Dispersion self-energy of the electron

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Electron mass renormalization and the Lamb shift have been investigated using the dispersion self-energy formalism. If shifts of both the electromagnetic field and quantum-mechanical transitions frequencies are considered, absorption from the electromagnetic field is canceled by emission due to atomic fluctuations. The frequencies of all modes are obtained from the self-consistency condition that the field seen by the electron is the same as the field produced by the expectation value of current. The radiation present can thus be viewed as arising from emission and subsequent reabsorption by matter. As developed here, the numerical predictions of dispersion theory are identical to those of quantum electrodynamics. The physical picture implied by dispersion theory is discussed in the context of semiclassical theories and quantum electrodynamics.

I. INTRODUCTION

There are several reasons for critical analysis of theories that treat charged particles quantum mechanically and the electromagnetic field classically according to Maxwell's equations. Such semiclassical theories have proved extremely useful where they give correct results,[1] and it is important to know their limitations. Semiclassical models, especially Jaynes's neoclassical theory[2] (NCT), have stimulated examination of the foundations of quantum electrodynamics[3] (QED) and motivated some very fundamental experiments.[4] While QED is undeniably very successful in its quantitative predictions, it is conceptually and mathematically difficult. In NCT, spontaneous emission is attributed to the radiation reaction, and there is no need to introduce vacuum fluctuations. Jaynes has estimated that the energy flux per square centimeter in vacuum fluctuations for modes only down to the Compton wavelength is greater than the total flux from our sun.[5] However, field modes down to this wavelength are required to explain the Lamb shift, and it is important to establish whether or not these vacuum fluctuations are physically real.[6]

The high accuracy of the predictions of QED implies that a quantitatively correct theory will give the same results as QED in the situations tested experimentally to date, but the conceptual problems still remain. It is important to know the limitations of semiclassical theories so that they can be applied to those problems where they give correct results. Of more fundamental importance is the possibility that quantization of the electromagnetic field is redundant once the charged particles are described quantum mechanically. While experiments on nonlocal effects[4] seem to exclude all semiclassical theories, it is difficult to completely discredit a theory until its exact nature is known.

There are several formulations that describe the interaction of quantum particles with a classical electromagnetic field. Jaynes has discussed[7] theories that justify use of the semiclassical approach for large assemblies of atoms or molecules with a well-defined dipole moment to serve as the source of the field. In his own NCT he assumes that the charge currents of an individual atom are the source of a classical electromagnetic field. Crisp and Jaynes[2] substituted the vector potential derived from the current back into the Schrödinger equation to obtain a system of nonlinear equations for probability amplitudes in the presence of the electron's self-field. They concluded that spontaneous emission and the Lamb shift can result from the radiation reaction field of the atom itself. Recently, Barut and co-workers have developed a self-energy formulation[8,5] that gives similar predictions to QED. They eliminate the field, as does Jaynes, and obtain an action integral that involves only the wave function. From this the interaction energy is obtained. They apply their theory to a wide variety of problems, including the Lamb shift and spontaneous emission.

There is another semiclassical formulation that appears to be closely related to theories mentioned above.[9] Rather than eliminating the electromagnetic field, the particle wave function is eliminated. The condition for the existence of self-consistent solutions gives an equation for the frequency shifts of the field modes. The interaction energy is then obtained from the sum of these frequency shifts. While equating the interacting energy to the zero-point-energy change of the field may seem to identify this method with QED, the use of the semiclassical approach results in considerable simplification in the mathematics. As discussed by Boyer,[10] the use of the zero-point energy may not imply the same conceptual basis as that of QED. This approach has been extensively applied to dispersion forces, including retarded and nonretarded van der Waals interactions between atoms and macroscopic bodies,[9] and is in good agreement with experiment. It will be referred to here as dispersion theory. Using dispersion theory, Mahanty calculated the Lamb shift,[11] and obtained Welton's semiclassical result.[12] His objective was to develop a recipe for a semiquantitative estimation of radiative effects in a system for which rigorous QED calculations cannot easily be made.

He concluded that, with the size of the atom taken into account, the dispersion self-energy formalism is mathematically similar to NCT.[2] His equations are based on the dipole approximation, and he attributes the energy splitting to shifts in the electromagnetic frequencies.

