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# Parametric amplification in a low-density plasma sheath

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In this work we use the nonlinear properties of the sheath capacitance in a low-density plasma, whose purpose is to produce parametric amplification of a rf signal in the high-frequency band. The experiment has been carried out on the Linear Mirror Device LISA of the Universidade Federal Fluminense, where a helium plasma was produced using a radio-frequency source with a power that can be varied from 10 to 100 W. The experimental results show that it is practical to construct a parametric amplifier with high selective gain, which is very useful for the amplification of weak signals, where the gain factor and the relation between signal and noise are important. We find good agreement between the theoretical model proposed for the sheath capacitance and the experimental results.

## I. INTRODUCTION

In plasma-physics research, mainly in plasma diagnostics with resonant probes, attention is focused on the mechanisms that are important for the alternating current (ac) signals in the plasma sheath.

The resonant probe was studied first by Takayama, Ikegami, and Miyasaki.[1] The measured direct current (dc) component as a function of the frequency shows a resonant effect, which is the reason for this name. Preliminary experimental data measured with a resonant probe showed a resonance frequency equal to the local electron plasma frequency independent of the potential applied to the probe. Both of these conclusions are wrong and a correct interpretation was given by Harp.[2]

Aihara, Lampis, and Takayama[3] and later Rapozo *et al.*[4] measured the dependence of the sheath resonance on the applied rf voltage. The analysis of Rosa[5] of the sheath resonance as a function of transit time of the ions described the capacitive characteristic of the sheath, with the possibility of the negative admittance in the sheath resonance region.

In this work we consider the results obtained from the electrical model that we derived for the plasma sheath region, where the nonlinear characteristic of the sheath capacitance was used for the parametric amplification of the radio-frequency signals.

The technological application of these properties is very interesting for the amplification of low-frequency signals or very high frequency (VHF) used in satellite reception, where the parametric amplification in the low-noise-amplifier (LNA) class is indispensible and practically only exists in the microwave band that makes use of varicaps. The experimental data from which the plasma admittance profiles in this study are obtained and the sheath resonance frequency profile versus the applied rf voltage of the transmitting and receiving electrodes ($T$ and $R$), are obtained in a helium plasma produced by the rf source in the Linear Mirror Device LISA of the Universidade Federal Fluminense (Niterói, Rio de Janiero), as shown in Fig. 1. The experimental ensemble used to obtain the admittance profile versus frequency in the system is shown in Fig. 2. The main parameters of this device can be found in the work of Rapozo *et al.*[6] The helium plasma is weakly ionized ($<1\%$) and the pressure is $1.8\times10^{-4}$ Torr. A device whose main objective is to detect a possible parametric amplification of the sheath was constructed. In order to carry out the diagnostics, we have used a circular Langmuir probe, a Hitachi oscilloscope, and a linear wattmeter.

This work is organized as follows. In Sec. II the experimental devices are described. In Sec. III we present the electrical model for the plasma sheath and the electrodes $T$ and $R$ system and theoretical analysis. In Sec. IV we present the experimental results and analysis. The conclusions are discussed in Sec. V.

## II. EXPERIMENT

The experiment was carried out in the linear mirror machine LISA (Ref. 6) designed and constructed at the Max-Planck-Institut für Plasmaphysik (Garching, Germany).

The helium plasma is produced by a rf source in region $A$ in a cylindrical stainless-steel vessel (diameter 17 cm, length 255 cm) (Fig. 1). The rf source, whose power can be varied from 10 to 100 W, has been connected to a slow wave antenna (diameter 9.7 cm. length 22 cm, and pitch 1.5 mm); the forward of the power source has been measured with a linear wattmeter (model WL2300 Bird). The cylindrical vessel was separated in two regions, $A$ and $B$, by a metallic mesh isolated from the inner wall of the cylindrical vessel (Fig. 1). We use this metallic mesh for two main reasons; first it is used to shield the $B$ region from the rf fields, therefore there is no rf field in region $B$. Secondly we use the metallic mesh to act as an anode for electrons that are created in region $A$. The electrons are accelerated by the metallic mesh, from the high electron density (region $A$) to the low-density plasma (region $B$). The plasma density and the electron temperature are monitored by the radial Langmuir probe between the electrodes $T$ and $R$.

