Nonclassical properties of states generated by the excitations on a coherent state

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We introduce states defined by $|\alpha,m\rangle = a^{\dagger m}|\alpha\rangle$ up to a normalization constant, where $|\alpha\rangle$ is a coherent state and m an integer. We study the mathematical and physical properties of such states. We demonstrate phase squeezing and the sub-Poissonian character of the fields in such states. We study in detail the quasiprobability distributions and the distribution of the field quadrature. We also show how such states can be produced in nonlinear processes in cavities.

I. INTRODUCTION

It is well known that the radiation field in a coherent state $|\alpha\rangle$ has properties like the classical field.[1] The number fluctuations $(\Delta N/N)$ are of the order $1/\sqrt{N}$. On the other hand, the field in a Fock state $|m\rangle$ is strictly a quantum-mechanical field with no classical analog. In this paper we study properties of a state that is intermediate between the Fock state and the coherent state. We consider the state obtained by repeated application of the photon creation operator on the coherent state. Such a state has a nonzero-field amplitude and is shown to exhibit nonclassical properties like the squeezing in one of the quadratures of the field, and sub-Poissonian photon statistics. We calculate different quasiprobability functions for fields in such states. We show that the Glauber-Sudarshan P function[1,2] is singular. We also calculate the distribution function for one of the field quadratures. We study in detail the Wigner function for such states. Finally, we discuss how such states can be generated in nonlinear processes in cavities.

II. THE STATE $|\alpha,m\rangle$ AND ITS NONCLASSICAL PROPERTIES

We introduce the state $|\alpha,m\rangle$ defined by

$$|\alpha,m\rangle = \frac{a^{\dagger m}|\alpha\rangle}{(\langle\alpha|a^m a^{\dagger m}|\alpha\rangle)^{1/2}} , \tag{2.1}$$

where $|\alpha\rangle$ is a coherent state and m is an integer. In the limit $\alpha \to 0$ $(m \to 0)$ the state $|\alpha,m\rangle$ reduces the Fock state (coherent state). Thus, it is a state intermediate between the Fock state and the coherent state, and we may call such states as "photon-added coherent states." Note that the state $|\alpha,m\rangle$ is not the same as the state

$$D(\alpha)|m\rangle = \exp(a^\dagger\alpha - a\alpha^*)|m\rangle \tag{2.2}$$

associated with the displaced harmonic oscillator.[3-5] This is because the operators $D(\alpha)$ and $a^{\dagger m}$ do not commute.

The normalization constant for the state $|\alpha,m\rangle$ can be obtained by using normal ordering of the operator $a^m a^{\dagger m}$. This leads to

$$\langle\alpha|a^m a^{\dagger m}|\alpha\rangle = \sum_{p=0}^{m} \frac{(m!)^2}{[(m-p)!]^2 p!}|\alpha|^{2(m-p)} = L_m(-|\alpha|^2)m! , \tag{2.3}$$

where $L_m(x)$ is the Laguerre polynomial of order m defined by[6]

$$L_m(x) = \sum_{n=0}^{m} \frac{(-1)^n x^n m!}{(n!)^2(m-n)!} . \tag{2.4}$$

Thus, the state $|\alpha,m\rangle$ becomes

$$|\alpha,m\rangle = \frac{a^{\dagger m}|\alpha\rangle}{[m!L_m(-|\alpha|^2)]^{1/2}} . \tag{2.5}$$

The state $|\alpha,m\rangle$ in terms of Fock states can be written as

$$|\alpha,m\rangle = \frac{\exp(-|\alpha|^2/2)}{[L_m(-|\alpha|^2)m!]^{1/2}} \times \sum_{n=0}^{\infty} \frac{\alpha^n\sqrt{(n+m)!}}{n!}|n+m\rangle . \tag{2.6}$$

Thus, the states $|\alpha,m\rangle$ amount to a truncation of coherent states, i.e., all the Fock states $|0\rangle, |1\rangle, |2\rangle, \ldots, |m-1\rangle$ are removed in a particular fashion. The expansion also leads to the following results for the scalar products:

$$\langle\alpha,m|\alpha,n\rangle = \frac{\exp(-|\alpha|^2)}{[m!L_m(-|\alpha|^2)n!L_n(-|\alpha|^2)]^{1/2}} \times \sum_{p=0}^{\infty} \frac{|\alpha|^{2p}(m+p)!\alpha^{m-n}}{p!(m+p-n)!} , \tag{2.7}$$

$$\langle\beta,m|\alpha,m\rangle = \frac{L_m(-\beta^*\alpha)}{[L_m(-|\beta|^2)L_m(-|\alpha|^2)]^{1/2}} . \tag{2.8}$$

