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Effect of radiation trapping on the polarization of an optically pumped alkali-metal atomic beam

D. Peterson and L. W. Anderson
Department of Physics, Universty of Wisconsin–Madison, Madison, Wisconsin 53706
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Calculations of the limitations imposed by radiation trapping on the polarization of an alkali-metal atomic beam by optical pumping are presented. In an atomic beam radiation trapping limits the density of alkali-metal atoms that can be highly polarized by optical pumping. This density is about $\frac{1}{3}$ the density of an alkali-metal vapor that can be highly polarized by optical pumping. In a low magnetic field the maximum intensity of an atomic beam that can be optically pumped is a few particle milliamperes.

I. INTRODUCTION

Optical pumping can be used to polarize an alkali-metal vapor target or an alkali-metal atomic beam.[1] The absorption of circularly polarized resonance radiation followed by spontaneous emission results in the polarization of the alkali atoms. Radiation trapping can seriously limit the polarization of alkali atoms by optical pumping. Radiation trapping occurs when the density of alkali atoms is high enough that multiple scattering of light occurs for one or more radiative decay branches of the alkali vapor. Previous papers have examined theoretically the effect of radiation trapping on the polarization of an optically pumped alkali-metal vapor in both weak and strong magnetic fields.[2,3] Experiments agree well with the previous calculations.[4] In this paper we analyze theoretically the effect of radiation trapping on the polarization that can be produced by optical pumping of an alkali-metal atomic beam in either a weak or a strong magnetic field. The radiation trapping of an alkali-metal beam is somewhat different from the radiation trapping of an alkali-metal vapor target because the photon emission or absorption line shape of a beam depends strongly on the angle between the direction of the alkali-metal beam and the direction of the photon. The absorption linewidth is nearly the natural width in a direction perpendicular to the atomic beam and is comparable to the vapor Doppler width in a direction parallel to the atomic beam.

The use of optical pumping to produce a polarized alkali-metal atomic beam is of current interest. Polarized Na negative-ion beams have been formed as follows.[5] An atomic beam of Na is polarized by optical pumping in a low magnetic field. The polarized atomic beam is surface ionized by a heated tungsten surface. The resulting alkali-metal positive ions are nuclear polarized. The polarized positive ions are extracted and accelerated. These polarized positive ions are then partially converted into polarized negative ions by charge-changing collisions in an alkali-metal vapor target. A polarized alkali-metal atomic beam may also be useful as a target in a storage ring.[6] The calculations in this paper show that in a low magnetic field the possible flux of alkali-metal atoms that can be polarized by optical pumping is limited by radiation trapping to a few particle mA or less.

II. OPTICAL PUMING OF AN ALKALI-METAL ATOMIC BEAM

The atoms in an atomic beam have a large average component of velocity parallel to the atomic beam axis ($v_{\parallel} \simeq \sqrt{4kT/m}$ and a small average component of their velocity perpendicular to the atomic beam axis ($v_{\perp} \simeq 0$). For an atomic beam formed by effusion from an oven, the distribution in $v_{\parallel}$ is large, being comparable to the average value of $v_{\parallel}$. For an atomic beam formed by using a supersonic nozzle the distribution in $v_{\parallel}$ can be much smaller than the average value of $v_{\parallel}$ due to the adiabatic cooling that occurs as the beam emerges from the nozzle. In this paper we restrict our calculations to the effects of radiation trapping on the polarization by optical pumping of an alkali-metal atomic beam formed by effusion from an oven with only a few comments at the end of the paper on the effects of radiation trapping on the polarization by optical pumping of an alkali-metal beam formed using a supersonic nozzle.

