Generating displaced and squeezed number states by a general driven time-dependent oscillator

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In this paper we examine the possibility of generating displaced and squeezed number states out of number states by the general driven time-dependent oscillator. Using the evolution-operator method developed by Cheng and Fung [J. Phys. A **21**, 4115 (1988)], we investigate the evolution of a number state of a general driven time-dependent oscillator as well as its squeezing property. Our analyses indicate that the wave function of the time-dependent oscillator starts as a number state at $t=0$ and evolves as a displaced and squeezed number state at a later time.

I. INTRODUCTION

In the past few years displaced and squeezed vacuum states (or simply squeezed states) of the electromagnetic field have been widely studied, both theoretically and experimentally.[1,2] These are states that have reduced fluctuations in one field quadrature, when compared with coherent states. They are of considerable importance these days owing to their prospective application, for instance, in optical communication, interferometry, gravitational-wave detection, Rydberg atoms, and so on.[3–6] Recently, considerable attention has been paid to generalizations of the squeezed states, namely the displaced and squeezed number states.[7] The displaced and squeezed number state is defined by

$$|z,\alpha,n\rangle = S(z)D(\alpha)|n\rangle = S(z)D(\alpha)\frac{a^{\dagger n}}{\sqrt{n!}}|0\rangle , \tag{1}$$

where $S(z)$ is the squeeze operator, given by

$$S(z)=\exp[\tfrac{1}{2}(za^{\dagger 2}-z^*a^2)] , \tag{2}$$

and $D(\alpha)$ is the displacement operator, given by

$$D(\alpha)=\exp(\alpha a^\dagger-\alpha^* a) . \tag{3}$$

For $n=0$, the displaced and squeezed number state reduces to the well-known squeezed state. The squeezing properties related to $|z,\alpha,n\rangle$ states can be directly deduced from those for the displaced and squeezed vacuum $|z,\alpha\rangle$, with the change of $\hbar/2$ to $(\hbar/2)(2n+1)$. It is thus evident that for every n the state $|z,\alpha,n\rangle$ may have, under appropriate values of z, one of the quadrature variances less than $\hbar/2$, corresponding to a coherent state.

How can we realize displaced and squeezed number states in physical terms? Let us consider a simple harmonic oscillator of unit mass (or a single cavity mode of the electromagnetic field) with Hamiltonian

$$H=\frac{p^2+\omega^2q^2}{2}=\hbar\omega(a^\dagger a+\tfrac{1}{2}) , \tag{4}$$

where a is the annihilation operator, given by

$$a=\frac{\omega q+ip}{\sqrt{2\hbar\omega}} . \tag{5}$$

Suppose that the oscillator is in a number state $|n\rangle$, which is an eigenstate of the Hamiltonian operator. Then a simple way of realizing the displaced and squeezed number state is to add, beginning at some moment of time, a term $\frac{1}{2}\beta^2q^2-\lambda q$ to the harmonic-oscillator Hamiltonian,[8] so that at all later times we have

$$H'=\hbar\Omega(A^\dagger A+\tfrac{1}{2})-\epsilon , \tag{6}$$

where

$$\begin{aligned} A&=\frac{\Omega(q-q_0)+ip}{\sqrt{2\hbar\Omega}} ,\\ \Omega&=(\omega^2+\beta^2)^{1/2} ,\\ q_0&=\frac{\lambda}{\Omega^2} ,\\ \epsilon&=\frac{\lambda^2}{2\Omega^2} . \end{aligned} \tag{7}$$

Here H' is just the Hamiltonian of a displaced and stiffened harmonic oscillator, and A is the corresponding annihilation operator. It is not difficult to show that A and $A^\dagger$ are linearly related to a and $a^\dagger$, i.e.,

$$\begin{aligned} A&=\frac{\omega+\Omega}{2\sqrt{\omega\Omega}}a+\frac{\omega-\Omega}{2\sqrt{\omega\Omega}}a^\dagger-\sqrt{\Omega/2\hbar}q_0 ,\\ A^\dagger&=\frac{\omega-\Omega}{2\sqrt{\omega\Omega}}a+\frac{\omega+\Omega}{2\sqrt{\omega\Omega}}a^\dagger-\sqrt{\Omega/2\hbar}q_0 . \end{aligned} \tag{8}$$