Finally, we note that the state $|\alpha,m\rangle$ can be written as a superposition of the displaced harmonic-oscillator coherent states as follows:

$$
\begin{aligned}
a^{\dagger m}|\alpha\rangle &= a^{\dagger m}D(\alpha)|0\rangle \\
&= D(\alpha)D^{-1}(\alpha)a^{\dagger m}D(\alpha)|0\rangle \\
&= D(\alpha)(a^{\dagger}+a^{*})^{m}|0\rangle \\
&= \sum_{p=0}^{m}\begin{bmatrix} m \\ p \end{bmatrix}(\alpha^{*})^{m-p}\sqrt{p!}D(\alpha)|p\rangle \ .
\end{aligned} \tag{2.9}
$$

We next examine fluctuation characteristics of the radiation field which is in the state (2.1).

A. Squeezing properties of the state $|\alpha,m\rangle$

Let us consider the field quadrature x defined by

$$x=\frac{a\exp(i\phi)+a^{\dagger}\exp(-i\phi)}{2} \ . \tag{2.10}$$

The mean value of x in the state $|\alpha,m\rangle$ is

$$\langle x\rangle = |\alpha|\cos(\theta+\phi)\frac{L_m^{(1)}(-|\alpha|^2)}{L_m(-|\alpha|^2)} \ , \tag{2.11}$$

where $L_m^{(k)}(x)$ is the associated Laguerre polynomial defined by[6]

$$L_m^{(k)}(x)=\sum_{n=0}^{m}\frac{(m+k)!}{(m-n)!n!(k+n)!}(-x)^n \ , \quad k>-1 \ . \tag{2.12}$$

The fluctuations in x can also be expressed in terms of Laguerre polynomials. Calculations show that

$$
\begin{aligned}
(\Delta x)^2 &= \langle x^2\rangle - \langle x\rangle^2 \\
&= (\{L_m^{(2)}(-|\alpha|^2)L_m(-|\alpha|^2)-[L_m^{(1)}(-|\alpha|^2)]^2\}2|\alpha|^2\cos[2(\theta+\phi)]-2[L_m^{(1)}(-|\alpha|^2)]^2|\alpha|^2-[L_m(-|\alpha|^2)]^2 \\
&\quad +2(m+1)L_{m+1}(-|\alpha|^2)L_m(-|\alpha|^2))/4[L_m(-|\alpha|^2)]^2 \ .
\end{aligned} \tag{2.13}
$$

We show in Fig. 1 the quantity $S_x=4(\Delta x)^2$ as a function of the parameter $|\alpha|$ for different values of m. We choose the phases such that $\theta+\phi=\pi$. For $m=0$ (coherent state) the value of S_x in Eq. (2.13) becomes equal to one. Also, for $m=0$ and $\alpha=0$ (vacuum state), the variance as given by Eq. (2.13) equals one as expected. For $m\neq 0$, $\alpha=0$, Eq. (2.13) reduces to

$$S_x=2m+1 \ . \tag{2.14}$$

For $m\neq 0$, $\alpha\neq 0$, Fig. 1 shows values of S_x less then one implying that the quadrature x of the field is squeezed. We get almost 50% squeezing over a wide range of parameters.

B. Sub-Poissonian-character of the field

Next we study the number distribution of the field in the state $|\alpha,m\rangle$. From Eq. (2.10) the probability of finding n photons in the field is given by

$$
\begin{aligned}
p(n) &= |\langle n|\alpha,m\rangle|^2 \\
&= \frac{|\langle n-m|\alpha\rangle|^2\frac{n!}{(n-m)!}}{L_m(-|\alpha|^2)m!} \ ,
\end{aligned}
$$

i.e.,

$$p(n)=\frac{n!|\alpha|^{2(n-m)}\exp(-|\alpha|^2)}{[(n-m)!]^2L_m(-|\alpha|^2)m!} \ , \tag{2.15}$$

which is zero for $n<m$. This distribution is found to have variance which is less than that for a Poisson distribution. To see this we calculate the parameter Q defined by[7]

$$Q(\alpha,m)=\frac{\langle(a^{\dagger}a)^2\rangle-\langle a^{\dagger}a\rangle^2}{\langle a^{\dagger}a\rangle} \ . \tag{2.16}$$