For an atomic beam formed by effusion from an oven at a temperature T the beam intensity (in atom/s) with velocities between v and $v+dv$ is given by

$$I(v)=2I_0 v^3 \exp(-v^2/\alpha^2)/\alpha^4 , \tag{1}$$

where $\alpha=(2kT/m)^{1/2}$ and I_0 is the total intensity integrated over all velocities.[7] The density of atoms in the atomic beam (in atoms/cm^3) with velocities between v and $v+dv$ is given by

$$\rho(v)=I(v)/(vA)=2I_0 v^2 \exp(-v^2/\alpha^2)/\alpha^4 A , \tag{2}$$

where A is the cross-sectional area of the atomic beam. An atomic beam can be polarized by optical pumping with the light beam making an arbitrary angle with the atomic-beam axis. Of course, there must be a static magnetic field parallel to the light beam in order to maintain the polarization produced by the optical pumping. In this paper we consider the two cases where the optical-pumping light beam is either nearly antiparallel (or paral-

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lel) to the atomic-beam axis or is perpendicular to the atomic-beam axis.

In order to simplify the problem we assume that all the atoms in the atomic beam have zero component of velocity perpendicular to the atomic-beam axis. This perfect collimation of the atomic beam produces the most serious possible radiation trapping by the atomic beam. We consider a ^{23}Na atomic beam in order to make the discussion concrete.

A. Optical pumping in a weak magnetic field with the light beam parallel to the atomic-beam axis

For optical pumping in a weak magnetic field with the light beam antiparallel (or parallel) to the atomic beam the absorption line shape seen by the optical-pumping light beam is obtained by substituting $v=\pm c[(\nu-\nu_0)/\nu_0]$ into Eq. (2) where the + sign corresponds to the light beam antiparallel to the atomic beam and the − sign corresponds to the light beam parallel to the atomic beam. The resulting normalized absorption line shape seen by the optical-pumping light is given by

$$\hat{g}=\frac{4\lambda^3}{\sqrt{\pi}}\left[\frac{m}{2kT}\right]^{3/2}(\nu-\nu_0)^2\exp-\left[\frac{mc^2(\nu-\nu_0)^2}{2kT\nu_0^2}\right]. \tag{3}$$

The absorption line shape seen by the optical-pumping light has maxima for $\nu-\nu_0=\pm\nu_0(2kT/mc^2)^{1/2}$. The + and − signs again correspond, respectively, to the light beam antiparallel or parallel to the atomic beam. The full width at half maximum (FWHM) for the absorption line shape seen by the optical-pumping light is $\Delta\nu_B\cong 1.7\nu_0(2kT/mc^2)^{1/2}$. For a ^{23}Na atomic beam formed by effusion from an oven at $T=600$ K the absorption line shape seen by the optical-pumping light beam has a FWHM of $\Delta\nu_B=1.89\times10^9$ Hz. For a ^{23}Na beam in a weak magnetic field the absorption line shape for each transition out of either of the ground hyperfine levels will have a line shape given by Eq. (3) and with a width $\Delta\nu_B$. Since the ground-level hyperfine separation of ^{23}Na is $\Delta\nu_{\mathrm{HFS}}=1.77\times10^9$ Hz the light source used for optical pumping of a ^{23}Na atomic beam either antiparallel (or parallel) to the atomic beam in a weak magnetic field must provide frequency coverage of about $\Delta\nu\cong\Delta\nu_{\mathrm{HFS}}+\Delta\nu_B=3.66\times10^9$ Hz. The static magnetic field must be parallel to the light beam.

In an atomic beam the collision rate is small so that the relaxation rate T_1^{-1} is very small. The polarization that can be obtained is therefore determined by the intensity and the frequency distribution of the optical-pumping light, the light-atom interaction time, and the atomic-beam density and geometry, which determine radiation trapping by the beam. The light-atom interaction time is the time for an atom to cross the optical-pumping light beam.

