Phase sensitivity in two-photon optical bistability

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A two-photon optical bistability equation is derived for a three-level V-type atom in a cavity. We consider the bistable states of the purely absorptive type and investigate the influence of the phase difference of the two cavity field modes on bistability characteristics due to the nonlinear coupling between atomic transitions and fields. Very different results are found for the three cases of phase difference we considered. The bistability is found to diminish when the two modes are in phase, in contrast to the two-level case. In addition, we also discuss the effect of longitudinal and transverse relaxation times on the bistability and how the bistability operation characteristics of one mode can be controlled by adjusting the other.

I. INTRODUCTION

Optical bistability has been a subject of extensive research in recent years.[1–8] Of particular interest is the two-photon bistability phenomenon in the case of a three-level atom near the resonance[9–12] that has been observed experimentally.[1] Theoretical treatment usually employs the method of the two-photon vector model. The idea is to simplify the complicated three-level system by replacing it with an equivalent two-level system.

The degenerate two-photon optical bistability has been investigated by Zhu,[13] who has obtained an optically bistable state for a cascade three-level atom with arbitrary detunings of the middle level. On the other hand, because of the two-mode coupling, multistability phenomena may appear in a system of many-level atoms. Furthermore, the nondegenerate two-photon bistability equation has also been considered[14] for a cascade three-level atom. The asymmetry of the two-mode operation and tristability are found to be consequences of the energy-level asymmetry.

Based on the relative phase coherence of the two-mode radiation field, a mechanism for optical bistability[15] in a three-level atom of the V or Λ type has been proposed. On the other hand, it has been noted that[16] the nonlinear susceptibility can cause an intensity-dependent phase shift between the pump and the signal modes in two-photon devices and that this effect can be taken into account by means of an additional term in the interaction Hamiltonian. In the interaction of a three-level atom with the multimode cavity field, the presence of two-mode two-photon processes and higher-order transitions enhances the field-atom coupling strength. Such processes due to the cooperation effect among the atoms usually show remarkable nonlinearity as has been noted in two-photon lasers of three-level atomic systems with homogeneous[17] and inhomogeneous[18] broadening.

Since the phase difference between the two modes of the cavity field has important influence on multiphoton processes due to the atom-field nonlinear coupling, and since the higher-order nonlinear coupling has a strong effect on optical multistability, it is therefore of interest to investigate the significance of the change of the two-mode phase difference to the optical multistability. We consider in this article the two-mode two-photon optical bistability for a system of V-type three-level atoms in an optical cavity and study the higher-order effect of the atom-field nonlinear coupling on the bistability due to the relative phase change between the two-mode cavity fields. Characteristics of the purely absorptive bistability for various relative phases are discussed. The intensities of the two-cavity output modes are given as functions of the intensity of one incident mode, while the intensity of the other mode is fixed. The possibility of controlling the switching character of one mode by the other is also examined.

II. THEORY

Consider a system of N three-level atoms with V-type level configuration as shown in Fig. 1. The atoms are contained in an optical cavity with two modes of cavity fields. When the system is irradiated by two incident modes of pumping field and is interacting with the cavity fields, the total Hamiltonian can be written as

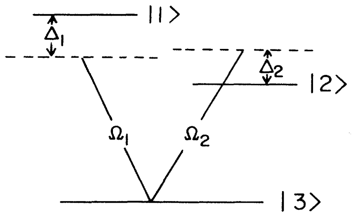

FIG. 1. Schematic diagram of a three-level V-type atom coupled with two-mode cavity fields of frequencies Ω_1 and Ω_2.

$$H=H_0+H_1+H_2 \ , \tag{1}$$

where the first term represents the total energy of noninteracting atoms and cavity fields

$$H_0=\sum_{n=1}^{N}\sum_{i=1}^{3}\hbar\omega_i A_{in}^\dagger A_{in}+\hbar\Omega_1 a_1^\dagger a_1+\hbar\Omega_2 a_2^\dagger a_2 \ . \tag{2a}$$

The second term gives the energy of interaction between the atoms and the cavity fields

$$H_1=\sum_{n=1}^{N}\hbar(g_1^* a_1^\dagger A_{3n}^\dagger A_{1n}+g_2^* a_2^\dagger A_{3n}^\dagger A_{2n}+\text{H.c.}) \ , \tag{2b}$$

and the third term describes the coupling between pumping and cavity fields

$$\begin{aligned}H_2&=i\hbar\kappa_1[a_1^\dagger\epsilon_1(t)-a_1\epsilon_1^*(t)]\\&\quad+i\hbar\kappa_2[a_2^\dagger\epsilon_2(t)-a_2\epsilon_2^*(t)] \ .\end{aligned} \tag{2c}$$

The notation is as follows. The operator $A_{ni}^\dagger$ (A_{ni}) creates (annihilates) the nth atom in the level i with energy $\hbar\omega_i$ and the operator $a_{1,2}^\dagger$ ($a_{1,2}$) creates (annihilates) a photon of the cavity field mode 1,2 of frequency $\Omega_{1,2}$. $g_{1,2}$ stands for the coupling constant in the interaction of the atom with the cavity field mode 1,2, and $\kappa_{1,2}$ are introduced just for convenience. The pumping fields of amplitudes $E_{1,2}$ and frequencies $\nu_{1,2}$ are given by $\epsilon_i(t)=E_i\exp(i\nu_i t)$, $i=1,2$.