It is the purpose of the present paper to further develop the predictions of semiclassical dispersion theory and compare it to NCT and other semiclassical theories, and to QED. In Sec. II a formulation of dispersion theory will be derived that includes all multipoles and both the quantum-mechanical transition frequencies and the natural frequencies of the medium. To provide a simple example, it will be applied to the polaron in Sec. III. Renormalization of electron mass and the Lamb shift will be considered in Sec. IV. In the final section, dispersion theory will be compared with semiclassical theories and with QED.

II. DISPERSION SELF-ENERGY OF AN ELECTRON

An electron in a medium interacts with its self-field. The electric and magnetic fields will be thought of as real and classical, while the electron is quantum mechanical and is described by its Schrödinger equation. It will be assumed that the system is at zero temperature. The self-field will be treated as a time-dependent perturbation. Four-vector current J and vector potential A will be defined for conciseness. The first three components are the three-vector Schrödinger current $\mathbf{j}$ and vector potential $\mathbf{A}$ and the four-vectors are $J=(\mathbf{j},ic\rho)$ and $A=(\mathbf{A},i\phi/c)$. Here ρ is charge density, ϕ is electric potential, c is the speed of light, and $i=\sqrt{-1}$. In SI (Système Internationale) units and the Lorentz gauge, the self-field contribution to the potential, A_s^α, satisfies

$$\nabla^2 A_s^\alpha(\mathbf{r},t)-\frac{1}{c^2}\frac{\partial^2 A_s^\alpha(\mathbf{r},t)}{\partial t^2}=-\mu J^\alpha(\mathbf{r},t)\ ,$$

where μ is magnetic permeability. The equation for the potential can be integrated, and the retarded solution is

$$A_s^\alpha(\mathbf{r},t)=\frac{\mu}{4\pi}\int d^3r' J^\alpha(\mathbf{r}',t-|\mathbf{r}-\mathbf{r}'|/c)\frac{1}{|\mathbf{r}-\mathbf{r}'|}\ . \tag{1}$$

The above form is useful for the physical interpretation, since it clearly represents the field due to current density at all points $\mathbf{r}'$. If the mode of wave vector $\mathbf{k}$ has frequency $\omega_\mathbf{k}$ and phase $\varphi_\mathbf{k}$, the solution can also be written in Fourier expanded form as

$$A_s^\alpha(\mathbf{r},t)=\frac{\mu}{V}\sum_\mathbf{k}\frac{1}{k^2-\omega_\mathbf{k}^2/c^2}\times\left[\int d^3r' J^\alpha(\mathbf{r}',\omega_\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{r}'}\right]e^{i\mathbf{k}\cdot\mathbf{r}}\ . \tag{2}$$

The potential can also be written formally as

$$A^\alpha(\mathbf{r},t)=\sum_\mathbf{k} A_\mathbf{k}^\alpha e^{i\mathbf{k}\cdot\mathbf{r}}\cos(\omega_\mathbf{k}t+\varphi_\mathbf{k})+A_0^\alpha(\mathbf{r},t)\ , \tag{3}$$

which is real provided $\mathbf{A}_{-\mathbf{k}}=\mathbf{A}_\mathbf{k}^*$. The term A_0^α is due to any external field that may be present.

The Hamiltonian that describes the electron in an electromagnetic field is

$$\hat{H}=-\frac{\hbar^2}{2m}\nabla^2-\frac{i\hbar e}{2m}(\nabla\cdot\mathbf{A}+\mathbf{A}\cdot\nabla)+\frac{e^2}{2m}A^2-e\phi\ .$$

The zero-order Hamiltonian, $\hat{H}_0$, will be taken to include all contributions except the self-field terms. Its eigenfunctions $\Psi_n(\mathbf{r},t)$ and eigenvalues E_n satisfy $\hat{H}_0\Psi_n=E_n\Psi_n$. The remainder of the Hamiltonian is

$$\hat{H}'=-\frac{i\hbar e}{2m}(\nabla\cdot\mathbf{A}_s+\mathbf{A}_s\cdot\nabla)+\frac{e^2}{2m}(A_s^2+2\mathbf{A}_0\cdot\mathbf{A}_s)-e\phi_s\ .$$