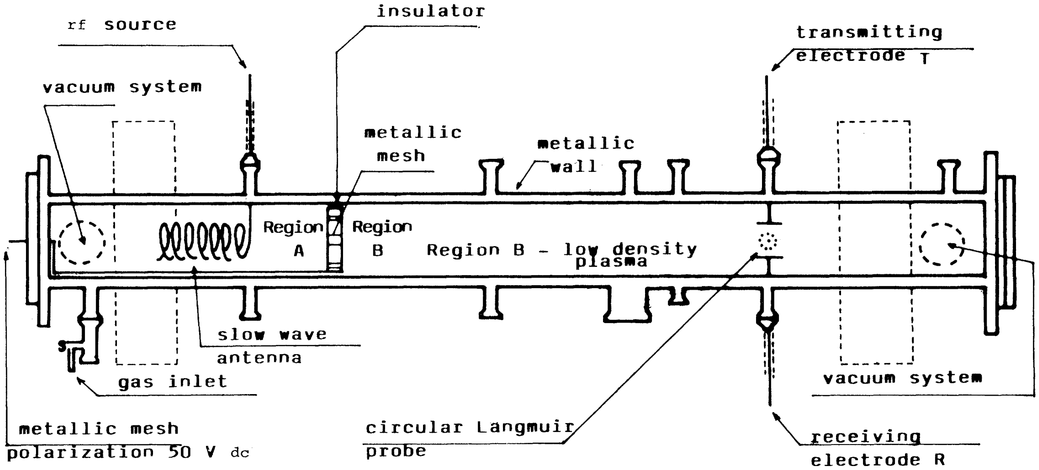

FIG. 1. The linear device LISA.

An oscillator ($G_1$) whose frequency can be varied from 190 kHz to 80 MHz and whose output voltage is varied between 0 and 7.0 V injects rf current into the low-density plasma (region $B$), through the movable plane radial electrode $T$, having a diameter of 3.0 cm (Fig. 1). The rf current transmitted across the plasma is received by the second radial electrode $R$ and it is fed to a 50-$\Omega$ load. The output voltage is measured by an oscilloscope.

For the electrode diameter fixed at 3.0 cm, we measured the sheath resonance frequency profile versus applied rf voltage $V$(rms) and the admittance profile versus frequency. In this experiment the distance between the circular radial electrodes $T$ and $R$ is kept fixed at 10 cm and the applied rf voltage was fixed at 3.0 V (rms).

Based on the results obtained by Rapozo *et al.*[4] for the sheath thickness profile versus the applied rf voltage $V$(rms) and the work of Rosa,[5] an experimental ensemble shown in Figs. 2 and 3 was built whose objective is to demonstrate possible parametric amplification due to the capactive characteristic of the sheath, a two-frequency $(\Omega,\omega_2)$ parametric amplifier where $G_1$ is the rf generator and $R_{g1}$ and $R_1$ are the internal resistance and load resistance of the generator, respectively. $L_1,C_1$ and $L_2,C_2$ are the tuning tank circuits that resonate at the angular frequencies $\omega_{r_1}$ and $\omega_{r_2}$ where $L_1$ and $L_2$ are the inductances and $C_1$ and $C_2$ the capacitances of the ensemble, respectively. $R_{g2}$ and $R_2$ are internal resistance and load resistance for the rf generator $G_2$ and $Z_p$ is a parametric impedance produced by the plasma in which we use the sheath capacitance as a parametric element.