The density operator ρ for the atom-field system satisfies the equation

$$\frac{\partial\rho}{\partial t}=-\frac{i}{\hbar}[H,\rho] \ . \tag{3}$$

To obtain the optical Bloch equations for field operators and the atomic density matrix elements, we replace the operators by their corresponding mean values by neglecting all the quantum fluctuations as usual. Thus,

$$\begin{aligned}\dot\alpha&=\text{Tr}(\dot\rho a_1)\\&=-[\kappa_1+i(\Omega_1-\nu_1)]\alpha-iNg_1\rho_{13}+\kappa_1E_1 \ ,\end{aligned} \tag{4a}$$

$$\begin{aligned}\dot\beta&=\text{Tr}(\dot\rho a_2)\\&=-[\kappa_2+i(\Omega_2-\nu_2)]\beta-iNg_2\rho_{23}+\kappa_2E_2 \ ,\end{aligned} \tag{4b}$$

$$\dot\rho_{12}=-\left[\frac{1}{T_1}+i(\Delta_1-\Delta_2)\right]\rho_{12}-ig_2^*\beta\rho_{32}+ig_1\alpha^*\rho_{13} \ , \tag{5a}$$

$$\dot\rho_{13}=-\left[\frac{1}{T_2}+i\Delta_1\right]\rho_{13}-ig_1^*\alpha(\rho_{33}-\rho_{11})+ig_2^*\beta\rho_{12} \ , \tag{5b}$$

$$\dot\rho_{23}=-\left[\frac{1}{T_2}+i\Delta_2\right]\rho_{23}-ig_2^*\beta(\rho_{33}-\rho_{22})+ig_1^*\alpha\rho_{21} \ , \tag{5c}$$

$$\dot\rho_{11}=i(g_1\alpha^*\rho_{13}-g_1^*\alpha\rho_{31})-\frac{1}{T_1}\rho_{11} \ , \tag{5d}$$

$$\dot\rho_{22}=i(g_2\beta^*\rho_{23}-g_2^*\beta\rho_{32})-\frac{1}{T_1}\rho_{22} \ , \tag{5e}$$

$$\begin{aligned}\dot\rho_{33}&=-i(g_1\alpha^*\rho_{13}-g_1^*\alpha\rho_{31})\\&\quad-i(g_2\beta^*\rho_{23}-g_2^*\beta\rho_{32})+\frac{1}{T_1}\rho_{33} \ ,\end{aligned} \tag{5f}$$

where we have defined the longitudinal and transverse relaxation times T_1 and T_2 of level 3, and the detuning parameters

$$\Delta_1=\Omega_1-(\omega_1-\omega_3) \ , \tag{6a}$$

$$\Delta_2=\Omega_2-(\omega_2-\omega_3) \ , \tag{6b}$$

for the cavity modes 1,2 from atomic resonance frequencies, respectively.

III. EQUATIONS OF BISTABILITY

In steady states, we set the left-hand side of Eqs. (4) to zero and find the amplitudes of the pumping fields

$$E_1=[i(\Omega_1-\nu_1)/\kappa_1+1]\alpha+iNg_1\rho_{13}/\kappa_1 \ , \tag{7a}$$

$$E_2=[i(\Omega_2-\nu_2)/\kappa_2+1]\beta+iNg_2\rho_{23}/\kappa_2 \ . \tag{7b}$$

The matrix elements of the density operator satisfy a set of coupled equations that follow from Eqs. (5) by setting the time derivatives to zero,

$$\rho_{12}=(g_1\alpha^*\rho_{13}-g_2^*\beta\rho_{32})/\Theta_1=\rho_{21}^* \ , \tag{8a}$$

$$\rho_{23}=[g_1^*\alpha\rho_{21}-g_2^*\beta(\rho_{33}-\rho_{22})]/\Theta_3=\rho_{32}^* \ , \tag{8b}$$

$$\rho_{13}=[g_2^*\beta\rho_{12}-g_1^*\alpha(\rho_{33}-\rho_{11})]/\Theta_2=\rho_{31}^* \ , \tag{8c}$$

$$\rho_{11}=iT_1(g_1\alpha^*\rho_{13}-g_1^*\alpha\rho_{31}) \ , \tag{9a}$$

$$\rho_{22}=iT_1(g_2\beta^*\rho_{23}-g_2^*\beta\rho_{32}) \ , \tag{9b}$$

$$\rho_{33}=iT_1(g_1\alpha^*\rho_{13}-g_1^*\alpha\rho_{31})+iT_2(g_2\beta^*\rho_{23}-g_2^*\beta\rho_{32}), \tag{9c}$$

where we have defined

$$\Theta_1=\Delta_1-\Delta_2-i/T_1 \ , \tag{10a}$$

$$\Theta_2=\Delta_1-i/T_2 \ , \tag{10b}$$

$$\Theta_3=\Delta_2-i/T_2 \ . \tag{10c}$$

After some algebraic manipulation, we can eliminate the off-diagonal matrix elements and obtain the set of coupled equations for the three diagonal matrix elements