The linear terms in $\mathbf{A}$ can be expressed in terms of the Schrödinger current by integrating by parts or using the Hermitian properties of the momentum operator

$$\begin{aligned}\int d^3r(\Psi_n^*\mathbf{A}\cdot\nabla\Psi_{n'}+\Psi_n^*\nabla\cdot\mathbf{A}\Psi_{n'})&=\int d^3r\,\mathbf{A}\cdot(\Psi_n^*\nabla\Psi_{n'}-\Psi_{n'}\nabla\Psi_n^*)\\&=\frac{2mi}{e\hbar}\int d^3r\,\mathbf{A}\cdot\mathbf{j}_{nn'}\ ,\end{aligned}$$

where

$$\begin{aligned}&\mathbf{j}_{nn'}\equiv-\frac{e\hbar}{2mi}(\Psi_n^*\nabla\Psi_{n'}-\Psi_{n'}\nabla\Psi_n^*)\ ,\\&\rho_{nn'}\equiv-e\Psi_n^*\Psi_{n'},\quad\langle\Psi_n^*\hat{H}'\Psi_{n'}\rangle=-\langle A_s^\alpha J_{nn'}^\alpha\rangle\end{aligned} \tag{4}$$

if the A^2 terms are neglected. The angular brackets denote integration over three-dimensional space. The electron wave function can be expanded as

$$\Psi_n^{(1)}(\mathbf{r},t)=\sum_{n'} a_{n'}(t)\Psi_{n'}(\mathbf{r},t)\ , \tag{5}$$

where n denotes the ground state. If the expansion coefficients are evaluated using second-order perturbation theory, with the interaction turned on at $t=-\infty$ using a factor $e^{\epsilon t}$ that is set equal to unity after integrating, one obtains

$$a_{n'}(t)=\frac{1}{2\hbar}\sum_\mathbf{k}\int d^3r\left[\frac{A_\mathbf{k}^\alpha J_{n'n}^\alpha e^{i\mathbf{k}\cdot\mathbf{r}}}{\omega_{n'n}-\omega_\mathbf{k}}e^{-i(\omega_\mathbf{k}t+\varphi_\mathbf{k})}+\frac{A_\mathbf{k}^\alpha J_{n'n}^\alpha e^{i\mathbf{k}\cdot\mathbf{r}}}{\omega_{n'n}+\omega_\mathbf{k}}e^{i(\omega_\mathbf{k}t+\varphi_\mathbf{k})}\right]\ . \tag{6}$$

The current in Eq. (1) is approximately, from Eq. (5),

$$J^\alpha=J_{nn}^\alpha+\sum_{n'}[a_n^*(t)J_{n'n}^\alpha+a_n(t)J_{nn'}^\alpha]\ . \tag{7}$$

For self-consistency, the potential that acts on the electron should be the same as the potential produced by it. Substitution of Eqs. (4), (6), and (7) into Eq. (2) then gives an expression for the amplitudes of the self-field modes $A^{\alpha}_{\mathbf{k}}$, with the coefficients $a_{n'}$ eliminated. Equating this to Eq. (3) and replacing $A^{\alpha *}_{\mathbf{k}}$ with $A^{\alpha}_{-\mathbf{k}}$ and then $\mathbf{k}'$ with $-\mathbf{k}'$ in the $a^{*}_{n'}$ terms gives

$$\sum_{\mathbf{k},\alpha} A^{\alpha}_{\mathbf{k}}\left[\delta_{\mathbf{k},\mathbf{k}'}\delta_{\alpha,\beta}-\frac{\mu c^2}{\hbar V(k^2c^2-\omega^2_{\mathbf{k}})}\sum_{n'}{}'\left[\frac{\langle e^{i\mathbf{k}'\cdot\mathbf{r}'}J^{\beta}_{nn'}\rangle\langle e^{-i\mathbf{k}\cdot\mathbf{r}}J^{\alpha}_{n'n}\rangle}{\omega_{n'n}+\omega_{\mathbf{k}}}+\frac{\langle e^{-i\mathbf{k}'\cdot\mathbf{r}'}J^{\alpha}_{nn'}\rangle\langle e^{i\mathbf{k}\cdot\mathbf{r}}J^{\beta}_{n'n}\rangle}{\omega_{n'n}-\omega_{\mathbf{k}}}\right]\right]=0\ . \tag{8}$$

For any given $\mathbf{k}$, only the two transverse components of A and a third component, say $i\varphi/c$, are independent.