In order to estimate the effect of radiation trapping on the polarization that can be produced by optical pumping we simplify the problem by considering an "idealized" Na atom with zero nuclear spin. We label the ground $^2S_{1/2}$ states with $m=-\frac{1}{2}$ and $m=\frac{1}{2}$ as states 1 and 2, respectively, and the lowest $^2P_{1/2}$ excited states with $m=-\frac{1}{2}$ and $\frac{1}{2}$ as states 3 and 4, respectively. This same simplification and notation was used in Refs. 2 and 3. We take the Na atomic beam to be 0.5 cm in radius, and we assume the atomic beam has a constant density of the Na inside this radius and zero density outside this radius. We take $m=23$ u, $\lambda=589$ nm, and $A=6.1\times10^7$ s^{-1} for our idealized Na atom and we take $T=600$ K as the oven temperature. The low-field rate equations for the optical pumping of the idealized Na atomic and the approximations and methods used in the numerical solution of these equations are given in Ref. 3. The notation in this paper is the same as that used in Ref. 3 so that we need not reproduce the rate equations from Ref. 3 in this paper.

The normalized line shape g for the emission or absorption of radiation at an angle θ to the atomic-beam axis is obtained by solving $\nu=\nu_0(1\pm v\cos\theta/c)$ for v and substituting the resulting expression for v into Eq. (2) with the appropriate normalization. The normalized line shape obtained is

$$g=\frac{4\lambda^3}{\sqrt{\pi}}\left[\frac{m}{2kT}\right]^{3/2}\frac{(\nu-\nu_0)^2}{|\cos^3\theta|}\exp-\left[\frac{mc^2(\nu-\nu_0)^2}{2kT\nu_0^2|\cos^2\theta|}\right]. \tag{4}$$

This line shape depends on both $\nu-\nu_0$ and θ. Note that Eq. (4) differs from Eq. (3) in that Eq. (3) is the absorption line shape seen by the optical-pumping light and hence Eq. (3) is equal to Eq. (4) evaluated at $\theta=180°$. At $\theta=90°$ the line shape g is zero if $\nu\neq\nu_0$, i.e., the line shape at $\theta=90°$ is a δ function so that $g(\nu-\nu_0)=\delta(\nu-\nu_0)$. The FWHM of g is zero at $\theta=90°$. At $\theta=90°$ the correct line shape should be the natural Lorentzian line shape with a FWHM of $\Delta\nu_N=A/2\pi=10^7$ Hz. At $\theta=90°$ the line shape is not correctly given by g. Since $\Delta\nu_N\ll\Delta\nu_B$ the line shape g is correct over all angles θ except for a small range of angles θ near $\theta=90°$. The range of angles over which $\Delta\nu_N$ is greater than the FWHM of g is from $\theta=90°$ to $\tilde{\theta}=\arccos(\pm\Delta\nu_N/\Delta\nu_B)=\pm89.66°$. Thus g accurately represents the correct emission or absorption line shape except over a fraction of a degree near $\theta=90°$. Therefore we utilize g at all angles ignoring the fact that this overemphasizes the radiation trapping very near $\theta=90°$.

The question as to what magnetic field is considered weak is now addressed. The field is considered weak in our calculations when emission on one transition can be absorbed by another transition. For example, when the photon emitted by a transition from state 3 to state 1 can be absorbed by the transition from state 2 to state 4. In order that this be the case independent of the direction of emission, one must have $g_J\mu_B H/h<\Delta\nu_N$, where g_J is the ground-level gyromagnetic ratio factor, μ_B is the Bohr magneton, and H is the magnetic field. This restricts the field to less than about 3 G.