$$(1+iT_1A_1)\rho_{11}+iT_1B_1\rho_{22}+iT_1C_1\rho_{33}=0\ , \tag{11a}$$

$$iT_1A_2\rho_{11}+(1+iT_2B_2)\rho_{22}+iT_1C_2\rho_{33}=0\ , \tag{11b}$$

$$\rho_{11}+\rho_{22}+\rho_{33}=1\ , \tag{11c}$$

where

$$A_1=-\frac{2i|g_1|^2|\alpha|^2}{T_2|\Theta_2|^2}-\frac{|g_1|^2|\alpha|^2}{|\Theta_2|^2}\left[\frac{\Theta_2^*}{\Theta_1\Theta_2}g_1\alpha^*g_2^*\beta-\frac{\Theta_2}{\Theta_1^*\Theta_2^*}g_1^*\alpha g_2\beta^*\right]\ , \tag{12a}$$

$$B_1=\frac{1}{|\Theta_2|^2}|g_2|^2|\beta|^2\left[\frac{\Theta_2^*}{\Theta_1\Theta_3^*}g_1\alpha^*g_2^*\beta-\frac{\Theta_2}{\Theta_1^*\Theta_3}g_1^*\alpha g_2\beta^*\right]\ , \tag{12b}$$

$$C_1=-(A_1+B_1)\ , \tag{12c}$$

$$A_2=\frac{1}{|\Theta_3|^2}|g_1|^2|\alpha|^2\left[\frac{\Theta_3^*}{\Theta_1\Theta_2^*}g_1^*\alpha g_2\beta^*-\frac{\Theta_3}{\Theta_1^*\Theta_2}g_1\alpha^*g_2^*\beta\right]\ , \tag{13a}$$

$$B_2=-\frac{2i|g_2|^2|\beta|^2}{T_2|\Theta_3|^2}-\frac{|g_2|^2|\beta|^2}{|\Theta_3|^2}\left[\frac{\Theta_3^*}{\Theta_1\Theta_3}g_1^*\alpha g_2\beta^*-\frac{\Theta_3}{\Theta_1^*\Theta_3^*}g_1\alpha^*g_2^*\beta\right]\ , \tag{13b}$$

$$C_2=-(A_2+B_2)\ . \tag{13c}$$

It is noted that in deriving Eqs. (13), we have included nonlinear terms of the atomic dipole moment coupled with cavity field modes. In other words, two-photon processes of Raman-type transition[19] as well as four-photon processes are included in our calculation as can be seen from terms involving two or four coupling constants. These fourth-order transitions are ordinarily ignored in the literature. They can change the strength of the nonlinear coupling and cause phase sensitivity in the optical bistability.

While the coupling between the two-mode cavity field and the atomic dipole moment is highly nonlinear, the set of equations (11) is linear and can be easily solved. The solutions are

$$\rho_{11}=-[T_1^2B_1C_2+i(1+iT_1B_2)T_1C_1]/D\ , \tag{14a}$$

$$\rho_{22}=-[T_1^2A_2C_1-iT_1C_2(1+iT_1A_1)]/D\ , \tag{14b}$$

$$\rho_{33}=[(1+iT_1A_1)(1+iT_1B_2)+T_1^2A_2B_1]/D\ , \tag{14c}$$

where D is the determinant of coefficients in Eq. (11) and can be found after a tedious but straightforward calculation. The result is too long to be reproduced here.

From Eqs. (12)–(14), we find

$$\begin{aligned}\rho_{33}-\rho_{11}&=[1+iT_1(B_2-B_1)]/D\\&=[1+\mu_1+\mu_2\cos(\theta_1-\theta_2)\\&\quad+\mu_3\sin(\theta_1-\theta_2)]/D=\frac{f_1(\mu)}{D}\ ,\end{aligned} \tag{15a}$$

$$\begin{aligned}\rho_{33}-\rho_{22}&=[1+iT_1(A_1-A_2)]/D\\&=[1+\nu_1+\nu_2\cos(\theta_1-\theta_2)\\&\quad+\nu_3\sin(\theta_1-\theta_2)]/D=\frac{f_2(\nu)}{D}\ ,\end{aligned} \tag{15b}$$

where the parameters μ_i and ν_i are given by

$$\mu_1=(2T_1/T_2)|g_2|^2|\beta|^2/|\Theta_3|^2\ , \tag{16a}$$

$$\mu_2=\frac{|g_2|^2|\beta|^2}{|\Theta_1|^2|\Theta_3|^2}|g_1g_2\alpha\beta|\left[\frac{1}{|\Theta_3|^2}\left[\frac{4\Delta_2(\Delta_1-\Delta_2)}{T_2}-\frac{2(\Delta_2^2-1/T_2^2)}{T_1}\right]+\frac{1}{|\Theta_2|^2}\left[\frac{2\Delta_1\Delta_2}{T_1}-\frac{2\Delta_1(\Delta_1-\Delta_2)}{T_2}+\frac{2\Delta_2(\Delta_1-\Delta_2)}{T_2}+\frac{2}{T_1T_2^2}\right]\right]\ , \tag{16b}$$