The mean number of photons is given by

$$
\begin{aligned}
\bar{n} &= \langle a^{\dagger}a\rangle = \langle aa^{\dagger}\rangle - 1 \\
&= \frac{\langle\alpha|a^{m+1}a^{\dagger m+1}|\alpha\rangle}{L_m(-|\alpha|^2)m!}-1
\end{aligned} \tag{2.17a}
$$

or

$$\bar{n}=\frac{(m+1)L_{m+1}(-|\alpha|^2)}{L_m(-|\alpha|^2)}-1 \ . \tag{2.17b}$$

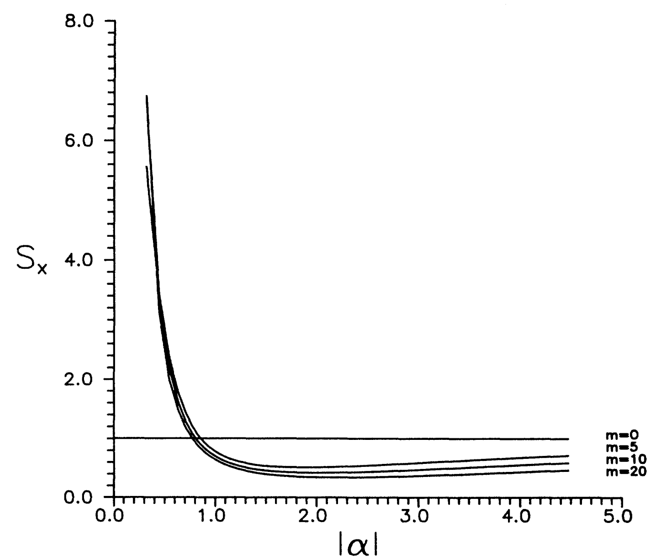

FIG. 1. Uncertainty in field quadrature x, S_x as a function of $|\alpha|$ for different values of m.

The second moment $\langle (a^\dagger a)^2 \rangle$ can be calculated by expressing $(a^\dagger a)^2$ in the antinormally ordered form

$$\langle (a^\dagger a)^2 \rangle = \langle a^2 a^{\dagger 2} - 3aa^\dagger + 1 \rangle \ . \tag{2.18}$$

The expectation values in (2.18) are now very simple to evaluate as

$$\langle \alpha, m | a^n a^{\dagger n} | \alpha, m \rangle = \frac{\langle \alpha | a^{n+m} a^{\dagger n+m} | \alpha \rangle}{m! L_m(-|\alpha|^2)} = \frac{(n+m)! L_{n+m}(-|\alpha|^2)}{m! L_m(-|\alpha|^2)} \ . \tag{2.19}$$

Thus, on combining Eqs. (2.16)–(2.19), we find the parameter

$$Q(\alpha,m) = \frac{\{[(m+2)L_{m+2}(-|\alpha|^2) - L_{m+1}(-|\alpha^2)](m+1)L_m(-|\alpha|^2) - [(m+1)L_{m+1}(-|\alpha|^2)]^2\}}{L_m(-|\alpha|^2)[(m+1)L_{m+1}(-|\alpha|^2) - L_m(-|\alpha|^2)]} \ . \tag{2.20}$$

In Fig. 2 we show the mean number of photons [Eq. (2.17b)] for different values of $|\alpha|^2$ and m. In Fig. 3 we display the parameter $Q(\alpha,m)$ as a function of $|\alpha|$ for different values of m. The values of $Q(\alpha,m)$ less than one signify the sub-Poissonian statistics of the field. We see that, for $m=0$ $Q(\alpha,0)=1$, corresponding to coherent state. For $\alpha=0$, $Q(0,m)=0$. For $\alpha\neq 0$, $m\neq 0$, we see that the field in the state $|\alpha,m\rangle$ exhibits a significant amount of sub-Poissonian statistics.

