Intensity fluctuations in a frequency down-conversion process with three-level atoms

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An ideal nondegenerate parametric oscillator is known to give perfect squeezing in the difference of the intensities of the signal and idler modes. In this paper, we look at a frequency down-conversion system comprising three-level atoms in a cavity. The squeezing in the intensity difference is calculated and found, in general, to be degraded by spontaneous emission. In the limit of large detunings, the system is found to behave as a nondegenerate parametric oscillator.

I. INTRODUCTION

There has recently been considerable research activity in demonstrating the quantum noise reduction or squeezing in the difference intensity of the signal and idler modes of a nondegenerate parametric oscillator. The squeezing in the intensity difference is a consequence of the simultaneous production of a pair of signal and idler photons. Although the uncorrelated transmission of the signal and idler photons through the cavity mirror diminishes the pair correlation for short times, for long times the pair correlation is recovered. This results in a dip in the spectrum of fluctuations for the difference current below the shot-noise level, approaching zero at zero frequency.

A number of theoretical papers have appeared that describe this effect,[1-3] and experiments have recently been carried out by Heidman *et al.*[4] and Nabors and Shelby[5] showing squeezing of 30% and 54%, respectively, below the shot-noise level in the above-threshold nondegenerate parametric oscillator.

In this paper, we wish to consider an ensemble of N three-level atoms in a ladder configuration interacting with three cavity modes of the electromagnetic field. The cavity is driven at the high-frequency (pump) and this produces some excitation of the atoms. Photons may be emitted into the signal and idler modes by cascade transitions. The cavity modes are detuned from the atomic transitions. While there are some analogies with the parametric oscillator, the proximity of the atomic levels will mean that some population will appear in the excited states and the correlation between the signal and idler photons will be degraded by spontaneous emission and the finite lifetime of the atomic levels. The experiments on the optical parametric oscillator were performed using a crystal with an optical nonlinearity where all transitions were assumed to be virtual. We shall determine in which limits the three-level atomic medium approaches the behavior of the parametric oscillator.

Recently the effects of spontaneous emission on the photon pairs emitted in four-wave mixing in a two-level atomic medium have been calculated by Zhang and Walls.[6] These calculations are relevant to a recent experiment by Vallett, Pinard, and Grynberg.[7]

The calculations presented in this paper consider a three-level medium and are based on theoretical techniques developed by Scully and Zubairy[8] to describe a correlated emission laser.

II. THE MODEL AND THE EQUATIONS OF MOTION

We consider a system of three-level atoms which are being injected in some lower state, $|c\rangle$, into a cavity at a rate r_c. The atomic levels are shown in Fig. 1. The transitions $|a\rangle$-$|b\rangle$ and $|b\rangle$-$|c\rangle$ are assumed to be dipole allowed, while the dipole-forbidden transition $|a\rangle$-$|c\rangle$ is induced by some external means (for example, by applying a strong magnetic field for a magnetic dipole-allowed transition). The $|a\rangle$-$|c\rangle$ transition is driven by an external field which is sufficiently strong that it may be treated semiclassically. The Rabi frequency is denoted by $\Omega e^{-i\phi}$. The other two transitions will be treated quantum mechanically but only to second order in the coupling constants. The atoms are interacting with three field modes: a pump, a signal, and an idler.

The Hamiltonian consists of a free part, an interaction part, and a part due to the dissipation in the system:

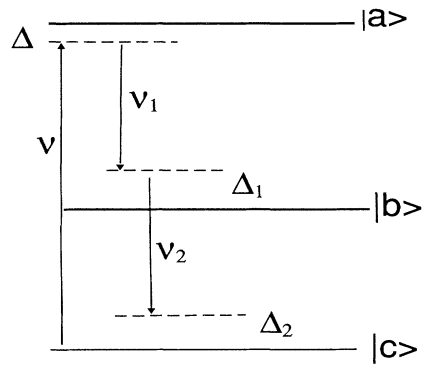

FIG. 1. The three-level system.

$$H = H_0 + V + H_{\text{bath}\,1} + H_{\text{bath}\,2} ,$$

$$H_0 = \sum_{i=a,b,c} \hbar\omega_i |i\rangle\langle i| + \hbar(\nu_1 a_1^\dagger a_1 + \nu_2 a_2^\dagger a_2) ,$$

$$V = \hbar g_1 (a_1 |a\rangle\langle b| + a_1^\dagger |b\rangle\langle a|) + \hbar g_2 (a_2 |b\rangle\langle c| + a_2^\dagger |c\rangle\langle b|) - \tfrac{1}{2}\hbar\Omega (e^{-i\phi - i\nu t}|a\rangle\langle c| + e^{i\phi + i\nu t}|c\rangle\langle a|) , \tag{1}$$

$$H_{\text{bath}\,1} = \sum_{i=1}^{2} (a_i \Gamma_i^\dagger + \Gamma_i a_i^\dagger) ,$$

where $|a\rangle$, $|b\rangle$, and $|c\rangle$ represent the upper, intermediate, and lower atomic states, respectively. The creation and annihilation operators a_1, $a_1^\dagger$, a_2, and $a_2^\dagger$ correspond to the signal and idler field modes of frequencies ν_1 and ν_2, respectively; g_1 and g_2 are the coupling constants associated with the transitions $|a\rangle$-$|b\rangle$ and $|b\rangle$-$|c\rangle$, respectively; and ν is the frequency of the strong pump field. Γ_1 and Γ_2 are the bath operators for the cavity damping of the signal and idler modes. The baths are assumed to be at zero temperature. $H_{\text{bath}\,2}$ represents the losses due to the spontaneous emission of the atoms. The spontaneous emission from all three levels is assumed to be the same, γ being the spontaneous emission rate into all other levels.