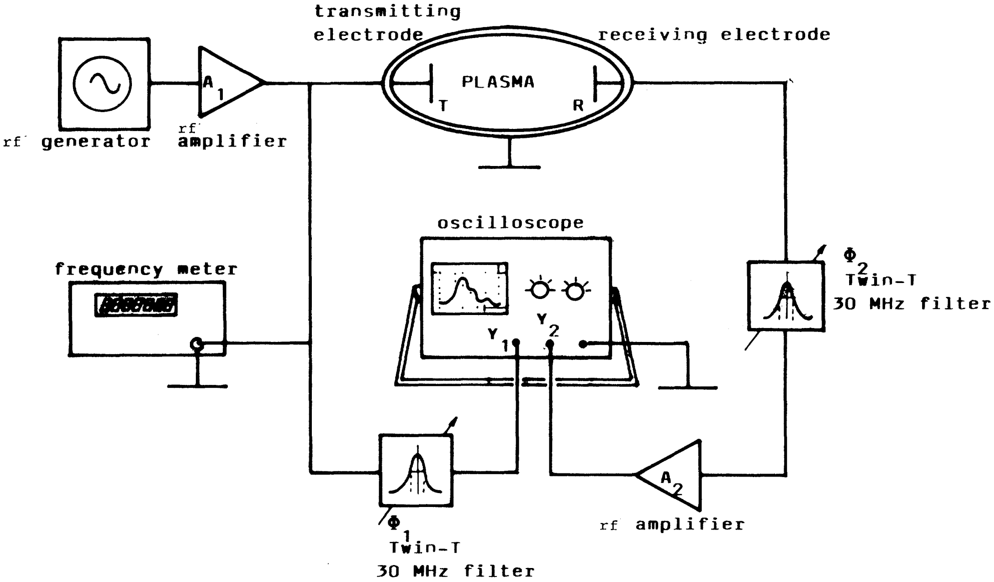

FIG. 2. Experimental ensemble used to obtain the admittance profile vs frequency between the electrodes $T$ and $R$.

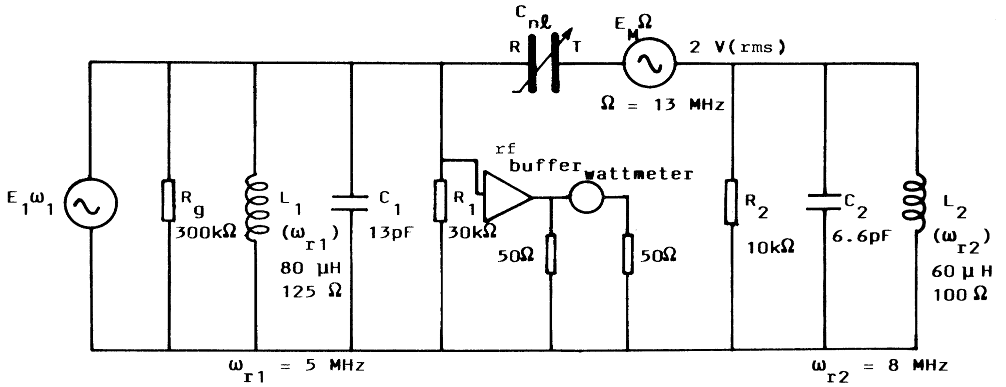

FIG. 3. Schematic circuit diagram of a parametric amplifier at two frequencies. $G_1, G_2 \rightarrow$ rf generators; $R_{g1}, R_{g2} \rightarrow$ internal resistances of the rf generators; $R_1, R_2 \rightarrow$ load resistances of the rf generators; $L_1, L_2 \rightarrow$ inductances of the system; $C_1, C_2 \rightarrow$ capacitances of the system; $Z_p \rightarrow$ parametric impedance produced by plasma; $\omega_{r_1} \rightarrow$ resonance frequency for the tank $L_1$ parallel to $C_1$; $\omega_{r_2} \rightarrow$ resonance frequency for the tank $L_2$ parallel to $C_2$, $T \rightarrow$ transmitting electrode; $R \rightarrow$ receiving electrode; $A \rightarrow$ to oscilloscope.

## III. PLASMA SHEATH ELECTRICAL MODEL AND THEORETICAL ANALYSIS

The aim of this section is to study the resonance and antiresonance of the admittance profile described by Rapozo *et al.*[4] The electrical model proposed for the system of transmitting ($T$) and receiving ($R$) electrodes, and the plasma sheath are shown in Fig. 4, where $C_E$ represent the capacitance between the electrodes $T$ and $R$, and $C_s$ is the sheath capacitance[5,7] which is defined by $C_s = \epsilon A / S$ ($A$ is the electrodes surface, $\epsilon$ is the dielectric constant, and $S$ is the sheath thickness); finally, $L_p$ and $R_p$ are the plasma inductance and the plasma resistance, respectively.