III. QUASIPROBABILITY DISTRIBUTIONS FOR THE FIELD IN THE STATE $|\alpha,m\rangle$

In this section we calculate different quasiprobability distributions for the state $|\alpha,m\rangle$. These distributions provide a convenient way of studying the nonclassical properties of fields.

A. *P* distribution

We first calculate the Glauber-Sudarshan P function associated with the state $|\alpha,m\rangle$. This function is defined by

$$|\alpha,m\rangle\langle\alpha,m| = \int P(z)|z\rangle\langle z| \frac{d^2 z}{\pi} \ , \tag{3.1}$$

where $|z\rangle$ is a coherent state.

The distribution $P(z)$ can be calculated using the inversion formula:[8]

$$\begin{aligned} P(z) &= \frac{\exp(|z|^2)}{\pi^2} \int d^2\beta \langle -\beta|\alpha,m\rangle\langle\alpha,m|\beta\rangle \exp[|\beta|^2 - (\beta z^* - \beta^* z)] \\ &= \frac{\exp(|z|^2)}{\pi^2 L_m(-|\alpha|^2) m!} \int d^2\beta (-\beta\beta^*)^m \exp[-|\alpha|^2 + (z-\alpha)\beta^* - (z-\alpha)^*\beta] \\ &= \frac{\exp(|z|^2 - |\alpha|^2)}{m! L_m(-|\alpha|^2)} \frac{\partial^{2m}}{\partial z^{*m} \partial z^m} \delta^{(2)}(z-\alpha) \ . \end{aligned} \tag{3.2}$$

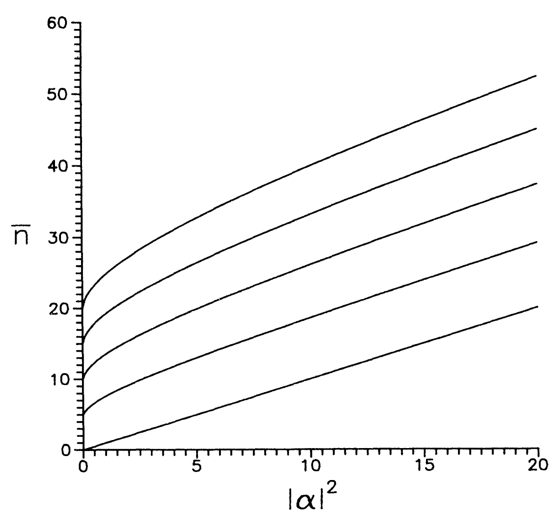

FIG. 2. Mean number of photons $\bar{n}$ as a function of $|\alpha|^2$ for different values of m.

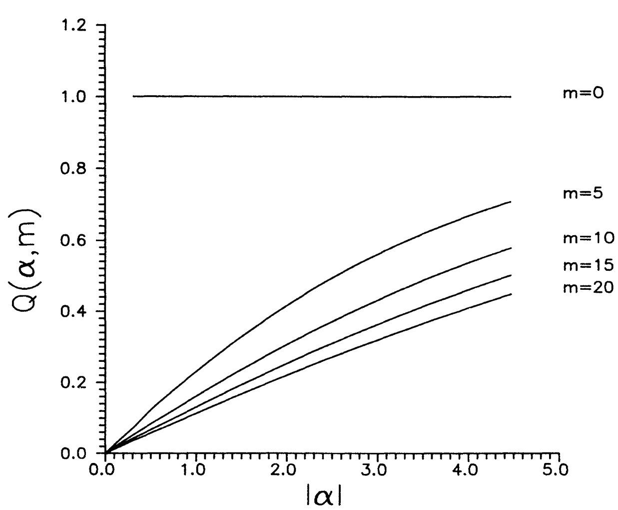

FIG. 3. $Q(\alpha,m)$ as a function of $|\alpha|$ for different values of m.

Thus, the quasiprobability distribution P is highly singular. This is quite typical of states exhibiting nonclassical character.