$$\mu_3=\frac{|g_2|^2|\beta|^2}{|\Theta_1|^2|\Theta_3|^2}|g_1g_2\alpha\beta|\left[\frac{1}{|\Theta_3|^2}\left[2(\Delta_1-\Delta_2)(\Delta_2^2-1/T_2^2)-\frac{4\Delta_1}{T_1T_2}\right]-\frac{1}{|\Theta_2|^2}\left[2\Delta_1\Delta_2(\Delta_1-\Delta_2)+\frac{2\Delta_2}{T_1T_2}+\frac{2(\Delta_1-\Delta_2)}{T_2^2}-\frac{2\Delta_2}{T_1T_2}\right]\right]\ , \tag{16c}$$

and ν_i are obtained from corresponding μ_i by making the replacements $g_1\to g_2$, $g_2\to g_1$, $\alpha\to\beta$, $\beta\to\alpha$, $\Delta_1\to\Delta_2$, $\Delta_2\to\Delta_1$, $H_1\to H_2$, and $H_2\to H_1$. The functions $f_1(\mu)$ and $f_2(\nu)$ are defined just for convenience. We note here that the phases of the cavity field modes 1 and 2 are explicitly included in Eqs. (15). They are introduced by

$$g_1\alpha=|g_1\alpha|e^{i\theta_1}\ , \tag{17a}$$

$$g_2\beta=|g_2\beta|e^{i\theta_2}\ . \tag{17b}$$

To find the amplitudes of the pumping fields, we need the off-diagonal density matrix elements ρ_{13} and ρ_{23}. It can readily be shown from Eqs. (8), however, that the first terms in (8b) and (8c) involve only higher-order coupling. Since the coupling constants are much smaller than unity, we can neglect the first term for all practical cases. Thus the pumping fields can be obtained by substituting Eqs. (15) in (8b) and (8c), which are then plugged in (7). That is,

$$E_1=\alpha\left[\left\{+\frac{N|g_1|^2}{\kappa_1 T_2|\Theta_2|^2}\frac{f_1(\mu)}{D}\right\}+i\left\{\frac{\Omega_1-\nu_1}{\kappa_1}-\frac{N\Delta_1|g_1|^2}{\kappa_1 D|\Theta_2|^2}f_1(\mu)\right\}\right], \tag{18a}$$

$$E_2=\beta\left[\left\{1+\frac{N|g_2|^2}{\kappa_2 T_2|\Theta_1|^2}\frac{f_2(\nu)}{D}\right\}+i\left\{\frac{\Omega_2-\nu_2}{\kappa_2}-\frac{N\Delta_1|g_2|^2}{\kappa_2 D|\Theta_1|^2}f_2(\nu)\right\}\right]. \tag{18b}$$

The multistability equations for a two-photon two-beam system then follow directly by taking the absolute square of (18). More explicitly, we have

$$|E_1|^2=|\alpha|^2\left[\left\{1+\frac{N|g_1|^2 f_1(\mu)}{\kappa_1 T_2 D|\Theta_2|^2}\right\}^2+\left\{\frac{\Omega_1-\nu_1}{\kappa_1}-\frac{N\Delta_1|g_1|^2}{\kappa_1 D|\Theta_2|^2}f_1(\mu)\right\}^2\right], \tag{19a}$$

$$|E_2|^2=|\beta|^2\left[\left\{1+\frac{N|g_2|^2 f_2(\nu)}{\kappa_2 T_2 D|\Theta_1|^2}\right\}^2+\left\{\frac{\Omega_2-\nu_2}{\kappa_2}-\frac{N\Delta_1|g_2|^2}{\kappa_2 D|\Theta_1|^2}f_2(\nu)\right\}^2\right] \tag{19b}$$

in which the effects on higher-order atomic transitions due to the relative phase of the cavity fields are already included. Such nonlinear interactions between the cavity field modes and multiatomic transitions have significant influence on the optical multistabilities as we shall see but have usually been ignored in the literature. It is observed from Eqs. (19) that the characteristic equations are symmetric with respect to the interchange of modes 1 and 2. A similar situation is found in the coherent Λ-type laser problem.[20] There is, of course, no such symmetry if the V-type atom is replaced by a Ξ-type atom.

IV. PURELY ABSORPTIVE OPTICAL BISTABILITY

In the case of pure absorption, we have $\Delta_1=\Delta_2=0$, $\Omega_1=\nu_1$, and $\Omega_2=\nu_2$. To simplify the notation, we introduce the dimensionless quantities

$$s_1=2|g_1\alpha|\sqrt{T_1T_2},\quad s_2=2|g_2\beta|\sqrt{T_1T_2}\ ,$$

$$r_1=2|g_1E_1|\sqrt{T_1T_2},\quad r_2=2|g_2E_2|\sqrt{T_1T_2}\ .$$

The intensities of the input laser field and the cavity field are then given by

$$I_{\rm in}^{(i)}=r_1^2,\quad I_i=s_i^2,\quad i=1,2\ . \tag{20}$$

Equations (19) then become

$$I_{\rm in}^{(i)}=I_i(1+2c_if_i/D_0)^2,\quad i=1,2\ , \tag{21}$$

where

$$c_i=N|g_i|^2T_2/2\kappa_i\ , \tag{22a}$$

$$D_0=1+\tfrac{3}{4}I_1I_2-\tfrac{3}{8}(I_1I_2)^{3/2}\cos(\theta_1-\theta_2)-\tfrac{3}{64}(I_1I_2)^2+(I_1+I_2)[1-\tfrac{1}{4}\cos(\theta_1-\theta_2)I_1I_2(1+T_2/T_1)]\ , \tag{22b}$$