The impedance $Z_p$ of the circuit (Fig. 4) is

$$Z_p = \frac{(R_p + iX_{L_p} - iX_{C_s})(-iX_{C_E})}{R_p + iX_{L_p} - iX_{C_s} - iX_{C_E}} , \tag{1}$$

where $i = -\sqrt{i}$, $X_{L_p}$ is the inductive reactance due to $L_p$, $X_{C_s}$ is the capacitive reactance due to $C_s$, and $X_{C_E}$ is the capacitive reactance due to $C_E$.

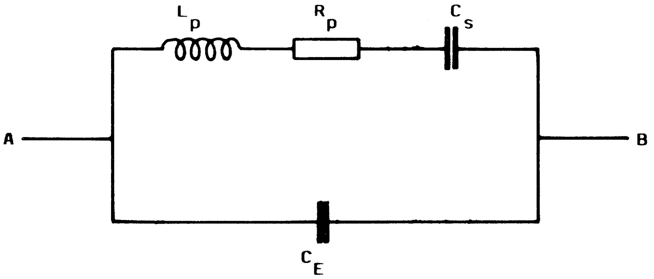

FIG. 4. Electrical model for the elements between the electrodes $T$ and $R$.

At resonance, $\mathrm{Im}[Z(\omega)] = 0$, so that we have

$$X_{C_E}(X_L - X_{C_S})(X_L - X_{C_t}) + X_{C_E} R_p^2 = 0 , \tag{2}$$

where $X_{C_t} = X_{C_s} + X_{C_E}$.

In a first approximation, we can assume $R_p = 0$ in (2), when calculating the resonance and antiresonance values, so that $X_L - X_{C_s} = 0$ and $X_L - X_{C_t} = 0$, which gives the roots for maximum admittance (sheath resonance) and for minimum admittance (plasma resonance), respectively,

$$\omega_a^2 = \frac{1}{LC_s} , \qquad \omega_b^2 = \frac{1}{LC_t} , \tag{3}$$

where

$$C_t = \frac{C_s C_E}{C_s + C_E} .$$

Figure 5 shows a qualitative admittance profile versus frequency $\omega$ for the proposed electrical model.

## IV. EXPERIMENTAL RESULTS AND ANALYSIS

In this work the main goal is to study the parametric amplification using the nonlinear properties of the sheath capacitance in a low-density helium plasma that was created in the metallic cylindrical vessel of the Linear Mirror Device LISA. More details about it can be found elsewhere in the works of Rapozo *et al.*[6]

The average electron density, measured with a circular Langmuir probe (diameter = 0.2 cm) between the radial electrodes (transmitting electrode $T$ and receiving electrode $R$) is $\bar{n} = 1.1 \times 10^6\ \mathrm{cm}^{-3}$.

Figure 6 shows the admittance profile versus frequency

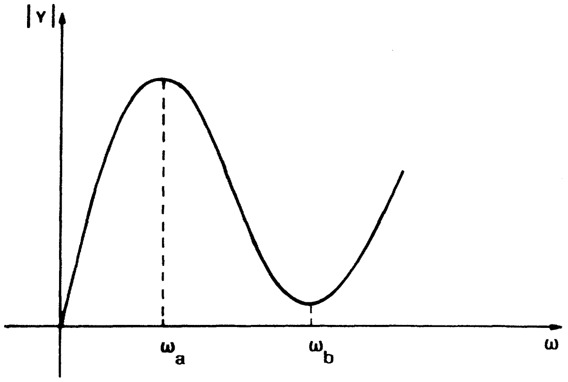

FIG. 5. Admittance profile vs frequency for the electrical model.