The normalized line shape g of Eq. (4) is used together with Eqs. (1)–(11) and Eqs. (A1)–(A15) in Ref. 3 to calculate the ground-level populations as a function of the light-atom interaction time. The initial conditions are taken as $n_1=n_2=\frac{1}{2}$. The rate equations are integrated numerically from time $t=0$ to a time equal to the light-atom interaction time yielding the state populations. The numerical integrations are carried out as described in Refs. 2 and 3. If the light beam and the atomic beam are exactly antiparallel (or parallel), the light-atom interaction time can be large. In the design of a practical system, however, one might have the light beam and the atomic beam cross at a small angle ψ. If the light beam has radius R_L, then the light-atom interaction time for an atom crossing the center of the light beam is approximately $2R/(v\sin\psi)$, where v is the most probable velocity of an atom in the atomic beam. We arbitrarily take the light-atom interaction time as 10^{-5} s in our calculations. Using a fixed-light interaction time ignores the distribution in velocities and in the position of atoms crossing the laser beam. We take the light-beam intensity as 0.25 W distributed uniformly in space over the light-beam diameter of 1 cm and distributed uniformly in frequency over $\Delta\nu_B$. Thus the light intensity per unit frequency is taken as 1.68×10^{-10} W/cm^2 Hz. We further assume that the light-beam intensity is constant along the direction of propagation and is not decreased by absorption. For the assumed light intensity the calculated values of the polarization as a function of the atomic-beam intensity are relatively insensitive to the value of the light-atom interaction time. We take the polarization of the laser beam as σ^- so that the atoms are pumped out of state 2. The calculated value of the polarization $P=(n_2-n_1)/(n_2+n_1)$ is determined for different total atomic densities in the atomic beam. The results of our calculations are shown in Fig. 1 where P is plotted as a function of ND, the product of N, the total density in the atomic beam, times D, the diameter of the atomic beam.

As can be seen with the assumed conditions one cannot produce a high polarization by optical pumping in an atomic beam with a diameter of 1 cm (a radius of 0.5 cm) if the atomic density is greater than about 10^{11} atoms/cm^3. The atomic-beam intensity corresponding to this density is estimated by multiplying the density by the ratio of the atomic-beam intensity to the atomic-beam density $(8kT/\pi m)^{1/2}$, times the cross-sectional area of the beam.[7] For a temperature of 600 K the value of $(8kT/\pi m)^{1/2}$ is 4.5×10^5 cm/s and for $R=0.5$ cm the value of the cross-sectional area of the beam is 0.78 cm^2. For these values the maximum beam intensity that can be highly polarized by optical pumping is 3.7×10^{16} atoms/s. This corresponds to about six particle milliamperes. It is noted that the polarization depends on ND. Thus the value of the polarization as a function of N for any D other than $D=1$ ($R=0.5$ cm) is easily obtained.

B. Optical pumping in a weak magnetic field with the light beam perpendicular to the atomic-beam axis

For optical pumping in a weak magnetic field with the light beam perpendicular to the atomic beam the absorption line shape that must be covered by the optical-pumping light consists of two absorption lines, one out of each of the ground hyperfine levels. Each of the two absorption line shapes is Lorentzian with $\Delta\nu_N=A/2\pi=10^7$ Hz.

In order to estimate for this case and for an idealized alkali-metal with $I=0$ the effect of radiation trapping on the polarization that can be produced by optical pumping, we again use the equations from Ref. 3. The problem of pumping with the light beam perpendicular to the atomic beam differs from that where the light beam is parallel to the atomic beam in that the normalized line shape for emission or absorption differs from the normalized line shape given in Eq. (4). We take the x direction as parallel to the atomic-beam axis and the z axis as parallel to the laser beam and hence the magnetic field. The normalized line shape for emission or absorption depends on the spherical coordinate angles θ and ϕ as well as $\nu-\nu_0$ and is given by

$$g=\frac{4\lambda^3}{\sqrt{\pi}}\left[\frac{m}{2kT}\right]^{3/2}\times\frac{(\nu-\nu_0)^2}{|\cos^3\phi||\sin^3\theta|}\exp-\left[\frac{mc^2(\nu-\nu_0)^2}{2kT\nu_0^2\sin^2\theta\cos^2\phi}\right], \quad (5)$$

where the angles θ and ϕ are such that $z/r=\cos\theta$, $x/r=\sin\theta\cos\phi$, and $y/r=\sin\theta\sin\phi$. The rate equations