$$f_{1,2}=1+\tfrac{1}{2}I_{2,1}\ . \tag{22c}$$

Substituting Eqs. (22) into (21) and making use of (20), we can rewrite explicitly the bistability equations in terms of the amplitudes. Thus,

$$\frac{r_1}{s_1}-1=2c_1(1+\tfrac{1}{2}s_2^2)\{1+\tfrac{3}{4}s_1^2s_2^2-\tfrac{3}{8}(s_1s_2)^3\cos(\theta_1-\theta_2)-\tfrac{3}{64}(s_1s_2)^4+(s_1^2+s_2^2)[1-\tfrac{1}{4}s_1^2s_2^2(1+T_2/T_1)\cos(\theta_1-\theta_2)]\}^{-1/2}\ , \tag{23a}$$

$$\frac{r_2}{s_2}-1=2c_2(1+\tfrac{1}{2}s_1^2)\{1+\tfrac{3}{4}s_1^2s_2^2-\tfrac{3}{8}(s_1s_2)^3\cos(\theta_1-\theta_2)-\tfrac{3}{64}(s_1s_2)^4+(s_1^2+s_2^2)[1-\tfrac{1}{4}s_1^2s_2^2(1+T_1/T_2)\cos(\theta_1-\theta_2)]\}^{-1/2}\ . \tag{23b}$$

These two equations, as expected, show complete symmetry with respect to the interchange of modes 1 and 2. It is also observed that the bistable operation of one mode depends on the intensity of the other through the factor $(1+\frac{1}{2}s_i^2)$. In what follows, we consider the characteristics of bistable operation for three different values of the relative phase.

(A) $\theta_1-\theta_2=\pi/2$. Equations (23) reduce, in this case, to

$$\frac{r_1}{s_1}-1=2c_1(1+\tfrac{1}{2}s_2^2)[1+\tfrac{3}{4}(s_1s_2)^2-\tfrac{3}{64}(s_1s_2)^4+s_1^2+s_s^2]^{-1/2}\ , \tag{24a}$$

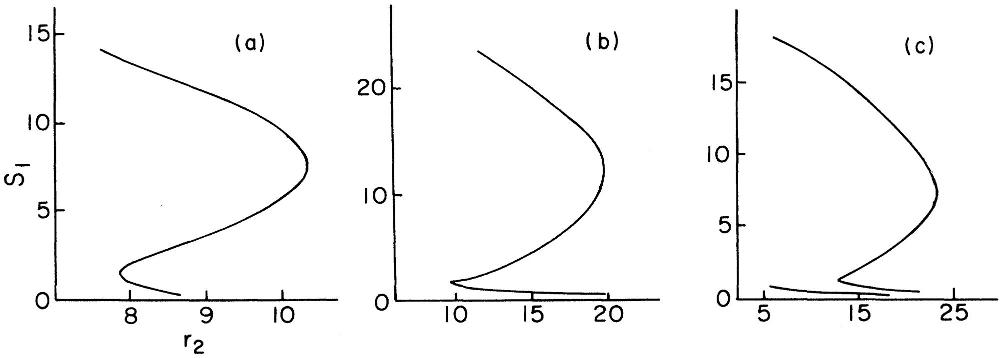

FIG. 2. Transmission curves of mode 1 vs the incident intensity of mode 2 for a fixed incident intensity of mode 1 and $\theta_1-\theta_2=\pi/2$. (a) $c_1=18$, $c_2=6$, $r_1=20$; (b) $c_1=18$, $c_2=6$, $r_1=30$; (c) $c_1=8$, $c_2=6$, $r_1=20$.

$$\frac{r_2}{s_2}-1=2c_2(1+\tfrac{1}{2}s_1^2)[1+\tfrac{3}{4}(s_1s_2)^2-\tfrac{3}{64}(s_1s_2)^4+s_1^2+s_s^2]^{-1/2} . \tag{24b}$$

It is seen that the intensities do not depend on the ratio T_2/T_1, a particular property of this case. In principle, each s_i has five roots. However, not all of them are physically meaningful. When one of the pumping fields, say, mode 1 has a fixed intensity, s_1 and s_2 are calculated as functions of r_2. Because of symmetry, the situation is the same if r_2 is fixed and r_1 varies. Numerical results are plotted in Figs. 2 and 3. We first note that s_1 shows much more remarkable bistability than s_2 when r_1 is fixed. A more careful examination of Fig. 2 reveals that the bistability operation of cavity mode 1 can be controlled by adjusting r_2 via the nonlinear interaction between cavity mode 2 and the coherent atomic transition. For the same r_1 and c_2, the range of bistability operation decreases with increasing c_1 as is seen from Figs. 2(a) and 2(c). However, the range may be restored as in Fig. 2(b) if r_1 is increased at the same time as c_1. On the other hand, the bistability range of s_2 remains small for reasonable choices of the parameters according to our numerical study. The situation is depicted in Figs. 3. Reasons for this may be understood as we consider other cases in the following.