B. Q function

The Q function is the absolute magnitude squared of the projection of a state of the field onto a coherent state.[9–11] Thus, the Q function for the field in the state $|\alpha,m\rangle$ will be defined by

$$Q(z)=\langle z|\alpha,m\rangle\langle\alpha,m|z\rangle \ . \tag{3.3}$$

The calculation shows that

$$Q(z)=\frac{|z|^{2m}}{m!L_m(-|\alpha|^2)}\exp(-|z-\alpha|^2) \ , \tag{3.4}$$

which is no longer centered at $z=\alpha$. Note that

$$|z|^{2m}[\exp(-|z-\alpha|^2)]$$

is the signature of the number (coherent) state. The Q function for the state $|\alpha,m\rangle$ is distinct from that of the two-photon coherent state.[12]

C. Wigner function

The Wigner function $W(z)$ associated with the state $|\alpha,m\rangle$ can also be evaluated in terms of the coherent-state matrix elements by using the formula[13]

$$W(z)=\frac{2}{\pi^2}\exp(2|z|^2)\int d^2\beta\langle-\beta|\alpha,m\rangle\langle\alpha,m|\beta\rangle \times\exp[2(\beta^*z-\beta z^*)] \ , \tag{3.5}$$

which, on simplification, reduces to

$$W(z)=\frac{2\exp(2|z|^2-|\alpha|^2)}{\pi^2 m!L_m(-|\alpha|^2)} \times\int d^2\beta(-\beta^*\beta)^m\exp[-|\beta|^2+\beta^*(2z-\alpha)-\beta(2z-\alpha)^*] \ . \tag{3.6}$$

The integral in Eq. (3.6) can be written as

$$\begin{aligned}W(z)&=\frac{2\exp(2|z|^2-|\alpha|^2)}{\pi m!L_m(-|\alpha|^2)}\frac{\partial^{2m}}{\partial\xi^{*m}\partial\xi^m}\frac{1}{\pi}\int d^2\beta\exp(-|\beta|^2+\beta^*\xi-\beta\xi^*) \quad \text{where } \xi=2z-\alpha\\&=\frac{2\exp(2|z|^2-|\alpha|^2)}{\pi m!L_m(-|\alpha|^2)}\frac{\partial^{2m}}{\partial\xi^{*m}\partial\xi^m}\exp(-|\xi|^2)\\&=\frac{2(-1)^m\exp(2|z|^2-|\alpha|^2)}{\pi m!L_m(-|\alpha|^2)}\exp(-|\xi|^2)L_m(|\xi|^2)m!\end{aligned} \tag{3.7}$$

and therefore the Wigner function for the state $|\alpha,m\rangle$ is

$$W(z)=\frac{2(-1)^m L_m(|2z-\alpha|^2)}{\pi L_m(-|\alpha|^2)}\exp(-2|z-\alpha|^2) \ . \tag{3.8}$$

It is clear from Eq. (3.8) that the Wigner function can become negative. This crosses zero whenever

$$L_m(|2z-\alpha|^2)=0 \ . \tag{3.9}$$

This is in contrast to the Wigner function for a coherent state. In Figs. 4(a) and 4(b) we show the Wigner function as a function of $z=x+iy$ for different values of m. We set $\phi=0$, $\theta=\pi$. This is in the light of our earlier finding that the phase squeezing was maximum for $\phi+\theta=\pi$. For $m=0$, the expression in Eq. (3.8) reduces to that for a coherent state. But for $m\neq0$, $W(z)$ shows minimum for some values of y in a fixed range of x. For example, $W(z)$ for $m=1$ is minimum at $y=0$ and for x in the range given by $0.13<x<1.87$ for $\alpha_1=2$, $\alpha_2=0$ ($\alpha=\alpha_1+i\alpha_2$). This is due to the presence of Laguerre polynomial in the numerator in Eq. (3.8). Figure 4(b) also shows regions where the Wigner function is negative.

D. The distribution of the field quadrature x

The probability distribution $p(x)$ associated with the field quadrature x can be obtained from Eq. (3.8). For simplicity we set $\phi=0$. The distribution $p(x)$ is defined by[14]

$$p(x)=\int_{-\infty}^{\infty}W(x+iy)dy \ . \tag{3.10}$$

On using Eqs. (3.8) and (3.10) and $\alpha=\alpha_1+i\alpha_2$, we get

$$p(x)=\frac{2\exp[-2(x-\alpha_1)^2](-1)^m}{\pi L_m(-|\alpha|^2)} \times\int_{-\infty}^{\infty}dy\exp[-2(y-\alpha_2)^2] \times L_m((2x-\alpha_1)^2+(2y-\alpha_2)^2) \ . \tag{3.11}$$