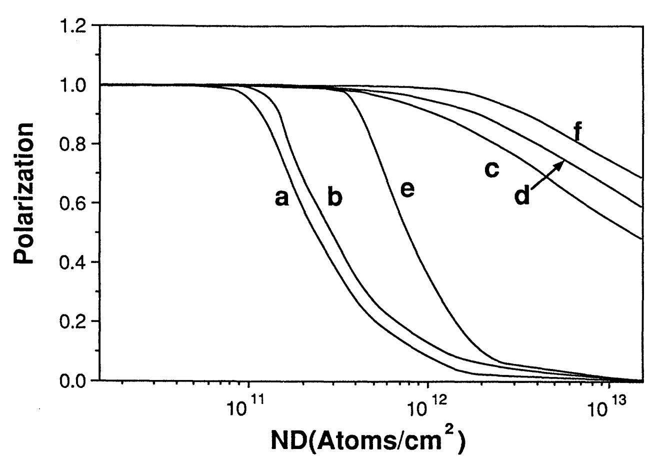

FIG. 1. A plot of the polarization P as a function of the product of the atomic-beam density times the atomic-beam diameter ND. The various curves are for the following situations: (a) optical pumping of an atomic beam parallel to the beam axis in a weak magnetic field; (b) optical pumping of an atomic beam perpendicular to the beam axis in a weak magnetic field; (c) optical pumping of an atomic beam parallel to the atomic axis in a strong magnetic field; (d) optical pumping of an atomic beam perpendicular to the atomic axis in a strong magnetic field; (e) optical pumping of a vapor target in a weak magnetic field; (f) optical pumping of a vapor target in a strong magnetic field. For optical pumping of the atomic beam the optical-pumping light-atom interaction time is taken as 10^{-5} s, and for optical pumping of the vapor cell the relaxation time is taken as 10^{-4} s. The optical-pumping light is assumed in all cases to cover uniformly the absorption line and to have an intensity per unit frequency of 1.68×10^{-10} W/cm^2 Hz.

are again solved numerically as a function of the time with the initial condition $n_1 = n_2 = \frac{1}{2}$ for different total densities in the atomic beam. In this situation the numerical solutions include an integration over ϕ as well as over r and θ. The integrations are over r from 0 to $R/(\cos^2\theta + \sin^2\theta \sin^2\phi)^{1/2}$, over θ from 0 to π, and over ϕ from 0 to 2π. The integrations over r and θ are carried out as described in Refs. 2 and 3, and the integration over ϕ is carried out using Gaussian integration of moments with 20 terms as described in Ref. 8. The state populations are determined for a time equal to the light-atom interaction time. If the light beam has a radius R_L, then the laser-atom interaction time is about $\tau = 2R_L/v$, where v is the most probable velocity for an atom in the atomic beam. We arbitrarily take the same light-atom interaction time 10^{-5} s as was used for the parallel-pumping calculations. We also assume the light intensity per unit frequency is the same as for the parallel-pumping calculations. The results of our calculation for n_1 and n_2 at different atomic-beam densities are combined to give P as a function of the density of the atomic beam. The results are again shown in Fig. 1. As can be seen the maximum value of the density times atomic-beam diameter for an atomic beam that can be highly polarized by optical pumping is nearly the same for perpendicular pumping as for parallel pumping, i.e., about $ND = 10^{11}$ atoms/cm^2. Also shown in Fig. 1 are calculations of the possible polarization for an idealized Na vapor target with a relaxation time of $T_1 = 10^{-4}$ s. The polarization of the vapor target is calculated as described in Ref. 3. The maximum density of an atomic beam that can be highly polarized by optical pumping using either parallel or perpendicular pumping is about a factor of 3 less than the density of a vapor target that can be highly polarized by optical pumping. These results are relatively insensitive to the light-atom interaction time for a light intensity as large as we have used.