(B) $\theta_1-\theta_2=0$. When the two cavity modes are in phase, Eqs. (24) become

$$\frac{r_1}{s_1}-1=2c_1(1+\tfrac{1}{2}s_2^2)\times\{1+\tfrac{3}{4}(s_1s_2)^2-\tfrac{3}{8}(s_1s_2)^3-\tfrac{3}{64}(s_1s_2)^4+(s_1^2+s_2^2)[1-\tfrac{1}{4}(s_1s_2)^2\times(1+T_2/T_1)]\}^{-1/2} , \tag{25a}$$

$$\frac{r_2}{s_2}-1=2c_2(1+\tfrac{1}{2}s_1^2)\times\{1+\tfrac{3}{4}(s_1s_2)^2-\tfrac{3}{8}(s_1s_2)^3-\tfrac{3}{64}(s_1s_2)^4+(s_1^2+s_2^2)[1-\tfrac{1}{4}(s_1s_2)^2\times(1+T_1/T_2)]\}^{-1/2} . \tag{25b}$$

Results for fixed r_1 are shown in Figs. 4 and 5. It is clear that the bistability is no longer as remarkable as in case

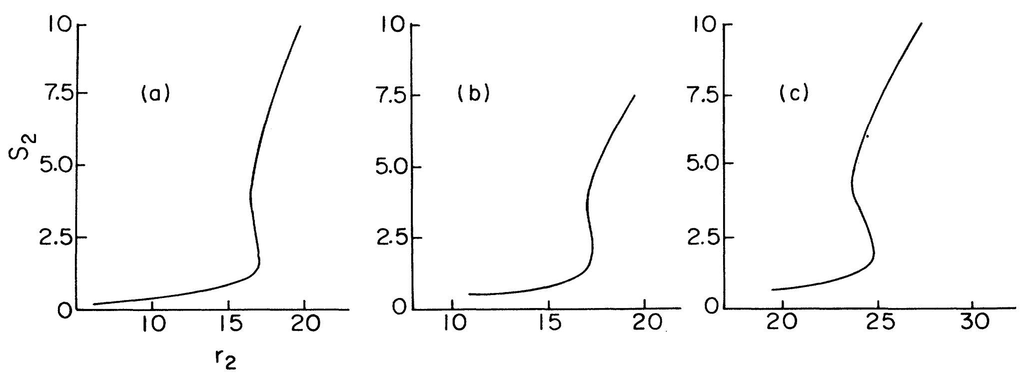

FIG. 3. Transmission curves of mode 2 vs the incident intensity of mode 2 for a fixed incident intensity of mode 1 and $\theta_1-\theta_2=\pi/2$. (a) $c_1=8$, $c_2=20$, $r_1=30$; (b) $c_1=8$, $c_2=20$, $r_1=20$; (c) $c_1=12$, $c_2=30$, $r_1=30$.

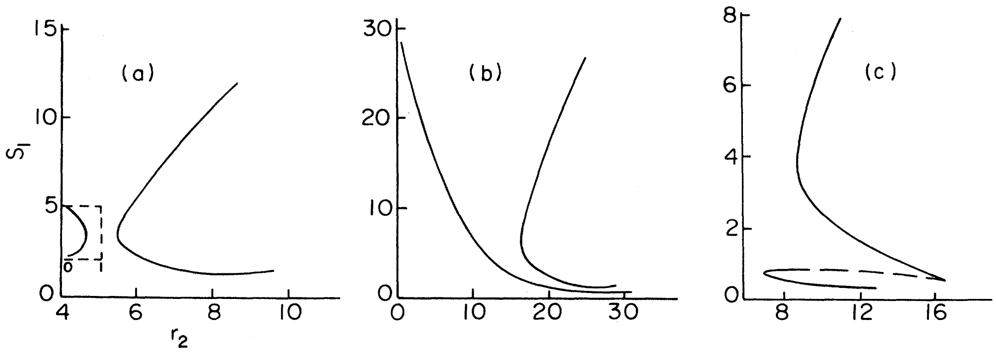

FIG. 4. Transmission curves of mode 1 vs the incident intensity of mode 2 for a fixed incident intensity of mode 1 and $\theta_1-\theta_2=0$. (a) $c_1=8$, $c_2=6$, $T_2/T_1=3$, $r_1=8$; (b) $c_1=8$, $c_2=6$, $T_2/T_1=3$, $r_1=30$; (c) $c_1=20$, $c_2=6$, $T_2/T_1=3$, $r_1=8$.