A similar system is treated by Ansari, Gea-Banacloche, and Zubairy.[9] They look at the degenerate case and the classical driving field is taken to be resonant with the atomic transition. They investigate the impact of the induced coherence between the upper and lower levels on the action of the system as a phase-sensitive amplifier and, when placed in a cavity, as a two-photon correlated emission laser. The master equation derived in Ansari, Gea-Banacloche, and Zubairy[9] is based on theoretical techniques derived by Scully and Zubairy[8] to treat a correlated emission laser. The method employed by Scully and Zubairy[8] involves solving for the probability amplitudes of the field-atom states and integrating the probability densities over all the atoms in the ensemble to obtain density-matrix elements. The atomic variables are eliminated by forming the reduced density matrix for the field ρ_F. This is done by tracing over the atomic states in the off-diagonal atom-field density matrix. The equation of motion for ρ_F, apart from the cavity loss terms, is

$$\langle n_1 n_2 |\dot{\rho}_F| n_1' n_2' \rangle = -\frac{i}{2}(V_{12}\rho_{21'} - \rho_{12'}V_{2'1'})_{\substack{n_1\to n_1+1 \\ n_1'\to n_1'+1}}^{\substack{n_2\to n_2+1 \\ n_2'\to n_2'+1}} - \frac{i}{2}(V_{21}\rho_{12'} + V_{23}\rho_{32'} - \rho_{21'}V_{1'2'} - \rho_{23'}V_{2'3'})_{\substack{n_2\to n_2+1 \\ n_2'\to n_2'+1}} - \frac{i}{2}(V_{32}\rho_{23'} - \rho_{32'}V_{2'3'}) . \tag{2}$$

The subscripts refer to the following atom-field states:

$$|1\rangle = |a, n_1 - 1, n_2 - 1\rangle ,$$
$$|2\rangle = |b, n_1, n_2 - 1\rangle , \tag{3}$$
$$|3\rangle = |c, n_1, n_2\rangle .$$

$\rho_{12'}{}_{\substack{n_2\to n_2+1 \\ n_2'\to n_2'+1}}$, for example, is the matrix element of ρ between the states $|1\rangle$ and $|2'\rangle$, where n_2 and n_2' are replaced by n_2+1 and $n_2'+1$, respectively.

As a first step to evaluating the reduced density-matrix elements $\rho_{12'}$, $\rho_{21'}$, $\rho_{23'}$, and $\rho_{32'}$ the wave vector $|\psi\rangle$ is expanded in terms of the eigenstates of the atom-field system:

$$|\psi\rangle = \sum_{i=a,b,c} \sum_{m_1,m_2} A_{im_1m_2} |i, m_1, m_2\rangle , \tag{4}$$

where $A_{im_1m_2}$ is the probability amplitude of finding the atom in state $|i\rangle$ and the fields of modes one and two in the states $|m_1\rangle$ and $|m_2\rangle$. The Schrödinger equation can be solved to give

$$\dot{A}_{an_1n_2} = -i(\omega_{an_1n_2} - i\gamma/2)A_{an_1n_2} + \frac{i}{2}\Omega e^{-i\phi - i\nu t} A_{cn_1n_2} , \tag{5}$$

$$\dot{A}_{cn_1n_2} = -i(\omega_{cn_1n_2} - i\gamma/2)A_{cn_1n_2} + \frac{i}{2}\Omega e^{i\phi + i\nu t} A_{an_1n_2} , \tag{6}$$

where $\omega_{im_1m_2} = \hbar\omega_i + \hbar(\nu_1 m_1 + \nu_2 m_2)$. Terms causing a change in population of the $|a\rangle$ and $|c\rangle$ levels due to transitions from the intermediate level have been dropped. This assumption holds as long as the pump field is sufficiently strong and the coupling is not too large. This gives equations of the form (5) and (6), which can be solved exactly. In the work of Scully and Zubiary[8] and Ansari, Gea-Banacloche, and Zubairy.[9] these equations are solved for the pump field on resonance with the atomic transition. We introduce the following detuning:

$$\nu = \omega_a - \omega_c - \Delta . \tag{7}$$

Making the transformation

$$A_{an_1n_2}=e^{-i(\omega_{an_1n_2}-i\gamma/2)t+i(\omega_{cn_1n_2}-i\gamma/2)t_0}\hat{A}_{an_1n_2}\ , \tag{8}$$

$$A_{cn_1n_2}=e^{-i(\omega_{cn_1n_2}-i\gamma/2)(t-t_0)}\hat{A}_{cn_1n_2}\ , \tag{9}$$

the solutions to (5) and (6) may be written as

$$\hat{A}_{an_1n_2}=\frac{\Omega}{2\mu}e^{-i\phi+(i/2)\Delta t}(e^{(i/2)\mu(t-t_0)}-e^{-(i/2)\mu(t-t_0)})\hat{A}^{F}_{n_1n_2}\ , \tag{10}$$