Equations (8) define a linear unitary transformation and may be written as

$$\begin{aligned} A&=U(r,s)aU(r,s)^\dagger ,\\ A^\dagger&=U(r,s)a^\dagger[U(r,s)]^\dagger , \end{aligned} \tag{9}$$

with

$$\begin{aligned} U(r,s)&=S(r)D(s) ,\\ r&=-\frac{1}{2}\ln\left[\frac{\omega}{\Omega}\right] ,\\ s&=-\sqrt{\Omega/2\hbar}q_0 . \end{aligned} \tag{10}$$

Clearly the unitary operator U is just the product of the squeeze operator S and the displacement operator D. The number state $|n\rangle'$ associated with H' may also be expressed as

$$|n\rangle' = U(r,s)|n\rangle = S(r)D(s)|n\rangle \ . \tag{11}$$

Hence the state $|n\rangle'$ is the desired displaced and squeezed number state with respect to the state $|n\rangle$.

The discussion presented above suggests that displaced and squeezed number states of a simple harmonic oscillator can be generated by displacing the oscillator and changing its frequency. It is therefore the purpose of this paper to examine the possibility of generating displaced and squeezed number states by the general driven time-dependent oscillator which is described by the Hamiltonian.[9]

$$H(t) = \frac{p^2}{2m(t)} + \frac{1}{2}m(t)[\omega(t)]^2x^2 - m(t)f(t)x \ , \tag{12}$$

where the mass parameter is taken as

$$m(t) = m_0 \exp\left[2\int \gamma(t)dt\right] \ , \tag{13}$$

and $\omega(t)$, $f(t)$, and $\gamma(t)$ are arbitrary functions of time. Using the evolution operator method developed by Cheng and Fung,[10] we investigate the evolution of a number state of a general driven time-dependent oscillator and discuss its squeezing property. An analytical example of a damped pulsating oscillator with variable frequency in the presence of an external driving force is examined, and implications of the results are discussed.

II. TIME-EVOLUTION OPERATOR

Consider a general driven time-dependent oscillator whose Hamiltonian takes the form

$$H(t) = H_0(t) + V(t) \ , \tag{14}$$

with

$$H_0(t) = \frac{p^2}{2m(t)} + \frac{1}{2}m(t)\omega(t)^2x^2 \ , \tag{15}$$

$$V(t) = -m(t)f(t)x \ , \tag{16}$$

where

$$m(t) = m_0 \exp\left[2\int \gamma(t)dt\right] \ , \tag{17}$$

and $\omega(t)$, $f(t)$, and $\gamma(t)$ are arbitrary functions of time. It is well known that $H_0(t)$ can be rewritten in terms of the su(1,1) generators as follows:[11]

$$H_0(t) = a_1(t)J_+ + a_2(t)J_0 + a_3(t)J_- \ , \tag{18}$$

where

$$J_+ = \frac{i}{2\hbar}x^2 \ , \tag{19}$$

$$J_- = \frac{i}{2\hbar}p^2 \ , \tag{20}$$

$$J_0 = \frac{i}{4\hbar}(px + xp) \ , \tag{21}$$

and

$$a_1(t) = -i\hbar m(t)\omega(t)^2 \ , \tag{22}$$

$$a_2(t) = 0 \ , \tag{23}$$

$$a_3(t) = -\frac{i\hbar}{m(t)} \ . \tag{24}$$

The operators J_+, J_0, and J_- form the su(1,1) Lie algebra:

$$[J_+, J_-] = -2J_0 \ , \tag{25}$$

$$[J_0, J_\pm] = \pm J_\pm \ . \tag{26}$$

The corresponding Schrödinger equation is

$$H_0(t)|\Psi_0(t)\rangle = i\hbar\frac{\partial}{\partial t}|\Psi_0(t)\rangle \ . \tag{27}$$

As usual, we will define the evolution operator $U_0(t,0)$ such that

$$|\Psi_0(t)\rangle = U_0(t,0)|\Psi_0(0)\rangle \ , \tag{28}$$

where $|\Psi_0(0)\rangle$ is the wave function at time $t=0$. Inserting (28) into (27) yields the evolution equation

$$H_0(t)U_0(t,0) = i\hbar\frac{\partial}{\partial t}U_0(t,0) \ , \tag{29}$$

$$U_0(0,0) = 1 \ . \tag{30}$$

Since J_+, J_0, and J_- form a closed Lie algebra su(1,1), the evolution operator can be expressed in the following form:

