Intensity correlation functions for dye lasers with white gain noise

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We consider a single-mode dye-laser model with gain parameter fluctuations modeled by a white noise. The decay of the intensity correlation function $C(t)$ is analyzed with the use of a method based on the combination of the low- and high-frequency expansions of the Laplace transform of $C(t)$. The correlation function is approximated by a superposition of two decaying exponentials. Numerical simulations show that this approximation is accurate. We analyze the relaxation of $C(t)$ in the domain of parameters, gain parameter close to the pump parameter, in which the white-gain-noise model reproduces the initial slow decay of $C(t)$. It is found that the intensity correlation function does not show two separated time scales as observed experimentally.

I. INTRODUCTION

The anomalous statistical properties of dye-laser light cannot be described by conventional laser theory based on the Langevin equation with spontaneous-emission noise. Pump noise seems to be responsible for these properties. A review of the experimental situation of dye-laser fluctuations has been given by Roy, Yu, and Zhu.[1] Theoretical studies on this problem have been also reviewed.[2] The experiments of Abate, Kimble, and Mandel[3] were the first to indicate the presence of anomalous fluctuations in dye lasers. Later experiments[4] suggested that these anomalies are due to fluctuations in the pump parameter. The standard theoretical model of dye-laser fluctuations includes pump fluctuations with a finite correlation time (colored noise). This model can be formally obtained replacing the loss parameter by a fluctuating quantity. We will refer to it as the loss-noise model. A variety of calculations and simulations exists for this model in the literature, either in the white-noise limit[5] for the fluctuating loss parameter or considering a finite correlation time.[6–13] Two important effects have been proposed as a clear signature of the presence of colored noise. One is the existence of a first-order-like transition in the most probable intensity value.[9,12] A second one is a very slow initial decay of the intensity correlation function.[7,8,13]

Experimental evidence has recently been reported that identifies the pump laser as the source of noise.[10] Then a natural alternative to the standard theoretical model is the consideration of fluctuations in the gain parameter.[14–17] It has been shown that in the white-noise limit this model describes correctly the anomalous intensity fluctuations.[15] It also predicts a first-order-like transition for the most probable intensity value.[15] As concerns the intensity correlation function, the initial slope takes very small values, much smaller than the ones for the white-noise limit of the loss-noise model.[15] Therefore some effects that, in the loss-noise model, are due to the finite correlation time of the pump noise[12,13] can be described by a white-noise model. However, numerical simulations seem to indicate that the decay of the intensity correlation function does not show two separated time scales.[15] These two time scales, which are observed in the experiments, can be obtained with colored loss-noise[13] and gain-noise[16] models.

In this paper we present an analysis of the relaxation of the intensity correlation function for the white-gain-noise model. To make this analysis we use a method[18,19] based on the combination of the low- and high-frequency expansions of the Laplace transform of the correlation function. The correlation function is approximated by a superposition of two decaying exponentials. Numerical simulations show that this approximation is accurate. It is found that when the gain parameter is close to the pump parameter, only one time scale is involved in the relaxation of the intensity correlation function. This range of parameters corresponds to the domain in which the intensity correlation function has an initial slow decay. Moreover, when two exponentials are used to approximate the intensity correlation function, the time scales are not clearly separated as they appear in the experiments.[13] Therefore, a colored noise model is required to describe the behavior of the intensity correlation function.

The paper is organized as follows. The gain-noise model is introduced in Sec. II. We present in Sec. III the double-expansion method, which is used in Sec. IV to analyze the relaxation of the intensity correlation function for the white-gain-noise model. A summary of conclusions is presented in Sec. V.

II. GAIN-NOISE MODEL

The gain-noise model for a single-mode dye laser on resonance is defined by the following stochastic equation for the intensity:[15]

$$\partial_{\bar{t}}\bar{I}=2\bar{I}\left[-\kappa+\frac{\Gamma}{1+\bar{I}}\right]+\frac{2\bar{I}\sqrt{Q}}{1+\bar{I}}\bar{p}(\bar{t})+D+(D\bar{I})^{1/2}\bar{q}(\bar{t}) , \quad (1)$$

where κ and Γ are the loss and gain parameters, respectively. The process $\bar{q}(\bar{t})$ represents the fluctuations due to spontaneous emission of strength D. It is taken to be a Gaussian white noise of zero mean. The process $\bar{p}(\bar{t})$ models fluctuations of the gain parameter of strength Q. In this paper $\bar{p}(\bar{t})$ is taken to be a Gaussian white noise of zero mean and correlation

$$\langle \bar{p}(\bar{t})\bar{p}(\bar{t}')\rangle=\delta(\bar{t}-\bar{t}') . \quad (2)$$

In the following we will consider situations above threshold where spontaneous-emission noise is known to have a very small effect.[1,2] We will therefore take the limit $D\to 0$ in (1).