$$\hat{A}_{cn_1n_2}=\frac{1}{2\mu}e^{-(i/2)\Delta t}[(\Delta+\mu)e^{(i/2)\mu(t-t_0)}+(-\Delta+\mu)e^{-(i/2)\mu(t-t_0)}]\hat{A}^{F}_{n_1n_2}\ , \tag{11}$$

where the atom is injected into the cavity at time t_0 in its lower state $|c\rangle$ and $\hat{A}^{F}_{n_1n_2}$ is the field probability amplitude evaluated at the initial time t_0. We also have

$$\mu=(\Delta^2+\Omega^2)^{1/2}\ . \tag{12}$$

For the intermediate level the equation of motion is

$$\dot{A}_{bn_1n_2}=-i(\omega_{bn_1n_2}-i\gamma/2)A_{bn_1n_2}-ig_1\sqrt{n_1}\,A_{an_1-1n_2}-ig_2\sqrt{n_2+1}\,A_{cn_1n_2+1}\ . \tag{13}$$

Following Scully and Zubairy[8] this can be integrated out, and the solutions for $A_{an_1-1n_2}$ and $A_{cn_1n_2+1}$ substituted in.

The reduced density-matrix element $\rho_{12'}$ can be obtained by summing the contributions $A_{an_1-1n_2-1}(t)A^{*}_{bn_1'n_2'-1}(t)$ of all the atoms which are injected at random times at a rate r_c:

$$\rho_{12'}=r_c\int_{-\infty}^{t}dt_0\,A_{an_1-1n_2-1}(t)A^{*}_{bn_1'n_2'-1}(t)\ . \tag{14}$$

In writing down the density matrix elements, a "coarse graining" approximation is being made. This assumes that the passage of a single atom will cause only small changes in the field. Because the field is not changed appreciably, $A^{F}_{n_1n_2}(t_0)$ can be replaced by $A^{F}_{n_1n_2}(t)$. This approximation will hold as long as the characteristic transit time for the atoms is small compared with the time scale over which the field changes. When (14) is evaluated, we have

$$\rho_{12'}=(n_1')^{1/2}\langle n_1-1,n_2-1|\rho_F|n_1'-1,n_2'-1\rangle T_{11}(n_2')^{1/2}\langle n_1-1,n_2-1|\rho_F|n_1'n_2'\rangle T_{12}\ , \tag{15}$$

where

$$T_{11}=\frac{i}{4}g_1r_c\left[\frac{\Omega}{\mu}\right]^2\left[\left[\frac{1}{\gamma}-\frac{1}{\gamma-i\mu}\right]\frac{1}{\gamma+i(\Delta_1+\Delta/2-\mu/2)}+\left[\frac{1}{\gamma}-\frac{1}{\gamma+i\mu}\right]\frac{1}{\gamma+i(\Delta_1+\Delta/2+\mu/2)}\right]\ , \tag{16}$$

$$T_{12}=\frac{i}{4}g_2r_c\frac{\Omega}{\mu^2}e^{-i\Phi}\left[\left[\frac{\Delta+\mu}{\gamma}-\frac{-\Delta+\mu}{\gamma-i\mu}\right]\frac{1}{\gamma-i(\Delta_2-\Delta_1-\Delta/2+\mu/2)}-\left[\frac{-\Delta+\mu}{\gamma}+\frac{\Delta+\mu}{\gamma+i\mu}\right]\frac{1}{\gamma-i(\Delta_2-\Delta_1-\Delta/2-\mu/2)}\right]\ . \tag{17}$$

Here $\Phi=\phi+\Delta_2t$, and the detunings Δ_1 and Δ_2 are given by (see Fig. 1).

$$\nu_1=\omega_a-\omega_b-\Delta-\Delta_1\ , \tag{18}$$

$$\nu_2=\omega_b-\omega_c+\Delta_1-\Delta_2\ . \tag{19}$$

In a similar manner, the matrix element $\rho_{32'}$ can be evaluated:

$$\rho_{32'}=(n_1')^{1/2}\langle n_1n_2|\rho_F|n_1'-1,n_2'-1\rangle T_{21}+(n_2')^{1/2}\langle n_1n_2|\rho_F|n_1'n_2'\rangle T_{22}\ , \tag{20}$$

where

$$T_{21}=\frac{i}{4}g_1r_c\frac{\Omega}{\mu^2}e^{i\Phi}\left[\left[\frac{1}{\gamma}-\frac{1}{\gamma-i\mu}\right]\frac{\Delta+\mu}{\gamma+i(\Delta_1+\Delta/2-\mu/2)}-\left[\frac{1}{\gamma}-\frac{1}{\gamma+i\mu}\right]\frac{-\Delta+\mu}{\gamma+i(\Delta_1+\Delta/2+\mu/2)}\right]\ , \tag{21}$$

$$T_{22}=\frac{i}{4}g_2 r_c\frac{1}{\mu^2}\left[\left[\frac{\Delta+\mu}{\gamma}+\frac{-\Delta+\mu}{\gamma-i\mu}\right]\frac{\Delta+\mu}{\gamma-i(\Delta_2-\Delta_1-\Delta/2+\mu/2)}+\left[\frac{-\Delta+\mu}{\gamma}+\frac{\Delta+\mu}{\gamma+i\mu}\right]\frac{-\Delta+\mu}{\gamma-i(\Delta_2-\Delta_1-\Delta/2-\mu/2)}\right]. \tag{22}$$

