

Physics of photosensitive-grating formation in optical fibers

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We study the physics of photosensitive-fiber-grating formation in optical fibers, assuming a local bleaching model, and demonstrate that sustained grating growth is possible. Good agreement with our experimental results is obtained after including fiber heating effects and assuming a reasonable two-photon absorption cross section. The grating is predicted to be chirped in space.

I. INTRODUCTION

It has been some years since the discovery that certain germania (GeO_2)-doped glass optical fibers can show a photosensitive response to a single-longitudinal-mode argon laser beam, leading to the formation of light-induced Bragg-matched phase gratings.¹ Despite a recent resurgence of interest in this phenomenon, there are still many questions surrounding the growth of these gratings. An early experiment provided evidence that the process begins with two-photon absorption (TPA) of the argon laser beam.² Later it was suggested by Meltz *et al.* that this TPA results in the bleaching of an ultraviolet defect absorption band of the glass, resulting in a modification of the index of refraction in the visible region of the spectrum.^{3,4} Recent work has built on this picture, adding the suggestion that new absorption bands are formed which contribute to the index change, perhaps even dominating over the simplest bleaching effect.⁵ In either case we shall generically refer to this approach as a "bleaching picture." Such a picture is an example of a local model: a model in which the modification of the index of refraction of the glass is a function only of the local intensity of the light. This is in contrast to, for example, a photorefractivelike model, as recently suggested by Payne,⁶ which involves carrier transport over macroscopic distances. An early objection to local models was raised by Bures, Lapierre, and Pascale, who pointed out that, in coupled-mode theory, the intensity modulation of the light within a real dielectric constant grating of fixed phase would always be 90° out of phase with the grating.⁷ It was therefore not clear how such a grating could grow. This led them to introduce an *ad hoc* phase shift of 90° into their phenomenological growth equations. In a photorefractivelike picture the requisite phase shift is implicit, but such a model predicts a modulation depth of the grating (in steady state) independent of the intensity of the laser that formed it.⁶ This is in sharp contrast with experiment.⁶

In this paper we will discuss the growth dynamics of photosensitive-grating formation based on a local bleaching model. We demonstrate numerically that a local model can lead to sustained growth of the grating, be-

cause the grating phase is not fixed in time or space, as assumed by Bures, Lapierre, and Pascale.⁷ Furthermore, as a consequence of the local nature of the model, we show that one would expect oscillatory behavior during the grating formation, which we observed experimentally. To more accurately reproduce the behavior of our experimental growth curves we find it necessary to take into account laser heating of the fiber, and in the process draw a conclusion about the relative signs of the thermal and two-photon bleaching changes in the index of refraction. Our conclusion is in accord with recent measurements.⁸ This paper is a detailed exposition of results that we have previously reported in brief.⁹

After a short theoretical introduction (Sec. II), in which we develop the basic equations used in our model system and calculate the expected behavior, we turn in Sec. III to a discussion of the experimental system and results. Then, in Sec. IV we discuss the role of heating in modifying the calculated behavior and show good agreement with experiment. In Sec. V we establish that the parameters that we use in the model are reasonable. Our conclusions are summarized in Sec. VI.

II. THEORETICAL BACKGROUND

We calculate the propagation of light within the fiber grating using the usual coupled-mode equation formalism.¹⁰ Since our fiber is single mode and maintains polarization, we ignore the transverse profile of the electric fields and just deal with plane waves. The total electric field within the fiber is written as

$$\mathcal{E}(z,t) = \frac{1}{2}E(z,t)e^{-i\omega t} + \text{c.c.} \quad (1)$$

We assume that the electric field consists of a forward- and a backward-propagating component,

$$E(z,t) = E_+(z,t)e^{ikz} + E_-(z,t)e^{-ikz}, \quad (2)$$

where k is the wave vector of the light in the medium (using the initial dielectric constant of the fiber). If a weak phase grating grows in the glass, the dielectric constant can be written as a sum of a slowly varying component and a rapidly varying component of spatial period $2k$; the