Since the absorption cross section at line center is very large for pumping perpendicular to the atomic-beam axis one might wonder whether it is possible to maintain even approximately a constant intensity of the pumping light across the atomic beam if the atomic beam is dense enough that radiation trapping is important. It is possible to make the fractional absorption small by using a laser light source detuned from line center by an appropriate amount. In this case the light intensity can be nearly constant across the entire atomic beam. In order that the absorption be adequate for optical pumping, it is necessary that the laser intensity be larger than is needed for optical pumping at line center by $g(0)/g(\nu-\nu_0)$, where $\nu-\nu_0$ is the frequency detuning of the laser. Thus although our calculations are carried out for an intensity per unit frequency of 1.68×10^{-10} W/cm^2 Hz at line center, an actual experiment might require a laser detuned from the line center and with a narrow bandwidth and a larger value of the intensity per unit frequency in order to obtain the calculated optical pumping with nearly constant light intensity across the atomic beam. For the low-field absorption one must have absorption out of each ground hyperfine level. One might need to use two lasers, one detuned above the upper-ground hyperfine level and the other detuned by an equal amount below the lower-ground hyperfine level.

C. Optical pumping in a strong magnetic field with the light beam parallel to the atomic beam

For optical pumping in a high magnetic field with the optical-pumping light beam parallel to the atomic beam, the absorption line shape seen by the light beam is given by Eq. (3). In a high magnetic field H_0 the ^{23}Na ground Zeeman hyperfine energy levels are given by

$$E_{m_J,m_I} \cong g_J \mu_0 H_0 m_J + \Delta\nu_{\mathrm{HFS}} m_J M_I/2 \ ,$$

where m_J and m_I are, respectively, the electronic and nuclear magnetic quantum numbers and where $\Delta\nu_{\mathrm{HFS}}$ is the ground hyperfine splitting. In order to optically pump Na in a high magnetic field, one must have light coverage such that there is absorption out of each of the four states with quantum numbers $m_J = -\frac{1}{2}, m_I$ (or the four states with quantum numbers $m_J = \frac{1}{2}, m_I$). The absorption out of each state with $m_J = -\frac{1}{2}, m_I$ has a FWHM of $\Delta\nu_B$. The optical-pumping light must, therefore, provide a frequency coverage of about $\Delta\nu = \Delta\nu_B + 3\Delta\nu_{\mathrm{HFS}}/4 \simeq 3.2\times10^9$ Hz.

The magnetic field is considered strong for our calculations provided the light emitted on one transition cannot be absorbed by another transition. In order that this is the case for emission in all directions, one must have a magnetic field large enough that $g_J \mu_B H_0 \gg \Delta\nu_B$.

The rate equations for optical pumping of an "idealized" alkali-metal with zero nuclear spin in a high magnetic field are Eqs. (4)–(6) in Ref. 2. The normalized line shape for emission or absorption is given by Eq. (4) in this paper. The equations are solved numerically as a function of the time up to the light-atom interaction time for various total densities in the atomic beam. The polarization $P = (n_2 - n_1)/(n_2 + n_1)$ at a time equal to the light-atom interaction time is then determined as a function of the density in the atomic beam. The results for the same light-atom interaction time and for the same light intensity per unit frequency as for the low field cases are shown in Fig. 1. As can be seen an atomic beam can be highly polarized by optical pumping only for densities up to about $\frac{1}{3}$ the density to which a vapor target can be highly polarized by optical pumping.

D. Optical pumping in a high magnetic field with the laser beam perpendicular to the atomic beam

For optical pumping in a high magnetic field with the laser beam perpendicular to the atomic beam, the absorption profile that must be covered by the optical-pumping light consists of four lines each with natural line shape. These lines will correspond to the absorption from the four levels with quantum numbers $m_J = -\frac{1}{2}, m_I$ (or

$m_J = \frac{1}{2}, m_I$).

The rate equations for the optical pumping of an "idealized" Na atom with $I = 0$ are given in Eqs. (4)–(6) in Ref. 2. The normalized line shape for emission or absorption is given by Eq. (5). The rate equations are solved numerically as a function of the time from time $t = 0$ to a time equal to the light-atom interaction time for different total densities in the atomic beam. The polarization that can be obtained by optical pumping is then determined. The polarization as a function of the total density in the atomic beam is shown in Fig. 1. The polarization as a function of the total density for perpendicular pumping in a high field is similar to the polarization for parallel pumping.