The results can be substituted back into (2) and a master equation for the interaction part of the Hamiltonian obtained:

$$\dot{\rho}_F=-[\beta_{11}^* a_1 a_1^\dagger\rho_F+\beta_{11}\rho_F a_1 a_1^\dagger-(\beta_{11}+\beta_{11}^*)a_1^\dagger\rho_F a_1+\beta_{22}^* a_2^\dagger a_2\rho_F+\beta_{22}\rho_F a_2^\dagger a_2-(\beta_{22}+\beta_{22}^*)a_2\rho_F a_2^\dagger]$$

$$-[\beta_{12}^* a_1 a_2\rho_F+\beta_{21}\rho_F a_1 a_2-(\beta_{12}^*+\beta_{21})a_2\rho_F a_1]e^{i\Phi}-[\beta_{21}^* a_1^\dagger a_2^\dagger\rho_F+\beta_{12}\rho_F a_1^\dagger a_2^\dagger-(\beta_{12}+\beta_{21}^*)a_1^\dagger\rho_F a_2^\dagger]e^{-i\Phi}, \tag{23}$$

where

$$\beta_{11}=-ig_1 T_{11}=\frac{i}{4}A_1\frac{\Omega'^2}{\mu'}\frac{1}{1+(\mu')^2}\left[-\frac{1+i\mu'}{1+i(\Delta_1'+\Delta'/2-\mu'/2)}+\frac{1-i\mu'}{1+i(\Delta_1'+\Delta'/2+\mu'/2)}\right], \tag{24}$$

$$\beta_{22}=-ig_2 T_{22}=\frac{1}{4}A_2\frac{1}{(\mu')^2}\left[\left[\Delta'+\mu'+\frac{-\Delta'+\mu'}{1-i\mu'}\right]\frac{\Delta'+\mu'}{1-i(\Delta_2'-\Delta_1'-\Delta'/2+\mu'/2)}+\left[-\Delta'+\mu'+\frac{\Delta'+\mu'}{1+i\mu'}\right]\frac{-\Delta'+\mu'}{1-i(\Delta_2'-\Delta_1'-\Delta'/2-\mu'/2)}\right], \tag{25}$$

$$\beta_{12}=-ig_1 T_{12}e^{i\Phi}=\frac{1}{4}A_3\frac{\Omega'}{(\mu')^2}\left[\left[\Delta'+\mu'+\frac{-\Delta'+\mu'}{1-i\mu'}\right]\frac{1}{1-i(\Delta_2'-\Delta_1'-\Delta'/2+\mu'/2)}-\left[-\Delta'+\mu'+\frac{\Delta'+\mu'}{1+i\mu'}\right]\frac{1}{1-i(\Delta_2'-\Delta_1'-\Delta'/2-\mu'/2)}\right], \tag{26}$$

$$\beta_{21}=-ig_2 T_{21}e^{-i\Phi}=-\frac{i}{4}A_3\frac{\Omega'}{\mu'}\frac{1}{1+(\mu')^2}\left[\frac{(\Delta'+\mu')(1+i\mu')}{1+i(\Delta_1'+\Delta'/2-\mu'/2)}+\frac{(-\Delta'+\mu')(1-i\mu')}{1+i(\Delta_1'+\Delta'/2+\mu'/2)}\right], \tag{27}$$

and

$$A_1=\frac{r_c g_1^2}{\gamma^2},\quad A_2=\frac{r_c g_2^2}{\gamma^2},\quad A_3=\frac{r_c g_1 g_2}{\gamma^2}. \tag{28}$$

All primed variables have been scaled by the spontaneous emission rate γ. The real and imaginary parts of β_{11} and β_{22} are responsible for the nonlinear absorption and nonlinear dispersion of modes one and two, respectively. β_{12} and β_{21} provide the nonlinear coupling between the two modes.

The terms in the master equation due to the cavity losses are given by

$$\dot{\rho}_F=\sum_{i=1}^{2}\kappa_i([a_i\rho_F,a_i^\dagger]+[a_i,\rho_F a_i^\dagger]), \tag{29}$$

where κ_1 and κ_2 are the cavity linewidths of the two modes. The explicit time dependence in the master equation can be transformed out by using the unitary transformation

$$U=e^{i\Delta_2 a_2^\dagger a_2 t}. \tag{30}$$

The generalized P representation[10] can now be used to transform the operator master equation into a c-number Fokker-Planck equation, from which the stochastic differential equations can be written down as

$$\dot{\alpha}_1=-(\kappa_1-\beta_{11})\alpha_1+g\alpha_2^\dagger+\Gamma_{\alpha_1}, \tag{31}$$

$$\dot{\alpha}_2=-(\kappa_2-i\Delta_2+\beta_{22}^*)\alpha_2+h\alpha_1^\dagger+\Gamma_{\alpha_2}, \tag{32}$$

where

$$g=\beta_{12}e^{-i\phi}\ , \tag{33}$$

$$h=-\beta_{21}^{*}e^{-i\phi}\ , \tag{34}$$

and the noise terms have the following correlations:

$$\langle \Gamma_{\alpha_1}(t),\Gamma_{\alpha_1^{\dagger}}(t')\rangle = f\delta(t-t')\ , \tag{35}$$

$$\langle \Gamma_{\alpha_1}(t),\Gamma_{\alpha_2}(t')\rangle = h\delta(t-t')\ , \tag{36}$$

$$\langle \Gamma_{\alpha_1^{\dagger}}(t),\Gamma_{\alpha_2^{\dagger}}(t')\rangle = h^{*}\delta(t-t')\ . \tag{37}$$

$f=\beta_{11}+\beta_{11}^{*}$ is the phase-insensitive noise or fluorescence. h is the phase-sensitive noise.