The integral in Eq. (3.11) can be evaluated numerically. We have already seen that the quadrature x can show squeezing and thus the variance of the distribution $p(x)$ can be less than that for a coherent state. In Fig. 5 we show the distribution $p(x)$. Here we have chosen $\phi=0$, $\theta=\pi$. We take $\alpha_1=2$ and $\alpha_2=0$. We see that, as m is increased, the width of the distribution becomes narrower and narrower compared to that for the coherent state. The coherent state corresponds to $m=0$. Note that $p(x)$

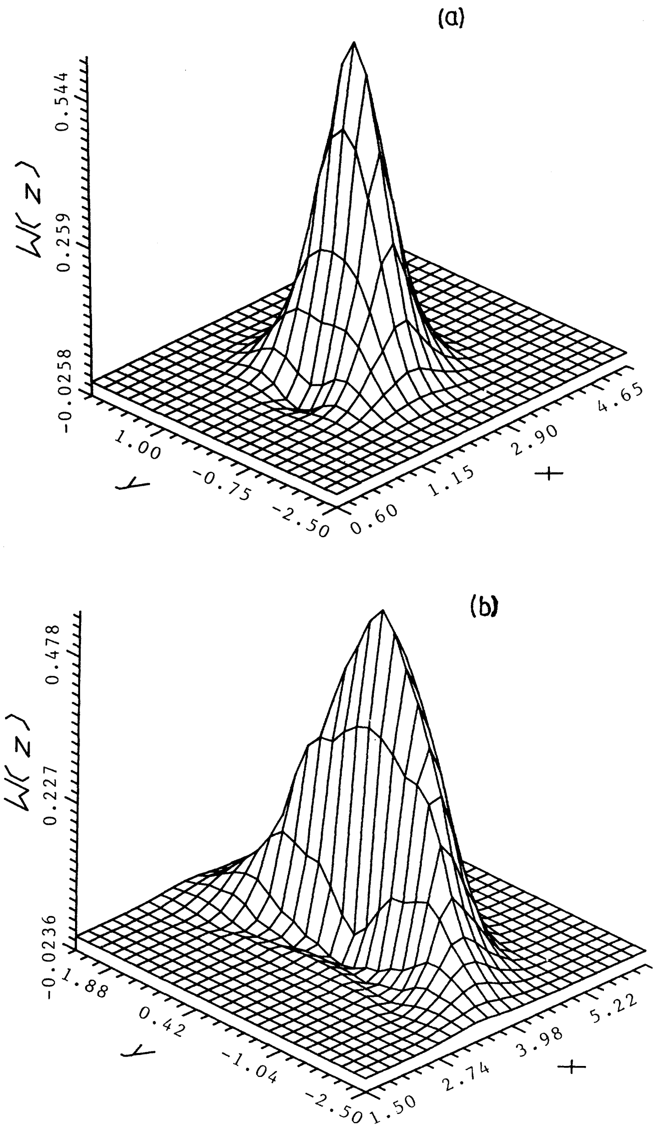

FIG. 4. (a) Wigner function $W(z)$ for $\alpha_1=2$, $\alpha_2=0$, and $m=1$. (b) Wigner function $W(z)$ for $\alpha_1=2$, $\alpha_2=0$, and $m=10$.

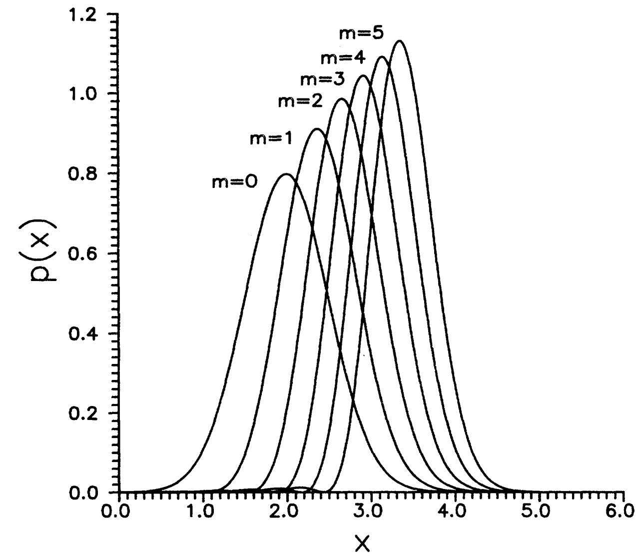

FIG. 5. Probability distribution $p(x)$ for $\alpha_1=2$, $\alpha_2=0$, and for different m values.