latter gives rise to the Bragg-matched reflection. We write

$$\Delta\epsilon(z,t) = \epsilon_0(z,t) + \epsilon_2(z,t)e^{2ikz} + \epsilon_2^*(z,t)e^{-2ikz}, \quad (3)$$

where $\Delta\epsilon$ represents the small modification in the background dielectric constant of the glass. We will take $\Delta\epsilon$ to be real,¹ consistent with the 95% reflection that we routinely observed in our fibers. Any higher spatial frequency components in $\Delta\epsilon$ are ignored, as they do not contribute to Bragg-matched reflection. At any instant in time the forward and backward electric-field envelope functions can be calculated by the usual coupled-mode formalism in the slowly varying envelope approximation.

$$\begin{aligned} \frac{\partial E_+(z,t)}{\partial z} &= \frac{ik}{2n^2} [\epsilon_0(z,t)E_+(z,t) + \epsilon_2(z,t)E_-(z,t)], \\ \frac{\partial E_-(z,t)}{\partial z} &= -\frac{ik}{2n^2} [\epsilon_0(z,t)E_-(z,t) + \epsilon_2^*(z,t)E_+(z,t)], \end{aligned} \quad (4)$$

where asynchronous terms have once again been dropped, and we have only considered the case of zero detuning of the laser frequency from the grating Bragg condition; $n = 1.5$ is the index of refraction of the glass. Equations (4) must be solved subject to the boundary condition appropriate for a beam incident on one end of the fiber, with both ends of the fiber in the air.

This is the standard formalism, which would likely be common to any proposed growth mechanism. We must now postulate a particular model for the evolution of the dielectric grating. Following the suggestions³ of Meltz *et al.*, we will assume that the dielectric constant is locally bleached by the two-photon absorption of the laser beam into the defect band of the GeO₂, centered at about 245 nm. For completeness we note that while this band is often attributed^{11,12} to an oxygen vacancy defect of the GeO₂ (in analogy with a similar band¹³ found in SiO₂), the precise microscopic origin of this band is still the subject of some discussion.^{14,15} But for us the key point is that such a band does exist, whatever its precise origin. One photon bleaching was clearly demonstrated in glassy GeO₂ by Cohen and Smith.¹⁶ The assumption here is that this band can also be significantly bleached through two-photon absorption. Through the Kramers-Kronig relation¹⁰ this leads to a change in the real part of the dielectric constant in the visible region of the spectrum. If we assume that the number of defect sites actually modified by two-photon absorption is a small fraction of the total number of sites, we can neglect depletion of the available sites and model the effect of the two photon absorption on the real part of the dielectric constant by the equation

$$\frac{\partial \Delta\epsilon(z,t)}{\partial t} = AI^2(z,t). \quad (5)$$

Here $I(z,t)$ is the local intensity of the light in the fiber, at any instant in time. The intensity is squared in this expression, which follows from the assumption that a two-photon-absorption process is dominant. Note that by allowing A to be either positive or negative this expression also serves to describe the more complicated case of the

creation of color center absorption bands in other regions of the spectrum, as recently discussed in Ref. 5. To complete the set of equations necessary to model this system, we note that the intensity of the light is related to the electric field by

$$I(z,t) = \frac{cn}{8\pi} |E_+(z,t)e^{ikz} + E_-(z,t)e^{-ikz}|^2. \quad (6)$$

In general, the parameter A in Eq. (5) is unknown to us, although we will attempt to estimate A in Sec. V below. In any case, we may absorb A into the time variable and then study the grating growth dynamics in scaled time units. Apart from this linear rescaling of the time axis, the only free parameter at our disposal is the precise length of the fiber (and hence the grating) in units of the wavelength of the light in the medium. We have used a typical fiber length of 30 cm, with a laser wavelength of 488 nm (in vacuum).

The result of a Runge-Kutta numerical solution¹⁷ to the set of coupled equations presented above is shown in Fig. 1 (solid line). Here we have plotted the transmission of the light through the fiber; the calculation conserves energy, and therefore the reflection increases correspondingly. The first striking feature is that the grating *can* in fact undergo sustained growth, eventually excluding most of the light from the fiber, in contrast to the suggestion of Bures, Lapiere, and Pascale. Their suggestion that a local model cannot lead to sustained growth in a coupled mode formalism is based on the assumption that the phase of the dielectric grating remains constant in time.