III. SPECTRUM OF INTENSITY FLUCTUATIONS

In the nondegenerate parametric oscillator, signal and idler photons are produced in pairs and are hence highly correlated. We wish to see whether this high correlation is maintained despite the spontaneous emission introduced by the atoms. In order to investigate this, the fluctation spectrum of the intensity difference between the signal and idler modes will be calculated. The procedure is to linearize the equations of motion and then calculate the spectrum.

Because the system is being driven below threshold, we form intensity variables[6]

$$\mathbf{I}=(\alpha_1^{\dagger}\alpha_1,\alpha_1\alpha_2,\alpha_1^{\dagger}\alpha_2^{\dagger},\alpha_2^{\dagger}\alpha_2)\ . \tag{38}$$

The equation of motion for the intensity variables can be obtained by applying the rules of Ito calculus.[11] We obtain

$$\dot{\mathbf{I}}=-\underline{A}\mathbf{I}+\mathbf{F}+\mathbf{\Gamma}\ , \tag{39}$$

where

$$\underline{A}=\begin{bmatrix} \tilde{\kappa}_1+\tilde{\kappa}_1^{*} & -g^{*} & -g & 0 \\ -h & \tilde{\kappa}_1+\tilde{\kappa}_2 & 0 & -g \\ -h^{*} & 0 & \tilde{\kappa}_1^{*}+\tilde{\kappa}_2^{*} & -g^{*} \\ 0 & -h^{*} & -h & \tilde{\kappa}_2+\tilde{\kappa}_2^{*} \end{bmatrix}\ . \tag{40}$$

The modified damping coefficients are defined as

$$\tilde{\kappa}_1=\kappa_1-\beta_{11}\ , \tag{41}$$

$$\tilde{\kappa}_2=\kappa_2-i\Delta_2+\beta_{22}^{*}\ . \tag{42}$$

$\mathbf{F}$ is a constant vector given by

$$\mathbf{F}=\begin{bmatrix} f \\ h \\ h^{*} \\ 0 \end{bmatrix}\ . \tag{43}$$

$\mathbf{\Gamma}$ is the noise term

$$\begin{bmatrix} \alpha_1^{\dagger}\Gamma_{\alpha_1}+\alpha_1\Gamma_{\alpha_1^{\dagger}} \\ \alpha_1\Gamma_{\alpha_2}+\alpha_2\Gamma_{\alpha_1} \\ \alpha_1^{\dagger}\Gamma_{\alpha_2^{\dagger}}+\alpha_2^{\dagger}\Gamma_{\alpha_1^{\dagger}} \\ \alpha_2^{\dagger}\Gamma_{\alpha_2}+\alpha_2\Gamma_{\alpha_2^{\dagger}} \end{bmatrix}\ , \tag{44}$$

and satisfies the correlation condition

$$\langle \mathbf{\Gamma}(t)\mathbf{\Gamma}(t')^{T}\rangle = \underline{D}\delta(t-t')\ , \tag{45}$$

where $\mathbf{D}$ is the diffusion matrix

$$\underline{D}=\begin{bmatrix} 2f\langle I_1\rangle & h\langle I_1\rangle+f\langle I_2\rangle & h^{*}\langle I_1\rangle+f\langle I_3\rangle & h^{*}\langle I_2\rangle+h\langle I_3\rangle \\ & 2h\langle I_2\rangle & f\langle I_4\rangle & h\langle I_4\rangle \\ & & 2h^{*}\langle I_3\rangle & h^{*}\langle I_4\rangle \\ & & & 0 \end{bmatrix}\ . \tag{46}$$

The equations of motion for the intensity variables are now linearized about the steady-state solutions. The steady-state solutions are given by

$$\langle I\rangle_{ss}=\underline{A}^{-1}\mathbf{F}\ . \tag{47}$$

Steady-state solutions exist only if they are stable under small perturbations. Stability is guaranteed if the real part of the eigenvalues of the coefficient matrix for the stochastic differential equations (31) and (32) are negative. The eigenvalues are given by

$$\lambda_{1,2}=-\tfrac{1}{2}(\tilde{\kappa}_1+\tilde{\kappa}_2^{*})\pm\tfrac{1}{2}[(\tilde{\kappa}_1-\tilde{\kappa}_2^{*})^2-4\beta_{12}\beta_{21}]^{1/2}\ , \tag{48}$$

$$\lambda_{3,4}=\lambda_{1,2}^{*}\ . \tag{49}$$

The squeezing will be maximized when the imaginary parts of the eigenvalues vanish. A nonzero imaginary part leads to a mixing of the quadratures, and this can feed fluctuations from the noisy quadrature into the squeezed quadrature. The eigenvalues are purely real when

$$\Delta_2+\mathrm{Im}(\beta_{22}-\beta_{11})=0\ , \tag{50}$$

$$\mathrm{Im}[(\tilde{\kappa}_1-\tilde{\kappa}_2^{*})^2-4\beta_{12}\beta_{21}]=0\ , \tag{51}$$

$$\mathrm{Re}[(\tilde{\kappa}_1-\tilde{\kappa}_2^{*})^2-4\beta_{12}\beta_{21}]\geq 0\ . \tag{52}$$