for the coherent state is Gaussian with a width $\frac{1}{2}$:

$$p(x)=\frac{2}{\sqrt{2\pi}}\exp[-2(x-\alpha_1)^2],\quad m=0\ . \tag{3.12}$$

Finally, we note that, for $\alpha_2=0$, the integral (3.11) can also be written as

$$p(x)=\frac{2\exp[-2(x-\alpha_1)^2](-1)^m}{\pi L_m(-|\alpha|^2)} \times \sum_{p=0}^{m}\sum_{q=0}^{p}\frac{(-4)^p m!(x-\alpha_1/2)^{2(p-q)}}{p!(m-p)!q!(p-q)!}\frac{\Gamma(q+\frac{1}{2})}{2^{q+1/2}}\ . \tag{3.13}$$

IV. PRODUCTION OF THE STATE $|\alpha,m\rangle$

We have seen the important properties like phase squeezing and sub-Poissionian statistics of the photon-added coherent states $|\alpha,m\rangle$. The question now arises of how such states can be produced in practice. In what follows, we discuss a possible scheme.

Consider the passage of the excited atoms through a cavity. We model atoms as two-level atoms. The atom makes a transition from the excited state to the ground state by emitting a photon.[15,16] Let the initial state of the atom-field system be $|\alpha\rangle|e\rangle$, where $|\alpha\rangle$ is the coherent state of the field. The interaction Hamiltonian has the form

$$H=\hbar(gS^{\dagger}a+g^*S^{-}a^{\dagger})\ . \tag{4.1}$$

Since the coupling constant g is generally small, the state at time t can be approximated by

$$|\psi(t)\rangle\cong|\alpha\rangle|e\rangle-\frac{iHt}{\hbar}|\alpha\rangle|e\rangle\ , \tag{4.2}$$

which is valid for interaction times such that $gt\ll 1$. On using Eq. (4.1), Eq. (4.2) reduces to

$$|\psi(t)\cong|\alpha\rangle|e\rangle-ig^*a^{\dagger}|\alpha\rangle|g\rangle\ . \tag{4.3}$$

From Eq. (4.3) we observe that, if the atom is detected to be in the ground state $|g\rangle$, then the state of the field is reduced to $a^{\dagger}|\alpha\rangle$, i.e., to $|\alpha,1\rangle$. Thus, we can, in principle, produce the state $|\alpha,1\rangle$.

Note that, if we send a ground-state atom through the cavity, then state of the combined system will be

$$|\psi(t)\rangle\approx|\alpha\rangle|g\rangle-igt\alpha|\alpha\rangle|e\rangle\ . \tag{4.4}$$

Thus, the detection of the atom in the excited state leaves the field in the coherent state $|\alpha\rangle$ itself indicating that the field in such a situation has no nonclassical character.

The above discussion also points out the very fundamental distinction between absorption and emission processes. We create nonclassical character in emission[17] and not in absorption.

An extension of the above arguments to the multiphoton processes would imply that the state $|\alpha,m\rangle$ can be produced in multiphoton emission processes. For example, in a two-photon medium, Eq. (4.1) is replaced by a new Hamiltonian with $a \rightarrow a^2$. Thus, the above procedure for a two-photon medium[18] will result in the state $|\alpha,2\rangle$. Similarly, for an m-photon medium the state $|\alpha,m\rangle$ is obtained.

In the above we have considered the interaction of atoms for short times and thus the photon-added coherent state is produced with small probability. The state $|\alpha,m\rangle$ may also be produced by other methods such as those based on special state reduction and feedback methods.[19] For example, consider the process of parametric amplification in which the signal (a mode) and idler (b mode) are generated. These two modes are strongly correlated. Let us assume that initially the signal field is in the state $|\alpha\rangle$. One can show that, if the b the mode is measured in the Fock state $|m\rangle$, then the state of the a mode is reduced to $|\alpha,m\rangle$.

In conclusion, we have introduced a new class of states that are generated by the action of photon creation operator on a coherent state and shown the important nonclassical properties[20] such states possess.

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