for an applied rf voltage of 3.0 V(rms). The objective of this profile is to avoid that the amplifier signal of frequency ($f_1=5$ MHz) will be equal to the sheath resonance frequency $\omega_s$. The admittance characteristic shows a sheath resonance frequency equal to 8.5 MHz and an average electron plasma frequency, $\bar{f}_{Pe}\cong 10.5$ MHz. These are coupled by the relation given by Harp *et al.*,[8]

$$f_R=\bar{f}_{Pe}/\sqrt{2S/d}\ , \tag{4}$$

where $f_R$ is the resonance sheath frequency and $\bar{f}_{Pe}$ is the average electron plasma density given by

$$\bar{f}_{Pe}=0.9\times 10^4\sqrt{\bar{n}}\quad (\mathrm{Hz,cm^{-3}})\ . \tag{5}$$

The distance $d$ between the electrodes $T$ and $R$ is fixed at 10 cm. The total number of electrons in the volume between the electrodes $T$ and $R$ is assumed to be constant during the variation of the sheath thickness produced by the rf injection. The average electron plasma density $\bar{n}$ is given by

$$\bar{n}=n_0\left[\frac{1-2S_0/d}{1-2S/d}\right]\ , \tag{6}$$

where $n_0$ and $\bar{n}$ are the electron plasma density without and with rf injection and $S_0$ and $S$ are the thickness of the electron plasma without and with the applied rf voltage.[4]

The amplification caused by the parametric effect due to the nonlinear characteristic of the sheath capacitance resulted in a maximum power gain $K_p(\omega_1)$ of 5.7 as shown in Fig. 7, which also shows that $K_p(\omega_1)$ is asymmetric about the resonance. The absence of symmetry can be explained by Fig. 8, which is the superposition of $K_p(\omega_1)$, $|Z_p/Z_0|$, and $|Z_1/Z_0|$ normalized profiles versus frequency $f_1$, where $|Z_p/Z_0|$ represents the plasma-electrode impedance system and $|Z_1/Z_0|$ represents the signal circuit impedance. $Z_0$ is the minimum impedance value $(1/Y_0)$ and $Y_0$ (Fig. 6) is the maximum admittance value (sheath resonance); $Y_0$ or $Z_0$ are measured with the experimental ensemble shown in Fig. 2.

The classical analysis of the two-frequency parametric amplifier gives for the ensemble of Fig. 3 a negative con-

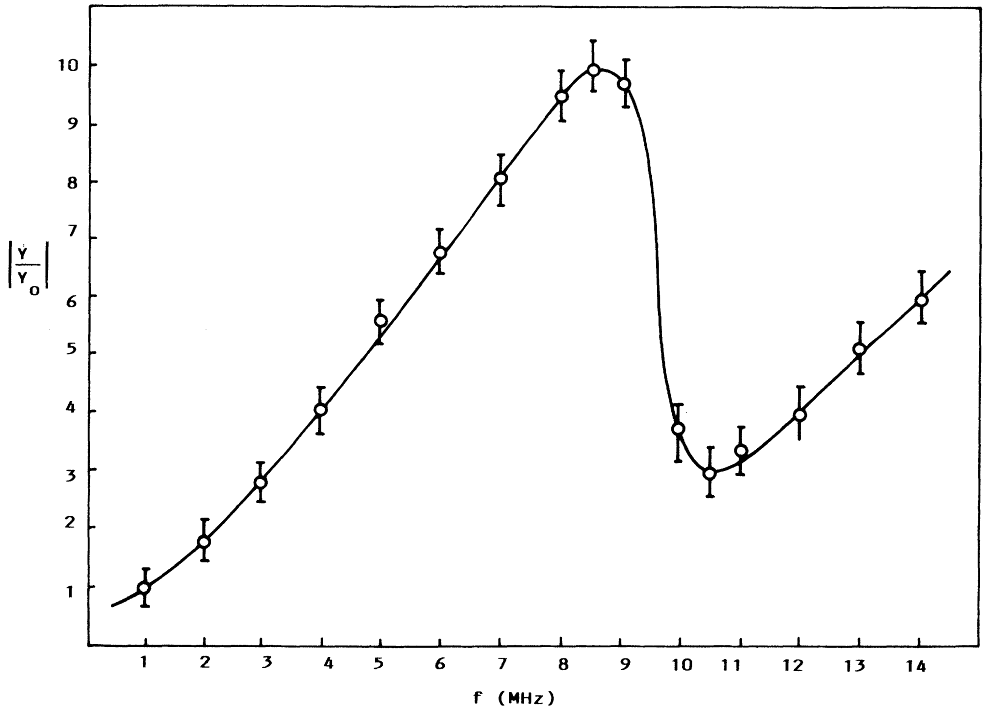

FIG. 6. Admittance profile vs frequency for an applied rf voltage of 3.0 V(rms).