The stationary spectrum for the correlation $\langle \mathbf{I}(\tau),\mathbf{I}(0)^{T}\rangle \equiv \langle \delta\mathbf{I}(\tau),\delta\mathbf{I}(0)^{T}\rangle$ is given by

$$\underline{S}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\tau e^{-i\omega\tau}\langle \mathbf{I}(\tau),\mathbf{I}(0)^{T}\rangle = \frac{1}{2\pi}(\underline{A}+i\omega\underline{I})^{-1}\underline{D}(\underline{A}^{T}-i\omega\underline{I})^{-1}\ . \tag{53}$$

An expression for the fluctuation spectrum of the intensity difference $S_{\Delta}(\omega)$ can be found in the work of Zhang and Walls.[6] From this a normalized spectrum is defined:

$$S_\Delta(\omega)=\int_{-\infty}^{\infty} d\tau e^{-i\omega\tau}\langle I_1(\tau)-I_2(\tau),I_1(0)-I_2(0)\rangle$$
$$=2(\kappa_1\langle I_1\rangle+\kappa_2\langle I_4\rangle)S(\omega)\ . \tag{54}$$

We write

$$S(\omega)=1+S_d(\omega)\ , \tag{55}$$

where

$$S_d(\omega)=\frac{4\pi}{\kappa_1\langle I_1\rangle+\kappa_2\langle I_4\rangle}$$
$$\times[\kappa_1^2 S_{11}+\kappa_2^2 S_{44}-\kappa_1\kappa_2(S_{14}+S_{41})]\ . \tag{56}$$

IV. RESULTS

The model of the three-level atom that we have used includes spontaneous emission from all levels, and this will degrade the correlation between the photon pairs. The other dissipative effect will be the nonlinear and cavity absorption of the two modes. Because spontaneous emission increases towards resonance, the correlation between the photon pairs should improve as we detune from the upper and intermediate levels. In fact, in the limit of large detunings from both the upper and intermediate levels, the system should behave as a nondegenerate parametric oscillator, for which perfect squeezing in the intensity difference can be obtained.[1-3]

In the limit of large detuning from the upper level, $\Delta' \gg \Omega'$ and $\Delta' \gg 1$, we have to first order in Ω'/Δ'

$$\beta_{11}=0\ , \tag{57}$$

$$\beta_{22}=A_2\frac{1-i(\Delta_1'-\Delta_2')}{1+(\Delta_1'-\Delta_2')^2}\ , \tag{58}$$

$$\beta_{12}=\tfrac{1}{2}A_3\frac{\Omega'}{\Delta'}\frac{1-i(\Delta_1'-\Delta_2')}{1+(\Delta_1'-\Delta_2')^2}\ , \tag{59}$$

$$\beta_{21}=\tfrac{1}{2}A_3\frac{\Omega'}{\Delta'}\frac{1-\Delta_1'/\Delta'-i\Delta_1'}{1+(\Delta_1')^2}\ . \tag{60}$$

For the parametric oscillator the coefficients governing the nonlinear coupling have the same phase. This implies that g and h should have the same phase in the limit of large Δ', as this can be achieved if $\Delta_1' \gg \Delta_2'$ and $\Delta_1' \gg 1$. The squeezing can be maximized by imposing the conditions (50)–(52). In the high detuning regime under consideration here, the first condition can be solved to give a first approximation to Δ_2:

$$\Delta_2'=\tfrac{1}{2}\Delta_1'\pm\tfrac{1}{2}\Delta_1'\left[1-\frac{4A_2}{\gamma(\Delta_1')^2}\right]^{1/2}\ . \tag{61}$$

The solution with the minus sign corresponds to two-photon resonance. The detuning is shifted slightly from its bare value for two-photon resonance because the strong driving field shifts the cavity resonances (ac Stark effect). If we now choose $\Delta_1'=-\Delta'/2$, the deterministic parts of the Langevin equations in the high detuning limit can be written in the following forms:

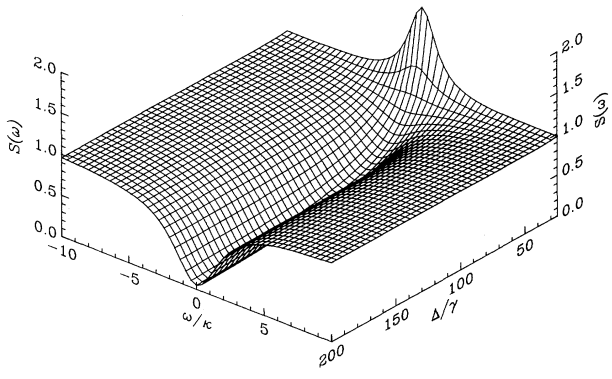

FIG. 2. The squeezing spectrum for fluctuations in the intensity difference as a function of the detuning from the upper level.

$$\dot{\alpha}_1=-\kappa_1\alpha_1-iA_3\frac{\Omega'}{(\Delta')^2}\alpha_2^\dagger\ , \tag{62}$$

$$\dot{\alpha}_2=-\left[\kappa_2+\frac{4A_2}{(\Delta')^2}\right]\alpha_2-iA_3\frac{\Omega'}{(\Delta')^2}\alpha_1^\dagger\ . \tag{63}$$

It should also be noted that both nonlinear coupling terms β_{12} and β_{21} depend linearly on the intensity of the driving field, as expected for a parametric oscillator.