$$U_0(t,0) = \exp[c_1(t)J_+]\exp[c_2(t)J_0]\exp[c_3(t)J_-] \ , \tag{31}$$

where $c_i(t)$ are to be determined. Then by direct differentiation with respect to time, we obtain

$$\frac{\partial}{\partial t}U_0(t,0) = [h_+(t)J_+ + h_0(t)J_0 + h_-(t)J_-]U_0(t,0) \ , \tag{32}$$

with

$$h_+(t) = \frac{dc_1}{dt} - c_1\frac{dc_2}{dt} + c_1^2\exp(-c_2)\frac{dc_3}{dt} \ , \tag{33}$$

$$h_0(t) = \frac{dc_2}{dt} - 2c_1\exp(-c_2)\frac{dc_3}{dt} \ , \tag{34}$$

$$h_-(t) = \exp(-c_2)\frac{dc_3}{dt} \ . \tag{35}$$

Substituting (18), (31), and (32) into (29), and comparing the two sides, we obtain after simplication

$$c_1(t) = m(t)\frac{\partial}{\partial t}\ln[F(t)], \quad c_1(0) = 0 \tag{36}$$

$$c_2(t) = -2\ln\left|\frac{F(t)}{F(0)}\right| \ , \tag{37}$$

$$c_3(t) = -F(0)^2\int_0^t du\frac{1}{m(u)F(u)^2} \ , \tag{38}$$

where $F(t)$ satisfies the differential equation

$$\frac{d^2F(t)}{dt^2}+\xi(t)\frac{dF(t)}{dt}+[\omega(t)]^2F(t)=0\ , \tag{39}$$

and

$$\xi(t)=\frac{\partial}{\partial t}\ln[m(t)]\ . \tag{40}$$

The second-order differential equation can be cast in the standard form such that

$$\left[\frac{d^2}{dt^2}+\lambda(t)\right]G(t)=0\ , \tag{41}$$

with

$$G(t)=g(t)F(t)\ , \tag{42}$$

$$\lambda(t)=\omega(t)^2-h(t)\ , \tag{43}$$

$$h(t)=\frac{1}{g(t)}\frac{d^2g(t)}{dt^2}\ , \tag{44}$$

$$g(t)=\sqrt{m(t)}\ . \tag{45}$$

Infeld and Hull have noted that most of the analytically solvable second-order differential equations involving a single variable that are of interest in electromagnetic and quantum theory can be transformed into this standard form.[12] Since $U_0(t,0)$ is known, the evolution operator U describing the whole system will be given by

$$U(t,0)=U_0(t,0)U_I(t,0)\ , \tag{46}$$

where $U_I(t,0)$ satisfies the evolution equation

$$i\hbar\frac{\partial}{\partial t}U_I(t,0)=H_I(t)U_I(t,0)\ , \tag{47}$$

$$U_I(0,0)=1\ , \tag{48}$$

with $H_I(t)$ being defined by

$$H_I(t)=U_0^\dagger(t,0)V(t)U_0(t,0)\ . \tag{49}$$

By straightforward evaluations of (49) we obtain

$$H_I(t)=-m(t)f(t)\exp\left[-\frac{c_2(t)}{2}\right]\left[x-\frac{c_3(t)}{2}p\right]\ . \tag{50}$$

In terms of the generators of the Heisenberg-Weyl algebra, $H_I(t)$ can be written as[13]

$$H_I(t)=b_1(t)e_1+b_2(t)e_2+b_3(t)e_3\ , \tag{51}$$

where

$$e_1=\frac{i}{\sqrt{\hbar}}p\ , \tag{52}$$

$$e_2=\frac{i}{\sqrt{\hbar}}x\ , \tag{53}$$

$$e_3=i\ , \tag{54}$$

and

$$b_1(t)=\frac{\sqrt{\hbar}}{i2}m(t)f(t)c_3(t)\exp\left[-\frac{c_2(t)}{2}\right]\ , \tag{55}$$

$$b_2(t)=i\sqrt{\hbar}m(t)f(t)\exp\left[-\frac{c_2(t)}{2}\right]\ , \tag{56}$$

$$b_3(t)=0\ . \tag{57}$$

The operators e_i form the Heisenberg-Weyl Lie algebra

$$[e_1,e_2]=e_3\ , \tag{58}$$

$$[e_1,e_3]=[e_2,e_3]=0\ . \tag{59}$$

Following similar procedure as shown above, the evolution operator $U_I(t,0)$ is found to be

$$U_I(t,0)=\exp[d_1(t)e_1]\exp[d_2(t)e_2]\exp[d_3(t)e_3]\ , \tag{60}$$

with

$$d_1(t)=\int_0^t b_1(u)du\ , \tag{61}$$

$$d_2(t)=\int_0^t b_2(u)du\ , \tag{62}$$

$$d_3(t)=-\int_0^t b_2(u)d_1(u)du\ . \tag{63}$$

Hence we have obtained an exact form of the time-evolution operator $U(t,0)$ of the general driven time-dependent oscillator.