While the form of Eq. (1) suggests that J should be thought of as a real current and charge distribution producing a real classical vector and scalar potential, it is not necessary to adopt this interpretation. Consistency with the generally accepted version of the quantum mechanics requires that this must be thought of only as a self-consistency requirement. It is not necessary to reinterpret quantum mechanics as is done in NCT to write these equations, although this possibility is not excluded. However, nothing introduced so far implies that $\mathbf{A}$ and ϕ give other than real classical fields.

Approximate solutions to Eq. (8) can be found by setting the frequency equal to kc or $\omega_{n'n}$ and $\mathbf{k}'=\mathbf{k}$ and combining terms with the same frequency; for example, the two components of the transverse modes. All terms that do not contain the inverse of the difference between $\omega_{\mathbf{k}}$ and the selected frequency are small by comparison. The resulting expression is exact for a free electron where modes with different wave vectors are not coupled. The equation for the frequencies ω that satisfy the self-consistency condition is then

$$\omega^2=\frac{1}{2}\left[k^2c^2+\omega^2_{n'n}\pm\left[(k^2c^2-\omega^2_{n'n})^2+\sum(\mu\omega_{n'n}/V)\langle e^{i\mathbf{k}\cdot\mathbf{r}}J^{\alpha}_{n'n}\rangle\langle e^{-i\mathbf{k}\cdot\mathbf{r}}J^{\alpha}_{nn'}\rangle\right]^{1/2}\right]\ . \tag{9a}$$

The sum is over modes with the same zero-order frequency. This looks like a system of coupled classical oscillators.[6] Field modes are pushed aside when new modes are inserted. If kc is not too close to $\omega_{n'n}$, the frequency shifts are

$$\Delta\omega_{kc}=\frac{\mu c^2}{V\hbar}\sum_{n',\alpha}\langle e^{-i\mathbf{k}\cdot\mathbf{r}'}J^{\alpha}_{nn'}\rangle\langle e^{i\mathbf{k}\cdot\mathbf{r}}J^{\alpha}_{n'n}\rangle\times[(kc)^2-\omega^2_{n'n}]^{-1}\frac{\omega_{n'n}}{kc}\ , \tag{9b}$$

$$\Delta\omega_{n'n}=-\frac{\mu c^2}{V\hbar}\sum_{\mathbf{k},\alpha}\langle e^{-i\mathbf{k}\cdot\mathbf{r}'}J^{\alpha}_{nn'}\rangle\langle e^{i\mathbf{k}\cdot\mathbf{r}}J^{\alpha}_{n'n}\rangle\times[(kc)^2-\omega^2_{n'n}]^{-1}\ . \tag{9c}$$

Equation (8) has a solution only if the corresponding determinant is zero, and manipulation of this determinant can be used to determine the frequencies $\omega_{\mathbf{k},l}$ required for self-consistency. The subscript $\mathbf{k}$ refers to the fact that these frequencies are wave-number dependent, while the subscript l indicates that more than one solution is possible for each $\mathbf{k}$. These frequencies are approximately its quantum-mechanical frequencies $\omega_{n'n}$ and the frequencies of the electromagnetic field kc.

In dispersion theory, the interaction energy is expressed in terms of a sum of frequency shifts as

$$\Delta E=\sum_{\mathbf{k},l}\hbar\Delta\omega_{\mathbf{k},l}[n(\omega_{\mathbf{k},l},T)+\tfrac{1}{2}]\ ,$$

where n is usually interpreted as number of bosons $[\exp(\hbar\omega/k_BT)-1]^{-1}$. It is conventional in dispersion theory to write this interaction energy in terms of a contour integral,[9] giving at zero temperature

$$\Delta E=\frac{\hbar}{4\pi i}\sum_{\mathbf{k},l}\oint d\omega\,\omega\frac{d}{d\omega}\ln\left[\frac{\omega-(\omega_{\mathbf{k},l}+\Delta\omega_{\mathbf{k},l})}{\omega-\omega_{\mathbf{k},l}}\right]$$

$$=-\frac{\hbar}{4\pi i}\sum_{\mathbf{k},l}\oint d\omega\ln\left[\frac{\omega-(\omega_{\mathbf{k},l}+\Delta\omega_{\mathbf{k},l})}{\omega-\omega_{\mathbf{k},l}}\right]$$

