and about 3.5% for evenly shared energy by positron and electron. We have followed the technique of Wright, Sud, and Kosik[7] to estimate the uncertainty in the remainder R, which leads to the uncertainty in the calculation to be less than 0.15%. The uncertainty in the present calculation is less than the DWBA calculation of Wright, Sud, and Kosik[7] as the remainder at $q=q_{\max}$ is less.

We compare our results with the results of Bethe and Heitler.[12] As expected the Coulomb distortion correction to the Bethe-Heitler result is significant in this energy range. We have also calculated total DWBA pair-creation cross section by using Gauss quadrature technique and using the spline interpolation to obtain the base points for integration. We present in Table III the results of our DWBA calculations, results of the semiempirical formulas of Overbo[9] and Maximon and Gimm,[10] results of Bethe and Heitler,[12] and experimental data. The experimental attenuation coefficient at 7.5 MeV photon energy has been obtained by interpolating the spline interpolation technique using the available experimental data. We have used the available experimental attentation coefficient data at the energies 5.3 MeV (Rosenblum, Shrader, and Warner[13]) 6.13 MeV (Paul[14]), 6.418 MeV (Moreh and Wand[15]), 7.279 and 7.646 MeV (Moreh, Saltzmann, and Wand[16]), and 10.0 MeV (Sherman *et al.*[17]) for $z=92$, and at the energies 5.435, 6.405, 7.725, and 10.833 MeV (Barlett and Donahue[18]) and 6.13 MeV (Paul[14]) for $z=50$. To estimate the uncertainty in the experimental data, we carried out the interpolation by omitting one of the experimental points and used instead experimental data as given in Table III. We found that the spline interpolation technique introduced uncertainty less than 0.001%. The estimated uncertainty in the experimental data at 7.5 MeV (for Sn 0.3% and for U 0.1%) has been obtained by considering the uncertainty in the experimental data. The experimental pair-production cross section is obtained by subtracting the atomic scattering cross section (σ_{sc}), photoelectric cross section σ_{ph}, and the triplet pair-production cross sections σ_{tr} from the total attenuation coefficient

$$\sigma_{\mathrm{pair}}=\sigma_{\mathrm{total}}-(\sigma_{\mathrm{sc}}+\sigma_{\mathrm{ph}}+\sigma_{\mathrm{tr}}) \ .$$

The subtracted atomic cross sections have been obtained by interpolation from the tables of Hubbell, Gimm, and Overbo.[11] We have obtained the distorted-wave pair-production cross section σ_{DWBA} for σ_{pair} by using the expression

$$\sigma_{\mathrm{pair}}=f_{\mathrm{rad}}(1-R)\sigma_{\mathrm{DWBA}} \ ,$$

where f_{rad} is the Mork-Olsen radiative correction and $1-R$ is the screening factor. The factor $f_{\mathrm{rad}}(1-R)$ has been obtained from the table of Hubbell, Gimm, and Overbo.[11] The screening factor is the sum of the Born approximation term expressed in terms of the form factor and the energy shift correction that is obtained by simply shifting the point Coulomb positron spectrum in energy (for details see Hubbell, Gimm, and Overbo[11]). Almost all contribution to the screening factor at 7.5 MeV photon energy is due to the Born approximation. The form factor approach does not predict the screening effect correctly in the low-energy region.[4] The radiation correction in the table of Hubbell, Gimm, and Overbo[11] has been obtained by extrapolation from the high-energy results by *ad hoc* extrapolation. We strongly suggest the

TABLE III. The total pair-production cross section for different Z values at 7.5 MeV photon energy in barns. The subscripts DWBA, expt, MG, OV, and BH correspond to the present DWBA result, interpolated experimental result, the Maximon-Gimm interpolation, the Overbo interpolation, and the Bethe-Heitler result.

Z	σ_{DWBA}	σ_{expt}	σ_{MG}	σ_{OV}	σ_{BH}
50	4.106	4.096±0.012	4.097	4.107	4.197
92	12.975	12.980±0.013	13.020	13.054	14.148

investigation of the screening effect in this energy region. We have used the table of Hubbell, Gimm, and Overbo[11] to evaluate the σ_{DWBA} as has been done recently by Sherman *et al.*[17] and Wright, Sud, and Kosik[7] in order to compare σ_{DWBA} with the available interpolating formulas. The cross section from the two interpolation formulas differ from each other by 0.24% of the Bethe-Heitler[12] cross section for Sn as well as for U. The DWBA results for $z=92$ are lower than the interpolated value by Maximon and Gimm[10] by 0.35% and are lower than that of Overbo[9] by 0.61%. On the other hand, our results are higher than the interpolated value obtained by Maximon and Gimm[10] by 0.2% for $z=50$, but they agree with the interpolated value of Overbo.[9] The experimental cross section is lower than the calculated DWBA cross section by 0.24% for $Z=50$ and is higher by 0.04% for $Z=92$. It may be noted here that the σ_{expt} given in Table III includes the contribution from the total photonuclear absorption cross sections. So the discrepancy between theory and the experimental data is expected to be larger than given here.

In conclusion we mention that the discrepancy between our DWBA calculations and the cross section obtained from the interpolating formulas at 7.5 MeV is less than the corresponding calculations at 10 and 20 MeV. However, the discrepancy is large and is found to increase with the atomic number. Finally, our calculation demonstrates the need for DWBA pair-production cross sections for a number of energy points between 5 and 10 MeV and for a number of nuclei to modify and improve the existing interpolating formulas.

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