III. SQUEEZING IN GENERAL DRIVEN TIME-DEPENDENT OSCILLATOR

Suppose we start with a number state at $t=0$:

$$|\Psi(0)\rangle=|n\rangle\ ; \tag{64}$$

that is, an eigenstate of the number operator $N\equiv a^\dagger a$,

$$N|n\rangle=n|n\rangle\ , \tag{65}$$

with

$$a=\frac{m_0\omega_0x+ip}{\sqrt{2m_0\hbar\omega_0}}\ , \tag{66}$$

$$m_0=m(t=0)\ , \tag{67}$$

$$\omega_0=\omega(t=0)\ . \tag{68}$$

The wave function at any later time will be represented by

$$|\Psi(t)\rangle=U(t,0)|\Psi(0)\rangle\ . \tag{69}$$

Now we can define a new operator $A(t)$ as

$$A(t)=U(t,0)aU^\dagger(t,0)\ , \tag{70}$$

and it is easy to see that the wave function $|\Psi(t)\rangle$ is a number state with respect to the new operator $\mathcal{N}(t)\equiv A(t)^\dagger A(t)$,

$$\mathcal{N}(t)|\Psi(t)\rangle=n|\Psi(t)\rangle\ . \tag{71}$$

Using (31), (46), (60), and (66), it can be shown that the original operator a is related to the new operator $A(t)$ by a Bogoliubov transformation plus a translation

$$A(t)=\eta_1(t)a+\eta_2(t)a^\dagger-\beta(t)\ , \tag{72}$$

with

$$|\eta_1|^2-|\eta_2|^2=1 , \tag{73}$$

where η_1 and η_2 are given by

$$\eta_1(t)=\tfrac{1}{2}\exp(-c_2/2)\left[1-c_1c_3+\exp(c_2)-\frac{ic_1}{m_0\omega_0}-ic_3m_0\omega_0\right] , \tag{74}$$

$$\eta_2(t)=-\tfrac{1}{2}\exp(-c_2/2)\left[1+c_1c_3-\exp(c_2)+\frac{ic_1}{m_0\omega_0}-ic_3m_0\omega_0\right] , \tag{75}$$

and

$$\beta=-\frac{1}{\sqrt{2m_0\omega_0}}(m_0\omega_0b_1-ib_2) . \tag{76}$$

Also, it is clear that the operators $A(t)$ and $A^\dagger(t)$ satisfy the commutation relation

$$[A(t),A^\dagger(t)]=1 \tag{77}$$

at all time t. In fact, apart from the presence of the c-number function $\beta(t)$, the operators $A(t)$ and $A^\dagger(t)$ take exactly the form of Yuen operators.[14] Furthermore, the evolution operator $U_0(t,0)$ can be written as a product of a squeeze operator and a rotation operator,[2] i.e.,

$$U_0(t,0)=S(z(t))R(\phi(t)) , \tag{78}$$

with

$$S(z)=\exp[\tfrac{1}{2}(za^{\dagger 2}-z^*a^2)] , \tag{79}$$

$$R(\phi)=\exp(-i\phi a^\dagger a) , \tag{80}$$

where

$$\cosh|z|=|\eta_1| , \tag{81}$$

$$\sinh|z|=|\eta_2| , \tag{82}$$

$$-\frac{z}{|z|}=\frac{\eta_2}{|\eta_2|}\frac{|\eta_1|}{\eta_1}=\frac{\eta_2}{|\eta_2|}\exp(-i\phi) , \tag{83}$$

and the operator $U_I(t,0)$ is just the Weyl displacement operator multiplied by a phase factor,[2] namely,

$$U_I(t,0)=\exp[i\theta(t)]D(\beta(t)) , \tag{84}$$

with

$$D(\beta)=\exp(\beta a^\dagger-\beta^*a) , \tag{85}$$

$$\theta=d_3+\frac{d_1d_2}{2} . \tag{86}$$

Therefore these results imply that the wave function $|\Psi(t)\rangle$ is a displaced and squeezed number state and its time evolution is a "displacement" and "squeezing" process. As a result, it can be concluded that the wave function starts as a number state at $t=0$ and evolves as a displaced and squeezed number state at a later time.