after integrating by parts. Equation (8) is of the form $|\delta_{i,j}+G_{i,j}|=0$, which can be diagonalized to give $|\delta_{i,j}(\omega-\omega_{i,j}-\Delta\omega_{i,j})/(\omega-\omega_{i,j})|=0$. The sum of the allowed frequencies shifts may be obtained from the logarithm of the trace of the matrix G as $-(\hbar/4\pi i)\oint d\omega\sum_1^{\infty}(1/r)\,\mathrm{Tr}(-G)^r$ or

$$\Delta E=\frac{\hbar}{4\pi i}\sum_r\frac{1}{r}\oint d\omega\,\mathrm{Tr}\left[\frac{\mu}{\hbar V(k^2-\omega^2/c^2)}\sum_{n'}{}'\left[\frac{\langle e^{i\mathbf{k}'\cdot\mathbf{r}'}J^{\beta}_{nn'}\rangle\langle e^{-i\mathbf{k}\cdot\mathbf{r}}J^{\alpha}_{n'n}\rangle}{\omega_{n'n}+\omega}+\frac{\langle e^{-i\mathbf{k}'\cdot\mathbf{r}'}J^{\alpha}_{n'n}\rangle\langle e^{i\mathbf{k}\cdot\mathbf{r}}J^{\beta}_{n'n}\rangle}{\omega_{n'n}-\omega}\right]\right]^r\ . \tag{10}$$

The subscript $\mathbf{k}$ has been dropped on ω, since the contour integral automatically selects a wave-vector-dependent frequency. The frequencies are positive, and thus the contour integral should enclose the positive but not the negative ω axis. Using the Lorentz gauge condition $\nabla\cdot\mathbf{A}+c^{-2}\partial\varphi/\partial t=0$, the longitudinal component $A^3_{\mathbf{k}}$ can be eliminated. The charge-current density continuity equation then implies that $J^3_{nn'}J^3_{n'n}+J^4_{nn'}J^4_{n'n}=[1-(\omega/kc)^2]J^4_{nn'}J^4_{n'n}$, which eliminates the pole at $\omega=kc$ for the longitudinal modes. This gives exactly what would have been obtained using the Coulomb gauge, since in this case the φ part of the solution is not retarded, so that $k^2-\omega^2/c^2$ is replaced by k^2, and the longitudinal field modes are generated entirely from φ.

III. LONGITUDINAL MODES AND THE FRÖHLICH POLARON

The interaction of an electron with the longitudinal-optical modes of a polar crystal is a good test of the formulation developed here. To quote Fröhlich, "this case provides a very simple example for a nonrelativistic field theory, and in view of its simplicity it might be expected to lead to the discovery of a number of new features of such fields and to the development of new methods."[13]

Since this interaction is longitudinal, only the fourth component in Eq. (10), that is, the φ part, contributes. The lattice is taken into account as a frequency-dependent dielectric function:

$$\epsilon(\omega)=\epsilon_0(\omega^2-\omega_{\rm LO}^2)/(\omega^2-\omega_{\rm TO}^2) .$$

Here $\omega_{\rm LO}$ and $\omega_{\rm TO}$ are the longitudinal- and transverse-optical frequencies, and ϵ_0 is the high-frequency dielectric function and is identified with the permittivity of vacuum for simplicity. If only the $r=1$ term is taken in Eq. (10),

$$\Delta E=-\frac{\hbar}{4\pi i}\oint d\omega\,\mathrm{Tr}\left[\frac{e^2}{\hbar V\mathbf{k}^2\epsilon_0}\frac{\omega^2-\omega_{\rm TO}^2}{\omega^2-\omega_{\rm LO}^2}\sum_{n'}{}'\left[\frac{\langle n|e^{i\mathbf{k}'\cdot\mathbf{r}'}|n'\rangle\langle n'|e^{-i\mathbf{k}\cdot\mathbf{r}}|n\rangle}{\omega_{n'n}+\omega}+\frac{\langle n|e^{-i\mathbf{k}'\cdot\mathbf{r}'}|n'\rangle\langle n'|e^{i\mathbf{k}\cdot\mathbf{r}}|n\rangle}{\omega_{n'n}-\omega}\right]\right]$$