(A). This is especially true for s_2 when r_1 is fixed as can be seen in Fig. 5. The situation is similar to what has been pointed out in general laser theory. When the two modes of the cavity field are in phase, the coupling between higher-order transitions and the field enhances the mode coupling in spite of the value of c_i, which is related to the cooperation parameters. Such enhanced coupling of cavity modes destroys the bistability which may occur in the two-level case. This is especially so when $c_1 \approx c_2$, the atom-field coupling for one mode helps the coupling for the other mode to saturate. Figure 5 shows that the cavity field mode 2 increases with the pumping field mode 2 and loses bistability character, because the interaction of atomic transitions with the cavity mode 2 saturates as r_2 increases.

(C) $\theta_1=\theta_2=\pi$. When the cavity field modes are out of phase, we have

$$\frac{r_1}{s_1}-1=2c_1(1+\tfrac{1}{2}s_2^2)\times\{1+\tfrac{3}{4}(s_1s_2)^2+\tfrac{3}{8}(s_1s_2)^3-\tfrac{3}{64}(s_1s_2)^4+(s_1^2+s_2^2)[1+\tfrac{1}{4}(s_1s_2)^2\times(1+T_2/T_1)]\}^{-1/2}\,, \tag{26a}$$

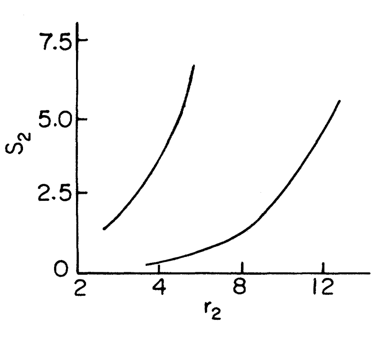

FIG. 5. Transmission curves of mode 2 vs the incident intensity of mode 2 for a fixed incident intensity of mode 1 and $\theta_1-\theta_2=0$; $c_1=8$, $c_2=6$, $T_2/T_1=1.25$, $r_1=8$.

$$\frac{r_2}{s_2}-1=2c_2(1+\tfrac{1}{2}s_1^2)\times\{1+\tfrac{3}{4}(s_1s_2)^2+\tfrac{3}{8}(s_1s_2)^3-\tfrac{3}{64}(s_1s_3)^4+(s_1^2+s_2^2)[1+\tfrac{1}{4}(s_1s_2)^2\times(1+T_1/T_2)]\}^{-1/2}\,. \tag{26b}$$

Results are plotted in Figs. 6. For both s_1 and s_2, the bistability character is clearly more noticable than the above two cases. When the cooperation parameters are close to one another as in Fig. 6(b), multistability occurs as expected. Even though the lower branch has only a relatively small range of variation and is separated from the upper branch by a small gap so that it may not be very meaningful in practice, nevertheless, it indicates the coupling of multitransition including the $|1\rangle \rightarrow |2\rangle$ transition with one mode of cavity fields causes negative feedback to the other mode. In Fig. 6(a), c_1 and c_2 are very different from one another while other parameters are similar to those in Fig. 6(b). The condition is apparently more favorable for mode 1 to operate in a tristable state than the other cases where $c_1 \approx c_2$. A similar situation is observed for s_2 versus r_2 plot as shown in Fig. 7. They all indicate that the feedback effect is enhanced by the coupling of mode 1 with the coherent transition. From the symmetry, we conclude directly that same results are expected for the variation of s_1 and s_2 with r_1 when r_2 is given.

V. ANALYSIS OF STABILITY

The stability of all steady-state solutions must be examined. This can be done by a standard linearized theory.[10] For simplicity, we discuss here only the $\Delta\theta=\pi/2$ case as an illustration. Since terms involving T_1 and T_2 do not appear in this case, it may be considered as a high-Q limit of more general cases in which the atomic-decay time is much shorter than the cavity-decay time. Other effects such as dispersion are neglected, and we are left only with variables r_i and s_i, $i=1,2$. The equations that govern the stability are then

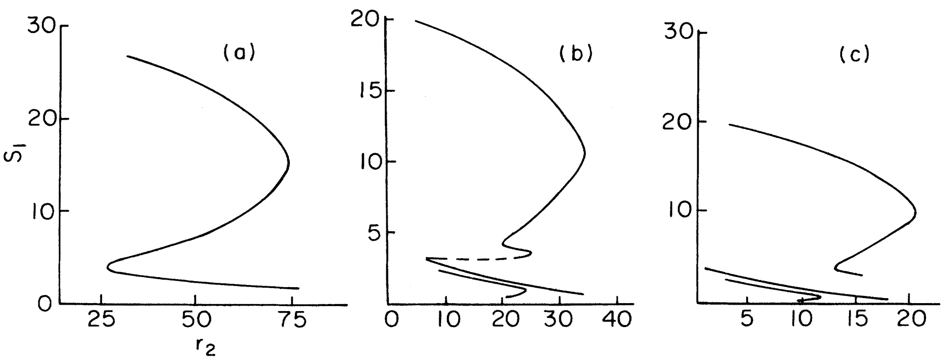

FIG. 6. Transmission curves of mode 1 vs the incident intensity of mode 2 for a fixed incident intensity of mode 1 and $\theta_1-\theta_2=\pi$. (a) $c_1=12$, $c_2=40$, $T_2/T_1=2$, $r_1=30$; (b) $c_1=30$, $c_2=40$, $T_2/T_1=2$, $r_1=20$; (c) $c_1=30$, $c_2=20$, $T_2/T_1=1.5$, $r_1=20$.