To see its squeezing property explicitly, we will compute the variances of x and p. Using the evolution operator $U(t,0)$ in Sec. II, it can be shown that the expectation values of these operators with respect to the wave function $|\Psi(t)\rangle$ is given by

$$\begin{aligned}\langle x\rangle&=\left[\frac{2\hbar}{m_0\omega_0}\right]^{1/2}\mathrm{Re}[(\eta_1-\eta_2)\beta^*]\\&=\left[\frac{2\hbar}{m_0\omega_0}\right]^{1/2}\mathrm{Re}[\exp(-c_2/2)(1-im_0\omega_0c_3)\beta^*] ,\end{aligned} \tag{87}$$

$$\begin{aligned}\langle p\rangle&=-(2m_0\hbar\omega_0)^{1/2}\mathrm{Im}[(\eta_1+\eta_2)\beta^*]\\&=-(2m_0\hbar\omega_0)^{1/2}\mathrm{Im}\\&\quad\times\left[\exp(-c_2/2)\left[\exp(c_2)-c_1c_3-\frac{ic_1}{m_0\omega_0}\right]\beta^*\right] .\end{aligned} \tag{88}$$

The corresponding fluctuations in x and p will then be

$$\begin{aligned}\Delta x&=\left[\frac{\hbar}{2m_0\omega_0}\right]^{1/2}(2n+1)^{1/2}|\eta_1-\eta_2|\\&=\left[\frac{\hbar}{2m_0\omega_0}\right]^{1/2}(2n+1)^{1/2}\\&\quad\times|\exp(-c_2/2)(1-im_0\omega_0c_3)| ,\end{aligned} \tag{89}$$

$$\begin{aligned}\Delta p&=\left[\frac{m_0\hbar\omega_0}{2}\right]^{1/2}(2n+1)^{1/2}|\eta_1+\eta_2|\\&=\left[\frac{m_0\hbar\omega_0}{2}\right]^{1/2}(2n+1)^{1/2}\\&\quad\times\left|\exp(-c_2/2)\left[\exp(c_2)-c_1c_3-\frac{ic_1}{m_0\omega_0}\right]\right| ,\end{aligned} \tag{90}$$

whence

$$\begin{aligned}\Delta x\,\Delta p&=\frac{\hbar}{2}(2n+1)|\eta_1+\eta_2||\eta_1-\eta_2|\\&=\frac{\hbar}{2}(2n+1)|1-im_0\omega_0c_3|\\&\quad\times\left|1-c_1c_3\exp(-c_2)-\frac{ic_1}{m_0\omega_0}\exp(-c_2)\right|\\&\geq\frac{\hbar}{2}(2n+1) .\end{aligned} \tag{91}$$

Immediately we see that

$$\Delta x\sim|\exp(-c_2/2)| , \tag{92}$$

$$\Delta p\sim|\exp(c_2/2)| . \tag{93}$$

So we obtain diminishing in the fluctuation of one operator at the expense of an increase in the fluctuation of the other operator. Thus, under appropriate values of the time-dependent parameters $\omega(t)$, $f(t)$, and $\gamma(t)$ of the general driven time-dependent oscillator, squeezing in one of the quadrature variances can be attained.

IV. EXAMPLE

For illustration we will consider the case of a damped pulsating oscillator with variable frequency driven by an external force. The combined effect of damping and pulsation is treated by means of the mass parameter[15]

$$m(t)=m_0\exp[2(\gamma t+\mu\sin\nu t)] \ . \tag{94}$$

Also, we will take the frequency $\omega(t)$ to be given by

$$[\omega(t)]^2=\Omega^2+\frac{1}{\sqrt{m(t)}}\frac{d^2\sqrt{m(t)}}{dt^2} \tag{95}$$

for some constant Ω. Then Eq. (41) becomes the equation of motion for a simple harmonic oscillator and can be solved easily. The desired solution for $F(t)$, which satisfies the initial condition $c_1(0)=0$, is given by