$$=-\frac{e^2(\omega_{\rm LO}^2-\omega_{\rm TO}^2)}{2V\epsilon_0\omega_{\rm LO}}\sum_{\mathbf{k}}\sum_{n'}{}'\frac{1}{\mathbf{k}^2}\frac{\langle n|e^{i\mathbf{k}\cdot\mathbf{r}'}|n'\rangle\langle n'|e^{-i\mathbf{k}\cdot\mathbf{r}}|n\rangle}{\omega_{n'n}+\omega_{\rm LO}}+\frac{e^2}{2V\epsilon_0}\sum_{\mathbf{k}}\sum_{n'}{}'\frac{1}{\mathbf{k}^2}\langle n|e^{i\mathbf{k}\cdot\mathbf{r}'}|n'\rangle\langle n'|e^{-i\mathbf{k}\cdot\mathbf{r}}|n\rangle . \quad (11)$$

The first term is due to the electron-lattice interaction, while the second term in Eq. (11) is the longitudinal self-energy of an electron in vacuum.

The electron-lattice interaction will be considered first. For a free electron, integration over plane-wave states $e^{i\mathbf{k}_n\cdot\mathbf{r}}/\sqrt{V}$ gives $\mathbf{k}_{n'}=\mathbf{k}_n+\mathbf{k}$, and the sum can be converted to an integral and evaluated to give

$$\Delta E=-\alpha\hbar\omega_{\rm LO}\,\mathrm{arcsin}(r_pk_n)/(r_pk_n) ,$$

where $r_p=(\hbar/2m\omega_{\rm LO})^{1/2}$ is the polaron radius and $\alpha=[e^2/(8\pi r_p)][1/\epsilon_0-1/\epsilon(0)]/\hbar\omega_{\rm LO}$ is the Fröhlich coupling constant. This result, in precise agreement with the second quantized Fröhlich theory, was obtained only after considering both the longitudinal-optical modes and the modes associated with the transition frequencies $\omega_{n'n}$.[14] If either of these frequencies is omitted, the term proportional to $(\omega_{n'n}+\omega_{\rm LO})^{-1}$ is wrong by a factor of 2 and an "emission" term, proportional to $(\omega_{n'n}-\omega_{\rm LO})^{-1}$, contributes even at zero temperature. The emission term drops out only because the divergent contribution to the energy change associated with the electron modes is exactly compensated by a contribution of opposite sign from the lattice modes.

The self-consistency condition, Eq. (8), does not directly determine the amplitude of the field modes, but only their frequencies. However, if the above result is to be consistent with the perturbation theory for the electron, the energy must be of the form $\Delta E=\hbar^{-1}\sum_{n'}{}'H'_{nn'}H'_{n'n}/(\omega_{n'n}\pm\omega)$. The potential that would give this energy shift is

$$\varphi(\mathbf{r},t)=[\hbar(\omega_{\rm LO}^2-\omega_{\rm TO}^2)/2V\epsilon_0\omega_{\rm LO}]^{1/2}\times\sum_{\mathbf{k}}(e^{i\mathbf{k}\cdot\mathbf{r}}/k)\exp[-i(\omega_{\mathbf{k}}t+\varphi_{\mathbf{k}})] .$$

This has the same amplitude as the potential in the usual Fröhlich Hamiltonian. The absence of a divergent term due to absorption of energy from the field at zero temperature is due to the cancellation of the contributions of the electron self-energy at frequency $\omega_{n'n}$ with those of the perturbation due to the lattice field at $\omega_{\rm LO}$. The potential is of the form of a dipole potential, being proportional to k^{-1}, and is due to dipoles that are polarized by the electron and act back on the electron.

To evaluate the second term in Eq. (11), the summation over n' can be performed using the completeness property of the wave functions corresponding to any zero-order Hamiltonian. $\sum_{n'}{}'\langle n|e^{i\mathbf{k}\cdot\mathbf{r}}|n'\rangle\langle n'|e^{-i\mathbf{k}\cdot\mathbf{r}}|n\rangle=\langle n|n\rangle$, since the $n'=n$ terms is zero for nonzero $\mathbf{k}$. The Coulomb self-energy of the electron is obtained. Equation (8) implies that only the frequencies approximately equal to $\omega_{n'n}$ satisfy the self-consistency condition and thus that the Coulomb self-energy, if nonzero, is associated with the atomic fluctuation frequencies.