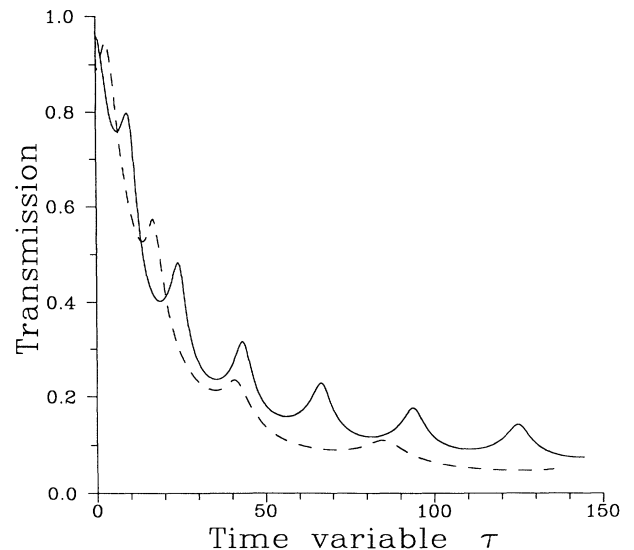


FIG. 1. Numerical simulation of photosensitive fiber transmission as predicted using Eqs. (4) and (5) (solid line, no heating). The dimensionless time variable $\tau = 5 \times 10^6 AI_0^2 t$, where I_0 is the laser intensity incident on the fiber. We have used a fiber length of 30 cm and a vacuum laser wavelength of 488 nm. To include laser heating [per Eq. (7), dashed line] and compare with the results shown in Fig. 2, we assume $B = 8.1 \times 10^{-13} \text{ cm}^2/\text{W}$ and $I_0 = 1.72 \times 10^6 \text{ W/cm}^2$.

In fact, our detailed calculations show that the phase is both time and space dependent, permitting grating growth. We discuss this further below. The coupled equations, although mathematically simple, lead to very rich behavior. The grating growth is not monotonic. Instead the transmission curve is subject to oscillatory behavior, with a time scale comparable to that of the overall reduction in fiber transmission. This appears to result from a delicate interplay between the phase and amplitude of the grating with the intensity profile of the light in the fiber. We do not fully understand the underlying physics of these oscillations. However, in the present model they appear to depend on the existence of Fresnel reflection from the *input* coupling end of the fiber, which will interfere with the light reflected from the grating; if we set the former equal to zero in the computer simulations, the numerical experiments yield a grating growth that is smooth. This was determined by setting the input face Fresnel reflection to zero in the computer model. The Fresnel reflection at the exit face is necessary to provide a component of backward wave in the fiber. Once the growth process begins it is possible to artificially eliminate the exit face Fresnel reflection from the calculation, in which case the numerical experiment indicates that growth stops but the oscillations continue. Thus in this model the oscillations appear to result in large part from an interference of the light reflected from the input face and that reflected from a moving grating.

Two points need to be made regarding the calculation. First, we have assumed that the fiber is precisely an integral number of wavelengths long. Varying this changes the phase of the oscillations on the growth curve, but does not otherwise result in significantly different behavior. Second, it is usual to assume that the Fresnel reflection from the fiber end faces is about 4%. In fact, by monitoring Fabry-Pérot oscillations in the transmission of a single-mode helium-neon laser probe while heating the fiber to vary the dielectric constant of the glass we find that the effective reflection depends on the quality of the cleave, and is more typically about 2%. This appears to be principally due to cleaves which are not perfectly perpendicular to the fiber axis, resulting in less than perfect coupling of the reflected light back into the fiber core. We have taken this into account in our calculations by modifying the Fresnel reflection coefficients accordingly. This leads to smaller oscillations, but does not otherwise qualitatively affect the results.