$$F(t)=\frac{A\cos\Omega t+B\sin\Omega t}{\sqrt{m(t)}} \ , \tag{96}$$

where A and B are some constants related by

$$\frac{B}{A}=\frac{\gamma+\mu\nu}{\Omega}\equiv\tan\phi \ . \tag{97}$$

With this $F(t)$ we can find the $c_i(t)$ of $U_0(t,0)$:

$$c_1(t)=-m(t)[\Omega\tan(\Omega t-\phi)+(\gamma+\mu\nu\cos\nu t)] \ , \tag{98}$$

$$c_2(t)=-2\ln\left|\frac{\cos(\Omega t-\phi)}{\cos\phi}\exp[-(\gamma t+\mu\sin\nu t)]\right| \ , \tag{99}$$

$$c_3(t)=-\frac{1}{m_0\Omega}\frac{\sin\Omega t\cos\phi}{\cos(\Omega t-\phi)} \ . \tag{100}$$

Once the $c_i(t)$ are found, the determination of the $d_i(t)$ of $U_I(t)$ will be trivial. Assuming a sinusoidal external driving force, say $f(t)=\sin(t)$ in Eq. (16), the $d_i(t)$ are given by

$$d_1(t)=\frac{i\sqrt{\hbar}}{2\Omega}\int_0^t d\tau\sin\tau\sin\Omega\tau\exp(\gamma\tau+\mu\sin\nu\tau)\left|\frac{\cos(\Omega t-\phi)}{\cos\phi}\right|\frac{\cos\phi}{\cos(\Omega t-\phi)} \ , \tag{101}$$

$$d_2(t)=\frac{i\sqrt{\hbar}m_0}{|\cos\phi|}\int_0^t d\tau\sin\tau|\cos(\Omega\tau-\phi)|\exp(\gamma\tau+\mu\sin\nu\tau) \ , \tag{102}$$

$$d_3(t)=\frac{\hbar m_0}{2\Omega|\cos\phi|}\int_0^t d\tau\int_0^\tau d\tau'\sin\tau\sin\tau'|\cos(\Omega\tau-\phi)|\sin\Omega\tau' \times\exp(\gamma\tau+\mu\sin\nu\tau)\exp(\gamma\tau'+\mu\sin\nu\tau')\left|\frac{\cos(\Omega\tau'-\phi)}{\cos\phi}\right|\frac{\cos\phi}{\cos(\Omega\tau'-\phi)} \ . \tag{103}$$

Now, using the above results, we can write Δq and Δp as follows:

$$\Delta q=\left[\frac{\hbar}{2m_0\omega_0}\right]^{1/2}X(t)\exp[-(\gamma t+\mu\sin\nu t)]\sqrt{2n+1} \ ,$$

$$\Delta p=\left[\frac{m_0\hbar\omega_0}{2}\right]^{1/2}Y(t)\exp(\gamma t+\mu\sin\nu t)\sqrt{2n+1} \ , \tag{104}$$

where

$$X(t)=\left[\frac{\cos^2(\Omega t-\phi)}{\cos^2\phi}+\frac{\omega_0^2}{\Omega^2}\sin^2\Omega t\right]^{1/2} \ , \tag{105}$$

$$Y(t)=\left[\left[\frac{\Omega^2[m(t)]^4}{\omega_0^2m_0^4}[X(t)]^2-\sin^2\Omega t\right][Z(t)]^2+\left[\frac{\cos\phi}{\cos(\Omega t-\phi)}-\sin\Omega tZ(t)\right]^2\right]^{1/2} \ , \tag{106}$$

$$Z(t)=\tan(\Omega t-\phi)+\frac{\gamma+\mu\nu\cos\nu t}{\Omega} \ . \tag{107}$$

It is clear that there is diminishing in the fluctuation of q, together with an increase in the fluctuation of p.

V. CONCLUSION

We have investigated the evolution of a number state of a general driven time-dependent oscillator as well as its squeezing property using the evolution operator method developed by Cheng and Fung. An analytical example of a damped pulsating oscillator with variable frequency in the presence of an external driving force was examined. Our analyses indicate that the wave function of the time-dependent oscillator starts as a number state at $t=0$ and evolves as a displaced and squeezed number state at a later time, and that its time evolution is just a displacement and squeezing process. Hence it can be concluded that displaced and squeezed number states can be generated out of number states by the general driven time-dependent oscillator.

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