We now look at how the squeezing spectrum varies with the detunings Δ' and Δ_1'. We choose $\Delta_1'=-\Delta'/2$ and Δ_2' such that the dispersive term vanishes. Rather than solve for Δ_2' exactly, we set it to zero, as it should only vary from this by a small amount.

The spectrum is obtained by inverting $\underline{A}$ numerically and evaluating the expressions (53) and (56). We check that the system is operating below threshold by ensuring that the real parts of the eigenvalues corresponding to the coefficient matrix of the equations of motion (31) and (32) are less than zero.

In Fig. 2 the spectrum for the fluctuations in the inten-

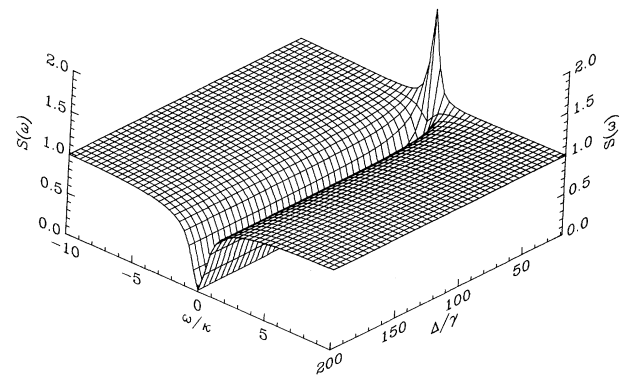

FIG. 3. The squeezing spectrum with unequal cavity-decay rates.

sity difference for different values of the scaled detuning Δ' is shown. The parameter values used are: $r_c = 10^{17}$, $g_1 = g_2 = 100$, $\gamma = 10^6$, $\Omega' = 1$, and $\kappa_1 = \kappa_2 = 10^6$. The noise frequency ω is scaled by the average of the cavity linewidths $\kappa = \frac{1}{2}(\kappa_1 + \kappa_2)$. The system operates below threshold for $\Delta' > 15.5$. It can be seen that the values of A_1, A_2, and A_3 are quite large, being 3 orders of magnitude larger than the cavity losses given by κ_1 and κ_2. In fact, if A_1, A_2, and A_3 are chosen to be about the same size as the cavity losses, only a small amount of squeezing is obtained.

In the work of Lane, Reid, and Walls,[2] it was seen that for $\kappa_1 \neq \kappa_2$, the central dip in the spectrum is narrowed. This phenomenon can be seen in Fig. 3, where, for a given detuning, the dc component is depressed compared with the symmetric case. The parameters used here are the same as in Fig. 2 except that $\kappa_1 = \kappa_2/5 = 10^6$. At a detuning of 200 atomic linewidths, 98% squeezing is obtained, compared with 95% in the symmetric case. As the detuning is increased the squeezing improves much more rapidly than the case with equal cavity linewidths. The squeezing has already reached 90% for $\Delta' \approx 60$, whereas for equal cavity linewidths the squeezing only reaches 90% when $\Delta' \approx 135$.

Figure 4 shows the variation of the zero-frequency component of the squeezing spectrum for $\Delta' = 200$ as a function of the ratio of the cavity linewidths. We obtain the best squeezing for κ_2 about five times greater than κ_1. The nonlinear absorption of mode two is much larger than that of mode one, as more atoms are in their lower state. Increasing the cavity linewidth of mode two increases the amount of light leaving the cavity, and hence lessens the relative nonlinear absorption of that mode. Of course, if κ_2 is increased too much, too much light escapes, and the ratio of nonlinear coupling effects to losses is worsened.

The parameters A_1, A_2, and A_3 govern nonlinear absorption in modes one and two and nonlinear coupling between the modes, respectively. They give a measure of the coherent to incoherent processes in the system. It has already been seen that the parameters must be large compared with the cavity losses if appreciable squeezing is to be obtained. The relative sizes of the parameters can also be adjusted to maximize the squeezing. In general, the nonlinear absorption of mode two is much larger than that of mode one, because the population of the lower state is much larger than that of the intermediate state. If the coupling g_1 is increased while g_2 is decreased such that the product $g_1 g_2$ remains the same, the nonlinear absorption of the two modes can be modified without changing the nonlinear coupling. In Fig. 5 the same parameters as in Fig. 2 are used except that $g_1 = 200$ and $g_2 = 50$. The squeezing is now about 98% at $\Delta' = 200$. We obtain 90% squeezing when $\Delta' \approx 70$.