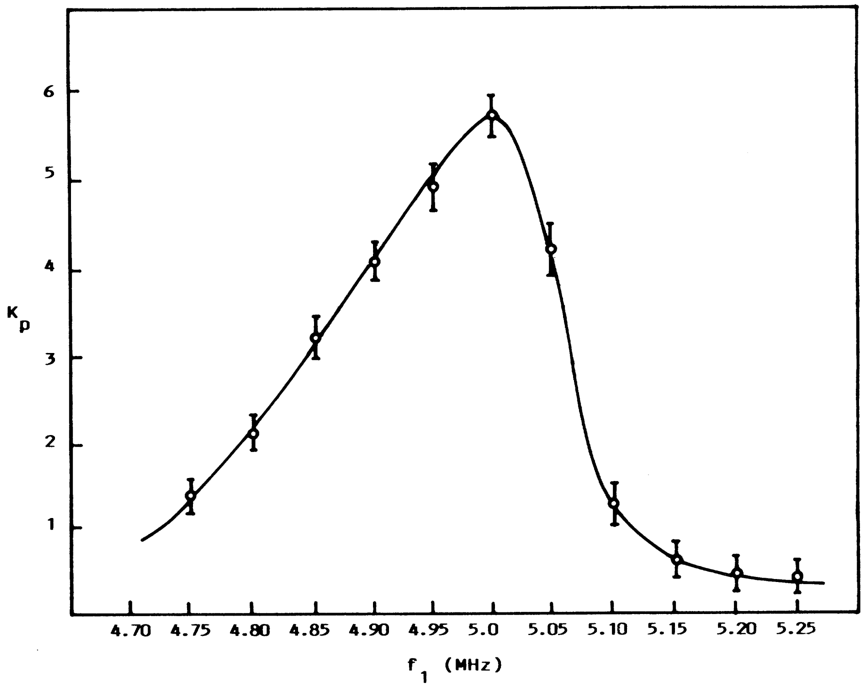

FIG. 7. Power gain profile $K_p$ vs input signal frequency close to the resonance.