$$\frac{ds_i}{dt}=r_i-s_i\left[1+\frac{2c_i(1+\frac{1}{2}s_i^2)}{1+\frac{3}{4}s_1^2s_2^2-\frac{3}{64}(s_1s_2)^4+s_1^2+s_2^2}\right]. \tag{27}$$

The steady-state solutions are given by Eqs. (24). Assume a small perturbation s_i around the steady-state value $s_i^{(o)}$ and substitute in Eq. (27), we find after linearizing the resulting equation and neglecting higher-order terms, that

$$\begin{bmatrix}\dfrac{ds_1}{dt}\\ \dfrac{ds_2}{dt}\end{bmatrix}=-\begin{bmatrix}F_{11} & F_{12}\\ F_{21} & F_{22}\end{bmatrix}\begin{bmatrix}s_1\\ s_2\end{bmatrix}, \tag{28}$$

where we have defined

$$F_{ij}=\left[\frac{\partial r_i}{\partial s_j}\right]_{s_j=s_j^{(o)}}. \tag{29}$$

The steady-state solution will be stable if the matrix F has eigenvalues with positive real parts. Thus the necessary and sufficient conditions for stability are

$$\mathrm{Tr}F>0 \quad\text{and}\quad \det F>0\ , \tag{30}$$

which, for Eqs. (24), take the forms

$$A^2>[c_1(1+\tfrac{1}{2}I_2)(I_1-I_2+\tfrac{3}{4}I_1I_2-\tfrac{9}{64}I_1^2I_2^2-1)+c_2(1+\tfrac{1}{2}I_1)(I_2-I_1+\tfrac{3}{4}I_1I_2-\tfrac{9}{64}I_2^2I_1^2-1)]\ , \tag{31}$$

$$\begin{aligned}&\left[1+\frac{2c_1}{A^2}(1+\tfrac{1}{2}I_2)(1-\tfrac{3}{4}I_1I_2+\tfrac{9}{64}I_1^2I_2^2-I_1+I_2)\right]\left[1+\frac{2c_2}{A^2}(1+\tfrac{1}{2}I_1)(1-\tfrac{3}{4}I_1I_2+\tfrac{9}{64}I_1^2I_2^2+I_1-I_2)\right]\\ &\quad-\frac{4I_1I_2}{A^4}[(1+c_1)+\tfrac{1}{2}I_1(\tfrac{3}{2}+c_1)-\tfrac{3}{32}I_1^2I_2(1+2c_1)-\tfrac{3}{64}I_1^2I_2^2]\\ &\quad\times[(1+c_2)+\tfrac{1}{2}I_2(\tfrac{3}{2}+c_2)-\tfrac{3}{32}I_1I_2^2(1+2c_2)-\tfrac{3}{64}I_1^2I_2^2]>0\ ,\end{aligned} \tag{32}$$

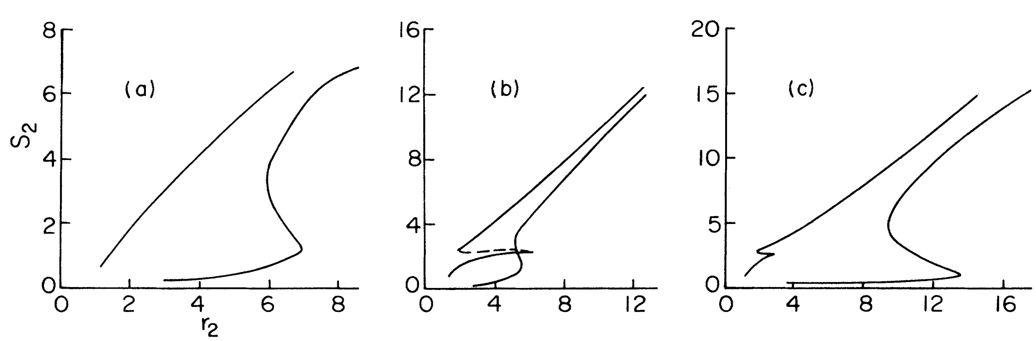

FIG. 7. Transmission curves of mode 2 vs the incident intensity of mode 2 for a fixed incident intensity of mode 1 and $\theta_1-\theta_2=\pi$. (a) $c_1=60$, $c_2=6$, $T_2/T_1=1.5$, $r_1=20$; (b) $c_1=8$, $c_2=6$, $T_2/T_1=1.25$, $r_1=8$; (c) $c_1=18$, $c_2=20$, $T_2/T_1=1.5$, $r_1=20$.

where $A=1+\frac{3}{4}I_1I_2-\frac{3}{64}I_1^2I_2^2+I_1+I_2$ and $I_i=(s_1^{(o)})^2$. All the solutions represented in Figs. 2 and 3 are stable solutions of Eqs. (24) as tested by (31). It is noted that as TrF or det$F$ approach zero, a critical slowing down[21] will occur. When $\Delta\theta=0$ or π, we are working with Eqs. (25) and (26) which involve T_i terms. The stability conditions become complicated and will not be discussed explicitly here.

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