E. Optical pumping of a ribbon-shaped atomic beam

The optically pumped, polarized alkali-metal ion source[5] uses an optically pumped atomic beam that is roughly circular in cross section, as do other proposed applications.[6] However one might wonder whether an atomic beam with a different cross-sectional shape might have different radiation-trapping limits. In this section we discuss briefly the limitations on the polarization that can be produced by optical pumping of a ribbon-shaped beam. For a ribbon-shaped atomic beam, a cross section of the beam is a rectangle with one side appreciably longer than the other side.

After carrying out some computer calculations we recognize that one can understand this problem as follows. Take the cross-sectional shape of the beam to be a rectangle with sides of a and b, where $b > a$. Consider the ribbon beam as made up of b/a small, side-by-side tubes each with a square, cross-sectional shape with the side of the square having a length a. The effect of radiation trapping of each square, cross-section tube on a side is close to the effect of radiation trapping by a circular, cross-section tube of diameter a. The effect of the other tubes that are side by side is to reduce the density that can be optically pumped to a high polarization by about a factor of 0.5. Thus for a ribbon-shaped atomic beam in a low magnetic field, one can optically pump the beam to a high polarization provided $\text{Na} \le 5\times 10^{10}$ atoms/cm^2, where a is the thin dimension of the atomic beam using either parallel or perpendicular pumping. Computer calculations indicate that this description is essentially correct. Thus by using a ribbon-shaped atomic beam, one can obtain a higher optically pumped atomic-beam intensity in atoms s than can be obtained in a circular, cross-section beam of diameter a. In fact the atomic-beam intensity for a ribbon beam is comparable to the intensity possible for a circular beam of diameter b. This advantage may, however, be offset for some applications by the necessity to optically pump a ribbon and by the rectangular shape of the polarized atomic beam. For example, for the polarized negative alkali-metal ion source of Ref. 5 the extracted alkali-metal ion beam might have a large emittance if the alkali-metal atomic beam were ribbon shaped.

III. CONCLUSIONS

Figure 1 shows plots of the polarization that can be obtained using optical pumping for an idealized Na atomic beam ($I = 0$) as a function of ND for a light-atom interaction time of 10^{-5} s. Also shown are plots of the polarization of an idealized Na vapor as a function of ND for a vapor with a relaxation time of $T_1 = 10^{-4}$ s. There are plots for both weak and strong magnetic fields. All plots assume an optical-pumping light intensity per unit frequency of 1.68×10^{-10} W/cm^2 Hz. Radiation trapping does limit the polarization in an atomic beam to a value that depends on the product of the total density in the atomic beam times the diameter of the beam. The polarization depends only weakly on whether the optical-pumping light is parallel or perpendicular to the atomic beam. This result seems reasonable since the radiation trapping depends primarily on the total density rather than on the direction of the pumping beam. The polarization that can be produced depends on the magnetic field. The polarization that can be produced in an atomic beam with a given value of ND is lower than can be produced in a vapor target of the same ND in either a weak or a strong magnetic field. This occurs because the optical-absorption cross section for an atomic beam is very large in a direction perpendicular to atomic-beam axis. The maximum value of the optical-absorption cross section for an atomic beam is proportional to $(\Delta\nu_B)^{-1}$ parallel to the direction of the atomic beam and is proportional to $(\Delta\nu_N)^{-1}$ perpendicular to the direction of the atomic beam. For a vapor the peak optical-absorption cross section is proportional to the inverse of the Doppler width, $(\Delta\nu_D)^{-1}$. For a ^{23}Na vapor at 600 K the Doppler width is $\Delta\nu_D = 1.7\times 10^9$ Hz. For an atomic ^{23}Na beam formed by effusion from the oven at 600 K the beam line width is $\Delta\nu_B = 1.89\times 10^9$ Hz. Since $\Delta\nu_B$ and $\Delta\nu_D$ are nearly the same and since $\Delta\nu_N \ll \Delta\nu_B$, it is clear that radiation trapping of an atomic beam must occur at lower density than for a vapor target. The fact that an atomic beam is severely radiation trapped at a density that is a factor of 3 lower than a vapor target is understood qualitiatively from the fact the photons are severely trapped perpendicular to the atomic-beam axis and can only escape when they have a considerable component of their wave vector along the atomic-beam axis.