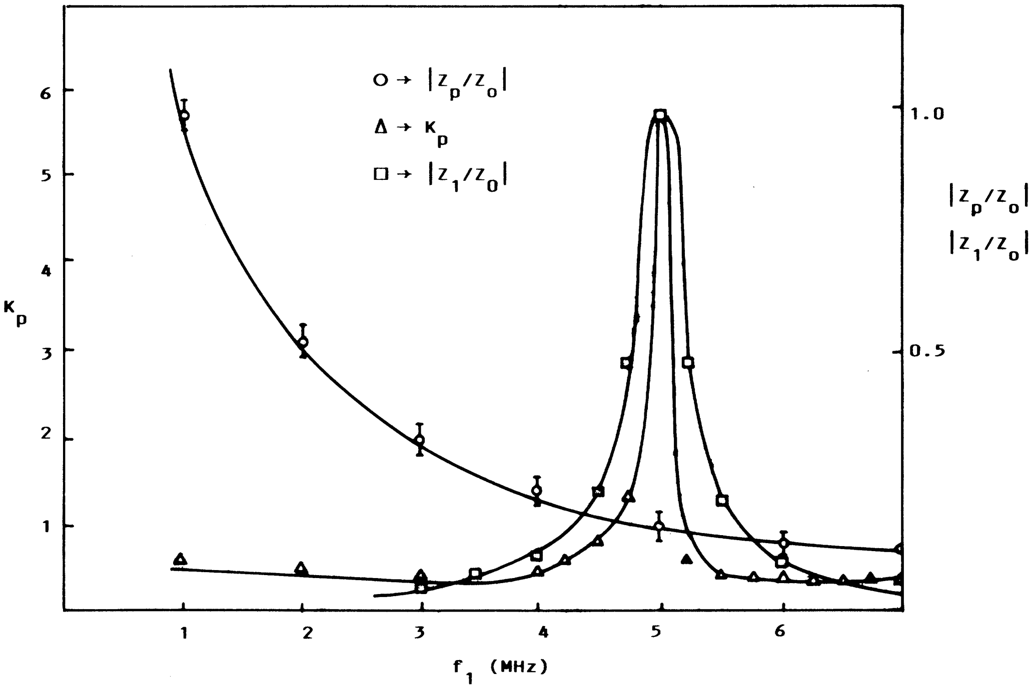

FIG. 8. Superposition of $K_p(\omega_1)$ and normalized profiles $|Z_p/Z_0|$ and $|Z_1/Z_0|$ vs frequency $f_1$.

ductance[9]

$$G_e = -\left[\frac{\Delta C}{2}\right]^2 \omega_1\omega_2|Z_2(\omega_2)| \tag{7}$$

when $Z_P$ is changed by a nonlinear capacitance. $\Delta C$ represents the variation of this capacitance due to the modulation with angular frequency $\Omega$ introduced by the rf generator $G_2$, $\omega_1$ is the frequency of the signal that needs to be amplified, and $\omega_2$ is the idler frequency. The power gain of the electrical circuit is given by[9]

$$K_p = \frac{1}{(1+G_e/2G_1)^2} , \tag{8}$$

where $G_1 = 1/R_1$ is the conductance of the signal circuit for resonance ($\omega_1 = \omega_{r_1}$) of the tank $L_1, C_1$.

In the two-frequency parameteric amplifier, which uses a nonlinear capacitance, a small variation in the frequency $\omega_1$ about the maximum $K_p$ leads to a symmetrical profile of the gain. In this work this is not observed because $Z_p$ cannot be considered as a single parametric capacitance.

The lack of symmetry in $K_p(\omega_1)$ can be seen, if we add the plasma conjugate complex impedance $Z_p^*(\omega_1)$ to the $Z_2(\omega_2)$ term (between the electrodes). In this case,

$$G_e = -\left[\frac{\Delta C}{2}\right]^2 \omega_1\omega_2[|Z_2(\omega_2)+Z_p^*(\omega_1)|] , \tag{9}$$

where $\omega_1 = 2\pi f_1$.

For resonance, $\omega_1 = \omega_{r_1}$ and $\omega_2 = \omega_{r_2}$, where $\omega_{r_1}$ and $\omega_{r_2}$ are the resonance frequencies for the tank circuit $L_1$ parallel to $C_1$ and $L_2$ parallel to $C_2$, respectively. In this situation, $|Z_2(\omega_2)| = R_2$ and

$$G_e = -\left[\frac{\Delta C}{2}\right]^2 \omega_1\omega_2[R_2+|Z_p^*(\omega_1)|] . \tag{10}$$

If the terms on the right-hand side of (1), except $X_{C_s}$, were equal to zero, i.e., if the plasma impedance $Z_p(\omega_1)$ were a pure nonlinear capacitive reactance, the profile of the power gain $K_p(\omega_1)$ close to the resonance frequency $\omega_{r_1}$ of $Z_1(\omega_1)$ would be symmetric. In our experiment, however, the additional components $R_p$, $L_p$, and $C_E$ of the circuit shown in Fig. 4 lead to an asymmetry of the profile of $K_p(\omega_1)$.

Figure 8 illustrates how $Z_p(\omega_1)$ affects $K_p(\omega_1)$. For frequencies slightly below $\omega_{r_1}$, both $|Z_1(\omega_1)|$ and $|Z_2(\omega_2)|$ increase, while $|Z_p(\omega_1)|$ decreases. Therefore the slope of $K_p(\omega_1)$ for $\omega_1 \lesssim \omega_{r_1}$ is reduced. For frequencies slightly above $\omega_{r_1}$, $|Z_2(\omega_2)|$ and $Z_p(\omega_1)|$ are decreasing and the magnitude of $G_e$ and consequently $K_p(\omega_1)$ decreases more rapidly with $\omega_1$ than for an ideal parametric amplifier.