III. EXPERIMENTAL RESULTS

An experimental growth curve is displayed in Fig. 2. The experimental set up has been described elsewhere.¹⁸ The fiber (Andrew Corporation) is single mode at 488 nm, maintains polarization and is the same type of fiber as was used in our previous work.¹⁸ The germania-doped core is elliptical, with a $1\ \mu\text{m} \times 2\ \mu\text{m}$ diameter. High-frequency oscillations on the growth curve, which are due to environmental perturbations,¹⁸ have been minimized by keeping the fiber slack. Residual oscillations were filtered out with a three-stage RC filter (18-Hz cutoff fre-

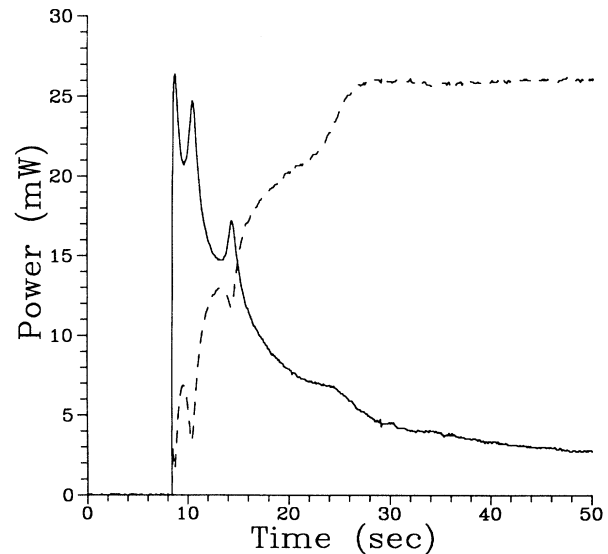


FIG. 2. Experimental results for our polarization maintaining single-mode fiber at 488 nm. A fiber length of 33 cm was used. Both the transmitted (solid line) and the reflected (dashed line) powers are shown. The powers stated are for the air side of the fiber cleaves.

quency) placed before the analog-to-digital converters.

In Fig. 2 we clearly observe low-frequency oscillatory behavior. Such behavior had been observed previously in a multimode fiber, where the interpretation is more complicated, but was dismissed as being of thermal origin.¹⁹ The evidence given at the time was that very low power preparation of the fiber (where heating of the fiber would presumably be minimized) resulted in smooth grating growth. Our own measurements at low powers do not lead to the disappearance of the oscillatory behavior, but rather to a lengthening of the period. In the curve shown in Ref. 19 the experiment has perhaps not proceeded far enough to be sure that oscillations would not be present.

Qualitatively, the oscillatory behavior that we observe is similar to that predicted by the simple bleaching model. However, the experimental results suggest a gradual damping of the oscillations with time, which does not come out of the simple model presented above. Further, we could not rule out the possibility of some fiber heating due to direct absorption of the argon laser beam, which could conceivably lead to some oscillatory behavior, as suggested by Lapiere, Bures, and Chevalier.¹⁹ Therefore we decided to incorporate heating into the system of coupled equations.

IV. HEATING EFFECTS

For our fiber parameters the transverse thermal diffusion time²⁰ (of order $1\ \mu\text{sec}$) is essentially instantaneous as compared with the grating formation time (approximately 10 sec). Also, the distance that heat can diffuse longitudinally on the time scale for grating formation is very long compared with the dielectric constant grating period ($0.2\ \mu\text{m}$). Therefore we assume that the

effect of linear absorption-induced heating is to modify only the $\epsilon_0(z,t)$ term of Eq. (3). In this spirit we make the replacement $\epsilon_0 \rightarrow \epsilon_0 + \epsilon_{th}$, where ϵ_{th} is a thermal contribution to the dielectric constant due to linear absorption of the laser beam by the fiber. We take this thermal contribution to be of the form

$$\begin{aligned} \epsilon_{th}(z,t) &= 2n \frac{dn}{dT} \Delta T(z,t) \\ &= B \langle I(z,t) \rangle, \end{aligned} \quad (7)$$