IV. TRANSVERSE MODES

The interaction energy is $\frac{1}{2}\hbar$ multiplied by the sum of the frequency shifts and should include all wave numbers and both the electron modes and the field modes. Equation (9a) is exact for a free electron and ΔE_t can be obtained from Eq. (9) or (10) and is

$$\Delta E_t=-\frac{\mu c}{2V}\sum_{n'}\sum_{\mathbf{k}}\frac{1}{k}\frac{\langle e^{i\mathbf{k}\cdot\mathbf{r}}J^\alpha_{n'n}\rangle_t\langle e^{-i\mathbf{k}\cdot\mathbf{r}}J^\alpha_{nn'}\rangle_t}{\omega_{n'n}+kc} .$$

The subscript t indicates that only transverse modes should be included. The integrals over current are, after including a factor $\sin^2\theta$ in the $\mathbf{k}$ integral to eliminate longitudinal modes,

$$\langle e^{i\mathbf{k}\cdot\mathbf{r}}J^\alpha_{n'n}\rangle_t\langle e^{-i\mathbf{k}\cdot\mathbf{r}}J^\alpha_{nn'}\rangle_t=(e\hbar/m)^2\mathbf{k}_n\cdot\mathbf{k}_n\delta_{\mathbf{k}_{n'},\mathbf{k}_n+\mathbf{k}}\sin^2\theta .$$

Approximately the denominator $\omega_{n'n}+kc$ as kc gives

$$\Delta E_t=-\frac{2}{3\pi}\left[\frac{e^2}{4\pi\epsilon_0}\right]\frac{\hbar^2k_n^2}{m^2c^2}\int_0^{k_{\max}}dk ,$$

in agreement with the lowest-order nonrelativistic treatment of electron mass renormalization.[15]

The Lamb-shift interaction is also identical to the nonrelativistic QED result, since the current integral is pro-

portional to the momentum integral for transverse modes,

$$\Delta E = -\frac{2}{3\pi}\frac{e^2\hbar^2}{4\pi\epsilon_0}\sum_{n'}{}' \frac{|\langle n|e^{i\mathbf{k}\cdot\mathbf{r}}\nabla|n'\rangle|^2}{m^2c}\int\frac{kdk}{\omega_{n'n}+kc},$$

from Eq. (9) or (10). There is no term proportional to $\omega_{n'n}-kc$, but it is surprising that this result has been obtained, since it is due solely to the balance between emission and absorption at both the electron transition frequencies and the field frequencies.

Consistency with perturbation theory for the electron requires that the amplitude of the potential seen by the electron is $(\hbar\mu c/2Vk)^{1/2}$ as in QED (Ref. 16) if all of ΔE_L is attributed to a field at frequency kc. It is a factor $\sqrt{2}$ smaller if contributions to this energy shift at frequencies kc and $\omega_{n'n}$ are considered separately. In the latter case, contributions at these two frequencies add for the terms that describe virtual transitions with photon emission and cancel for the photon absorbing transitions. Only the photon absorbing transitions can conserve energy, and thus only these could result in real transitions (if they were not canceled by the atomic fluctuations).. The field is associated only with the interaction, and note that it is the frequency shift rather than the frequency that was originally used to determine it. Proportionality to $1/\sqrt{k}$ came from the inverse distance dependence with retardation after inclusion of both the field and transition frequencies. While in NCT the atom is not correlated with its own radiation field, in dispersion theory this interaction is coherent. In the latter case, the product of vector potential times current is averaged, whereas in the former they are averaged separately. This may be the primary difference between NCT and dispersion theory. The amplitude of the field suggests that energy is conserved, for example, when radiation is emitted or absorbed, in contrast to the usual semiclassical picture where a classical field from a single emission process can stimulate more than one absorption event. Recently Crisp[17] concluded that in NCT "the equations of motion are found to have a constant of the motion that can be interpreted as stating that the sum of the atomic energy, energy of interaction, and the energy stored in the electromagnetic field is constant."