Figure 9 shows the evidence of the influence of $Z_p^*(\omega_1)$ on the power gain $K_p(\omega_1)$ of the system, where the frequency $\omega_1$ and the pumping frequency $\Omega$ were maintained constant in the resonance. The profile shown in Fig. 9 was obtained by the variation of the tuning filter across $C_2$.

The pump frequency was fixed and the tuning of the tank $L_1, C_1$ changed by the variation of $C_1$ as shown in Fig. 10.

The symmetry of $K_p$ occurred in both cases about the maximum value. This occurs due to the fact that the plasma impedance is constant ($\omega_1 =$ const).

## V. CONCLUSIONS

The results we have obtained show again the capacitive nature of the sheath with a parametric amplification of rf signals with frequencies of about 5 MHz, but we will need a more accurate study in order to improve the amplification factor and to extend the frequency band of this device. The pronounced drop of the amplification

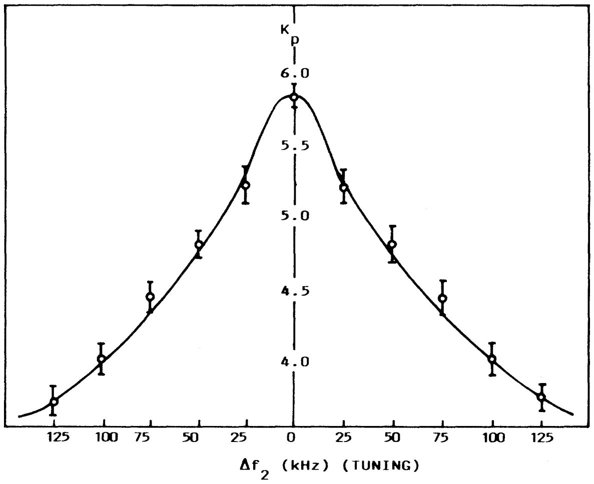

FIG. 9. Power gain profile $K_p(f_1)$ vs $\Delta f_2$.

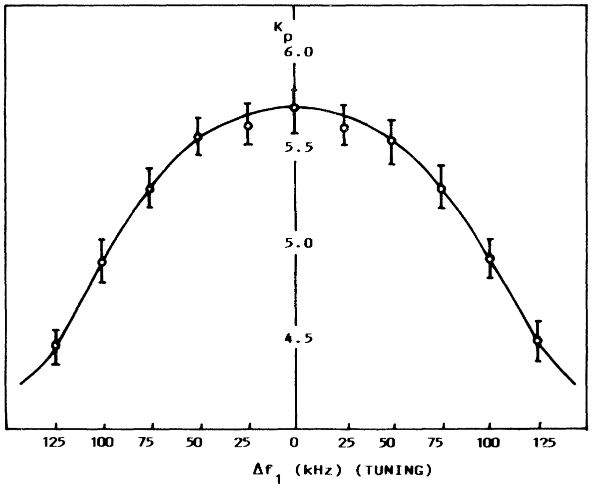

FIG. 10. $K_p(f_1)$ vs $\Delta f_1$ obtained by the variation of the tuning of the circuit tank $L_1$ parallel to $C_1$.

factor near and above the resonance frequency of the signal circuit shows the necessity of determining the nature of the gain when the signal and its tuning circuit respond with the sheath resonance frequency. An amplification obtained under these conditions would probably eliminate the dependence of the negative conductance on the reactive part of the impedance $Z_p$ of the plasma. This would enable us to study the variation of the sheath capacitance, which is fundamental for the amplification factor $K_p$.

It is necessary to study the behavior of this parametric amplifier at higher frequencies such as in the VHF band and in the microwave region, where there is a technological interest. This is the subject of a proposal for future work, provided we have a higher-density plasma and appropriate rf generators. Another possibility is to study the sheath formed at antennas immersed in a magnetized plasma. Parametric amplification in the sheath formed at an antenna could increase its gain, improving, this way, the transfer of power gain to the plasma in the rf heating process. The possibility to apply this nonlinear behavior of the sheath capacitance to a regular distribution nonlinear reactance line, forming a parametric TWT (traveling-wave tube)[7] is also planned to be considered in the future.

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