where $\langle I(z,t) \rangle$ represents the intensity in the fiber, after spatially averaging over the grating period but leaving the slow variation unaffected. The parameter B depends not only dn/dT , but also on the absorption coefficient of the fiber and the heat transfer to the surrounding air. However, these subtleties may be bypassed by resorting to experiment. Measuring the temperature rise in a fiber with its output end in index matching fluid, the quantity B can be extracted from Eq. (7) if dn/dT is known. Using a thermocouple in intimate thermal contact with the fiber cladding, and 85-mW incident laser power, we measured a steady-state temperature rise of 0.14 K. For pure fused silica²¹ $dn/dT = 1.0 \times 10^{-5} \text{ K}^{-1}$, but our fibers have an estimated 20 mol % GeO_2 doped into the core. For such a material we have been unable to obtain a value of dn/dT , but adding germania does not appear to significantly modify it from that of pure silica.²² We therefore use the value of dn/dT of pure silica. This leads to a value of $B = 8.1 \times 10^{-13} \text{ cm}^2/\text{W}$. (For estimates that require a conversion from power to intensity inside the fiber, we use the geometric area of the fiber core.)

In Fig. 1 we also plotted (dashed line) our predicted results with this B , for $A > 0$. We have taken the incident laser intensity $I_0 = 1.72 \times 10^6 \text{ W/cm}^2$, to compare with the experiment of Fig. 2. Note that the effect of adding a small heating term is to dampen the oscillations, in qualitative agreement with the experimental results. The relative sign of A and B is crucial. If we assume that A and B are of opposite sign the effect of the heating term is to sharpen the oscillations rather than dampen them. We remark that if we consider a fixed grating subjected to laser heating [that is, we set $A = 0$ in Eq. (5) but include the effect of the ϵ_{th}], we find that the numerical solutions of the equations do not show oscillations in the transmitted intensity. This argues against the conjecture¹⁹ that the oscillations are the result of a thermal effect. Rather, we find that they are an integral aspect of the grating growth, but are slightly *damped* by the thermal effects.

It is interesting to consider the spatial distribution of $\Delta\epsilon$ [Eq. (3)] after the phase grating has been formed. We show the slowly varying part of the dielectric constant $\epsilon_0(z)$ in Fig. 3. The time is fixed to correspond to the end of the run shown in Fig. 1 (dashed line), where the transmission has been reduced to about 5% of the starting value. Clearly the grating has forced the light to be concentrated near the entrance face of the fiber. The $\epsilon_0(z)$ indicates that the spatial average dielectric constant has been modified by the laser. This is a direct consequence of our assuming a local model, where the local

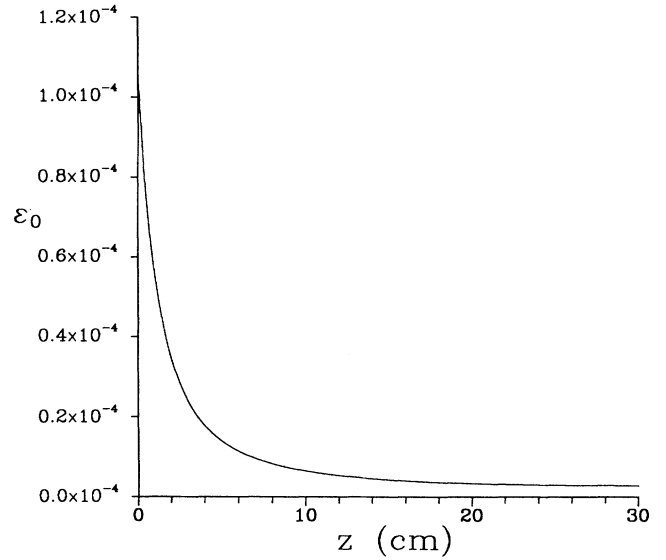


FIG. 3. Slowly varying part of the dielectric constant $\epsilon_0(z)$ of Eq. (3), as a function of position z along the 30-cm-long fiber. Here $z = 0$ corresponds to the entrance face of the fiber. Laser heating is taken into account as in Fig. 1.