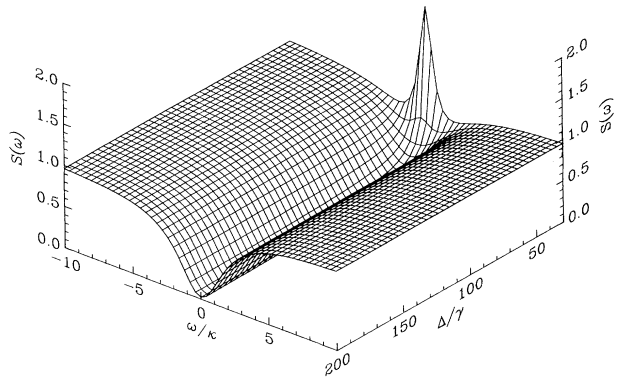

FIG. 5. The squeezing spectrum with unequal coupling constants.

Figure 6 shows the variation of the zero-frequency component of the squeezing spectrum for $\Delta' = 200$ as a function of the ratio of the coupling constants for the two transitions. If g_2 is decreased too much, g_1 becomes large, and the nonlinear absorption of mode one begins to play a role.

The dependence of the squeezing on the Rabi frequen-

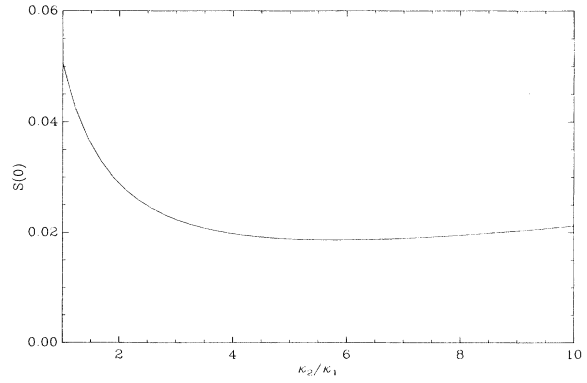

FIG. 4. The zero-frequency component of the squeezing spectrum for $\Delta' = 200$ as a function of the ratio of the cavity-decay rates.

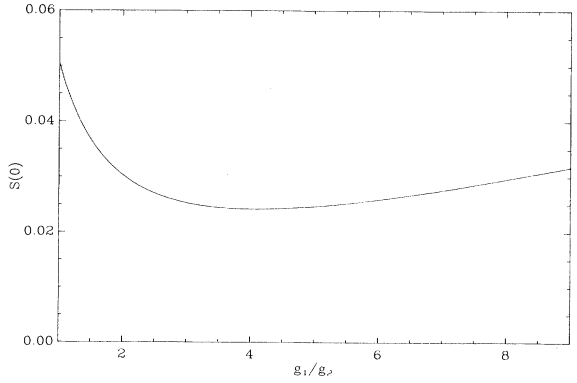

FIG. 6. The zero-frequency component of the squeezing spectrum for $\Delta' = 200$ as a function of the ratio of the coupling constants.

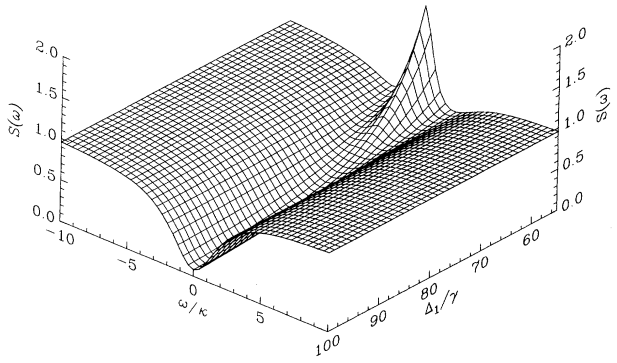

FIG. 7. The squeezing spectrum for a resonant driving field.

cy Ω' can easily be seen from (59) and (60). In the high detuning limit, the nonlinear coupling increases linearly with the scaled Rabi frequency. Therefore, so long as $\Omega' \ll \Delta'$, the squeezing can be maximized by increasing the scaled Rabi frequency.

Finally, a few comments should be made regarding the case where the driving field is resonant with the upper level. In such a situation, the spontaneous emission from the upper level will be appreciable, but this should not affect the squeezing as long as there is a large detuning from the intermediate level. This is because the spontaneous emission will add the same noise to both modes, and hence the difference in the intensities should be unaffected. The spectrum can be calculated in the same way as the previous spectra. Δ' and Δ_2' are taken to be approximately zero in the equations (24)–(27). They will be shifted slightly from zero due to the ac Stark effect and should be determined self-consistently from the conditions (50)–(52). Rather than do this, we set $\Delta'=\Delta_2'=0$ in (24)–(27) and evaluate Δ_2 using the first eigenvalue condition (50). The spectrum shown in Fig. 7 uses the same parameters as Fig. 2. It can be seen that being in resonance with the upper level does not affect the squeezing in the intensity difference that can be obtained.

V. CONCLUSION

In this paper, we have looked at a model for a three-level system that includes spontaneous emission. The three-level atoms are injected into a cavity and are driven by an external field, which is detuned from the transition between upper and lower atomic states. The cascade transitions are detuned from two cavity resonances, and it is the fluctuations in the intensity difference between these two modes which are of interest. It is found that the system behaves as an ideal parametric oscillator when on two-photon resonance and the fields are highly detuned from the upper and intermediate levels; that is, good squeezing can be obtained in the large detuning limit. It is also found that small improvements in the squeezing can be obtained by choosing the two cavity decay rates and the two coupling constants not to be equal. Finally, it is seen that the squeezing is not affected by driving the system on resonance, as the spontaneous emission adds correlated noise to both modes.

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