V. DISCUSSION

The predictions of dispersion theory developed here, including the Lamb shift and lack of a term proportional to $(kc-\omega_{n'n})^{-1}$ that would result in absorption from the zero-temperature vacuum field, are in agreement with experiment. They are also in agreement with QED. There is no divergence at $kc=\omega_{n'n}$, since these frequencies are shifted away from each other by their interaction. Since dispersion theory assumes that the electron-medium interaction energy is equal to the sum of the frequency shifts multiplied by $\frac{1}{2}\hbar$, it is possible that dispersion theory is completely equivalent to QED to the order calculated and in the situations considered here. If this is the case, its use is justified by the straightforward mathematics required and especially by the simplicity and directness of the physical interpretation that it suggests. It is not possible to be certain at this stage in the examination of the dispersion-theory formalism whether it is a true semiclassical theory or an alternate formulation of QED.

One of the key issues is why the interaction energy should be expressible as the sum of the frequency shifts multiplied by $\frac{1}{2}\hbar$. There appear to be several alternatives including the following. (i) All the modes are quantized, even in vacuum. (ii) Lorentz invariance requires that the field energy be linear in frequency for a classical or quantum theory, as proved by Boyer in his development of random electrodynamics.[18] (iii) An argument of the form of the fluctuation dissipation theorem[19] requires that the field energy be that of a system of bosons if there is a well-defined linear relationship between current and voltage so that impedance can be defined. In dispersion theory, the frequencies and hence the interaction energy are obtained from the self-consistency condition that the field produced by the electron is the field that it sees. This suggests that the electromagnetic field is the result of emission by the atom or free electron and does not necessarily imply a nonzero field in vacuum.

The physical model of the interaction process suggested by the present formulation of dispersion theory is different from the model in a second quantized theory. At each wavelength there are two modes that are important in the physics of the electron-medium interaction, the medium's field frequency and the electron's quantum-mechanical transition frequency. The former generates virtual transitions to higher-energy states, while the latter gives the transverse self-energy. Qualitatively and quantitatively correct results are obtained only if the effects of both these frequencies are taken into account. There is no need to assume that the atom cannot absorb energy from the field at zero temperature, since this effect is canceled by atomic fluctuations. In QED the picture is complicated by questions of the ordering of operators.[20] Milonni has identified the atom source modes with the transition frequencies and the field modes with the frequencies kc. In dispersion theory, both are the result of self-consistent emission and absorption by the source. Fain[21] has pointed out that a ground-state atom does not undergo spontaneous absorption due to vacuum field fluctuations because in this state they are exactly canceled by the atomic fluctuations. In the formulation of dispersion theory presented here it is more natural to suppose that the field fluctuations are also due to the atom.

Using the conventional line of reasoning in dispersion theory, where an energy of $\frac{1}{2}\hbar\omega$ per mode is assumed, the amplitude of the field can be obtained by assuming that, for a consistent theory, the same interaction energy should result from dispersion theory as from perturbation theory. By this argument, the electric potential seen by an electron in a polar crystal is the Fröhlich dipole potential that can be ascribed to polarization of the lattice by the electron itself. However, it is stripped of the usual phonon annihilation and creation operators. It is only after addition of contributions at the quantum-mechanical transition frequencies that the usual result is obtained and absorption is eliminated. For an electron in

vacuum, the field amplitude obtained from the above argument has the same $\sqrt{k}$ dependence as the amplitude in QED. However, the amplitude of the field at frequency kc can differ by a factor $\sqrt{2}$.

Although it has been assumed in the past that dispersion theory is equivalent to Jaynes's NCT, the quantitative predictions of these two formalisms differ. If the frequency shifts in Eq. (9) are compared with Jaynes's frequency shifts,[2] it can be seen that the former shifts do not contain the probability amplitudes. The difference in the expressions for the frequency shifts and interaction energies obtained here and in Barut's theory[8] as compared to NCT requires explanation.[22,23] In dispersion theory, the potential seen by the electron is not that of a moving point charge, but rather its square root. For the polaron, the physical interpretation is that the electron sees the dipole field of the lattice induced by the point charge itself. In the case of an electron in a vacuum that can support propagating transverse electromagnetic waves, the electric field seen by the electron is proportional to $\sqrt{k}$, and the electron sees the field of this electromagnetic wave. In NCT, the fluctuations of the atom are assumed to be uncorrelated with its own radiation field. As a consequence, the field and the current are averaged separately. In dispersion theory, the interaction energy, which is the product of vector potential multiplied by current, is averaged. This implies that the atom interacts coherently with its radiation field. The expression obtained for the interaction energy, Eq. (10), is very similar to Barut's.

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