change in the dielectric constant is a monotonic function of the intensity (which is positive definite). Of particular interest is the distribution of $\epsilon_2(z)$, which is the complex amplitude of the dielectric constant grating. We define $\epsilon_2(z) = |\epsilon_2(z)| \exp[-i\varphi(z)]$. In Fig. 4 we present $|\epsilon_2(z)|$ and in Fig. 5 we present $\varphi(z)$. It is a consequence of Eq. (5) that $\varphi = 0$ at the exit face of the fiber. From Fig. 5 we see that the gratings are strongly chirped. Over short segments the chirp is approximately linear. This may

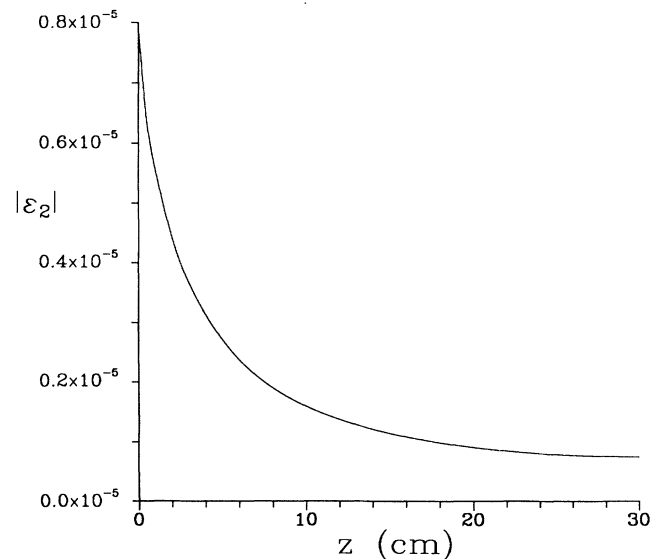


FIG. 4. Amplitude of the phase grating $|\epsilon_2(z)|$ as a function of position. Heating as in Fig. 1.

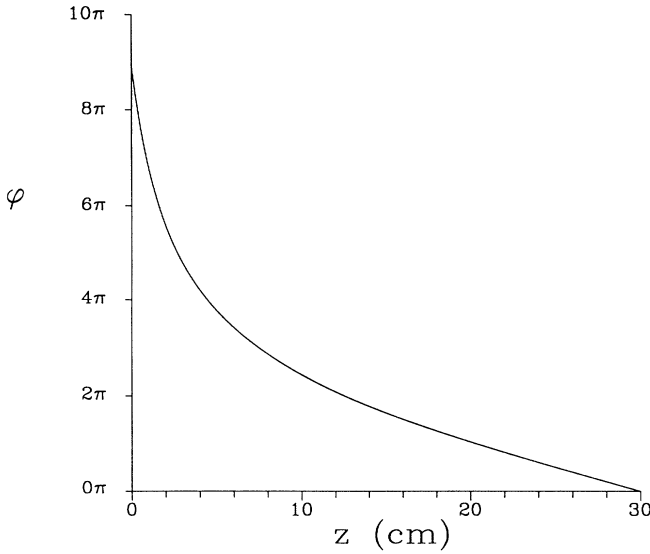


FIG. 5. Phase $\varphi(z)$ of $\epsilon_2(z) = |\epsilon_2(z)| \exp[-i\varphi(z)]$. Heating as in Fig. 1.

have interesting implications for pulse compression purposes.²³

Finally, we are left with the problem of determining whether the time scales for grating growth are in fact reasonable, given that TPA is likely to be a weak process. We turn to this problem in Sec. V.

V. MATERIAL EFFECTS

In general the parameter A of Eq. (5) will be related to the two-photon absorption cross section of the 245-nm band, as well as other material parameters which are at present unknown to us. We may, however, place a reasonable bound on A by the following considerations.