If an atomic beam is formed using a supersonic nozzle, the beam has a large average value of $v_\parallel$ but has a small distribution in $v_\parallel$ due to the adiabatic cooling as the beam emerges from the nozzle. In this situation the normalized line shape for absorption or emission will be narrow, and the optical-absorption cross section will be large, even parallel to the atomic beam axis. This leads to severe radiation trapping at atomic-beam densities even lower than for an atomic beam formed by effusion from an oven. In fact the density in a beam formed using a supersonic nozzle is limited by radiation trapping to a value that is comparable to an atomic beam formed by effusion from an oven at a temperature of T_1, where T_1 is the temperature of the beam after adiabatic cooling when it emerges from the supersonic nozzle.

The atoms in an atomic beam may have a significant component of velocity perpendicular to the atomic-beam axis except for very highly collimated atomic beams. If the component of velocity perpendicular to the atomic beam is greater than about 1% of the most probable component of velocity parallel to the atomic-beam axis, then the normalized line shape for absorption or emission perpendicular to the line shape is determined by the spread in the atomic velocities perpendicular to the atomic-beam axis. As the spread in atomic velocities perpendicular to the atomic-beam axis becomes large, the absorption cross section decreases. The density of the atomic beam, at which radiation trapping becomes important, increases toward the density at which radiation trapping becomes important in a vapor.

Radiation trapping limits the intensity of an atomic beam that can be polarized by optical pumping. As mentioned, the maximum atomic-beam intensity that can be optically pumped in a low magnetic field is a few particle milliamperes for an atomic beam that is 1 cm in diameter. In a high magnetic field somewhat higher atomic-beam intensities can be obtained.

Finally a brief discussion of the approximations utilized in the solution to the rate equations may be appropriate. There is no alkali-metal atom that has nuclear spin 0. The nuclear spin of ^{23}Na is $\frac{3}{2}$. The total angular momentum F of the Na atom is the vector sum of the electronic angular momentum plus the nuclear-spin angular momentum. For both the $3\,^2S_{1/2}$ level and the $3\,^2P_{1/2}$ level, $F=2$ or 1. In the $3\,^2S_{1/2}$ level, the $F=2$ and $F=1$ levels are separated by the ground-level hyperfine splitting. In the $3\,^2P_{1/2}$ level the $F=2$ and $F=1$ levels are separated by a hyperfine splitting of 1.92×10^8 Hz. As a result of these hyperfine splittings, the optical-absorption cross section for light of a given frequency ν is less than it would be if the alkali metal had zero nuclear spin and therefore no hyperfine structure. Thus, radiation trapping produces smaller effects for the real alkali-metal vapor than for a vapor of idealized alkali-metal atoms with $I=0$. It is expected that the polarization obtained by optical pumping at a given density can be somewhat higher than predicted by our calculations.

We have also made the assumption of uniform atomic-state densities n_1, n_2, n_3, and n_4 throughout the cylindrical atomic beam. Since radiation may escape the beam at $r=R$, the excited-state density is expected to be less near $r=R$ than at the axis of the cylinder. Our assumption of spatially flat-state densities presumes too large an excited-state density near $r=R$. This causes our calculations to overestimate the amount of depolarizing scattered radiation, thereby exaggerating the effects of radiation trapping. Thus, the assumption of uniform atomic-state densities also results in an underestimate of the polarization that can be produced by optical pumping.

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