The basic equation describing attenuation of the laser by TPA is

$$\frac{\partial I}{\partial z} = -\beta I^2, \quad (8)$$

where β is the usual TPA coefficient.²⁴ If N is the number density of defect sites which can undergo TPA, and σ_2 is the TPA cross section of the defects, then we may write

$$\beta = \frac{N\sigma_2}{\hbar\omega}, \quad (9)$$

where $\hbar\omega$ is the one-photon energy. Alternatively, if we assume that one defect is bleached for every pair of photons absorbed, then TPA attenuation of the laser beam may be expressed by

$$\frac{\partial I}{\partial z} = 2\hbar\omega \frac{\partial N}{\partial t}. \quad (10)$$

Combining Eqs. (8)–(10), we have

$$\frac{\partial N}{\partial t} = -\frac{N}{2(\hbar\omega)^2} \sigma_2 I^2. \quad (11)$$

Each defect that undergoes TPA is assumed to lead to a local change in the dielectric constant. We may write $\Delta\epsilon = 4\pi\Delta\alpha(N_{\text{total}} - N)$, where N_{total} is the total number of defect sites, $\Delta\alpha$ is the effective change in the linear polarizability of a defect which has undergone TPA, and we are ignoring local field effects. Finally, since the maximum change in the dielectric constant $\Delta\epsilon_{\text{max}}$ that we may expect in this simple picture is given by $4\pi N_{\text{total}}\Delta\alpha$, we have, from Eq. (11),

$$\frac{\partial(\Delta\epsilon)}{\partial t} = \frac{(\Delta\epsilon)_{\text{max}}}{2(\hbar\omega)^2} \sigma_2 I^2, \quad (12)$$

as long $\Delta\epsilon \ll (\Delta\epsilon)_{\text{max}}$ (i.e., ignoring depletion of the available defect sites). Equation (12) may be compared directly with Eq. (5) to obtain an explicit expression for A . With this expression we can now attempt to check whether the model is reasonable in view of the experimental growth time for the grating.

A rough comparison of Fig. 2 with Fig. 1 (dashed line) leads to an A of about $2 \times 10^{-19} \text{ cm}^4/\text{W}^2 \text{ sec}$. We can obtain an upper bound on what σ_2 must be taking as a lower bound for $\Delta\epsilon_{\text{max}}$ the actual value of $\Delta\epsilon$ in the fiber at the end of the experiment. Either by roughly estimating this $\Delta\epsilon$ from the experimental reflectivity by assuming a uniform grating,¹⁸ or by using the spatial average of ϵ_2 from the numerical experiment of Fig. 4 as an estimate of $\Delta\epsilon$, we find a $\Delta\epsilon \approx 2 \times 10^{-6}$. This leads to an upper bound of σ_2 about $3 \times 10^{-50} \text{ cm}^4 \text{ sec}$. To be even more conservative we note from Fig. 3 that the model calculation leads to a peak value of ϵ_0 of 1×10^{-4} (this calculation ignores possible defect depletion), leading to a loose upper bound of σ_2 of $2 \times 10^{-48} \text{ cm}^4 \text{ sec}$. Since typical values of σ_2 range from 10^{-49} to $10^{-51} \text{ cm}^4 \text{ sec}$ for a wide variety of molecules,²⁴ the model we have presented here for the grating formation is plausible. Furthermore, recent measurements⁸ of the index change in a fiber exposed to ultraviolet light at 266 nm show that the resulting index change has the same sign as dn/dT , which is consistent with our present findings.

VI. CONCLUSIONS

In conclusion, we have demonstrated through explicit calculation that a local bleaching model can lead to sustained grating growth in photosensitive fibers. Low-frequency oscillations in the transmission of light through the grating follow as a consequence. We have experimentally observed such oscillations, which in our model are a consequence of the grating formation dynamics, but are modified by direct laser heating of the fiber. After taking heating into account, we find good agreement between theory and experiment. The defect TPA cross section needed to predict the time scale for grating formation is found to be reasonable. The model therefore appears to be a viable alternative to photorefractivelike models, and we would like to suggest that the term “pho-

refractive" no longer be applied to this effect without justification. Finally, it follows as a necessary consequence of this model that the dielectric constant phase grating that is formed is substantially chirped in space.

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