Redefining the intensity variable, time scale, and introducing new parameters α, α_1, and α_2,

$$I=(\Gamma/Q)\bar{I}, \quad t=2Q\bar{t} , \qquad \alpha_1=\frac{\Gamma}{Q}, \quad \alpha_2=\frac{\kappa}{Q}, \quad \alpha=\alpha_1-\alpha_2 , \quad (3)$$

we obtain[15]

$$\partial_t I=I\left[-\alpha_2+\frac{\alpha_1}{1+I/\alpha_1}\right]+\frac{I}{1+I/\alpha_1}p(t) , \quad (4)$$

$$\langle p(t)p(t')\rangle=2\delta(t-t') . \quad (5)$$

This model has two independent parameters α_1,α_2. These two parameters correspond to the gain and loss parameters rescaled with respect to the noise intensity. The white-loss-noise model has only one independent parameter, $\alpha=\alpha_1-\alpha_2$, which corresponds to the pump parameter rescaled with respect to the noise intensity. The loss-noise model is obtained taking the limit $\alpha_1\to\infty$ with $\alpha=\alpha_1-\alpha_2$ fixed. In this limit the effects of saturation can be neglected. In the following we will analyze the white-gain-noise model for different values of the parameter α/α_1. This parameter gives information on the importance of saturation effects. When α/α_1 is close to zero, saturation effects are small and we recover the loss-noise model. On the contrary, when α/α_1 is close to 1, the predictions of the white-gain-noise model are different from those of the loss-noise model.

The Fokker-Planck equation for the intensity probability density associated with (4) and (5) is given in the Stratonovich interpretation by

$$\partial_t P(I,t)=LP(I,t) \equiv -\frac{\partial}{\partial I}v(I)P(I,t)+\frac{\partial^2}{\partial I^2}D(I)P(I,t) , \quad (6)$$

$$v(I)=I\left[-\alpha_2+\frac{\alpha_1}{1+I/\alpha_1}\right]+\frac{I}{(1+I/\alpha_1)^3} , \quad (7)$$

$$D(I)=\frac{I^2}{(1+I/\alpha_1)^2} . \quad (8)$$

The stationary solution of (6)–(8) is given by[15]

$$P_{\rm st}(I)=N\left[I^{\alpha-1}+\frac{I^\alpha}{\alpha_1}\right] \times\exp\left[\left[1-\frac{2(\alpha_1-\alpha)}{\alpha_1}\right]I-\frac{\alpha_1-\alpha}{2\alpha_1^2}I^2\right] . \quad (9)$$

This stationary solution is only valid in the region $\alpha>0$ that we consider here.

As mentioned in the Introduction, the white-gain-noise model describes correctly important experimental features that have been attributed to colored noise effects in the context of the loss-noise model. However, numerical simulations[15] indicate that the decay of the intensity correlation function does not show two separated time scales that typically appear for colored noise models. To analyze the relaxation of the intensity correlation function we use the double-expansion method. This method is presented in the following section.

III. THE DOUBLE-EXPANSION METHOD

Markovian correlation functions can be calculated with the double-expansion method. Here we are interested in the calculation of the normalized intensity correlation function $C(t)$,

$$C(t)=\frac{\langle I(t+s)I(s)\rangle-\langle I\rangle^2}{\langle I^2\rangle-\langle I\rangle^2} . \quad (10)$$

The method has been described in detail in Refs. 18 and 19. The basic idea is to consider a double expansion of the Laplace transform of any Markovian steady-state correlation function for both high and low frequencies, ω:

$$\tilde{C}(\omega)=\frac{1}{\omega}\sum_{k=0}^{\infty}\mu_k(1/\omega)^k , \quad (11)$$

$$\tilde{C}(\omega)=\sum_{k=0}^{\infty}(-1)^k T_k\omega^k/k! , \quad (12)$$

where the coefficients of the expansions are the derivatives at $t=0$,

$$\mu_k=\left.\frac{d^kC(t)}{dt^k}\right|_{t=0+} \quad (13)$$

and the so-called relaxation moments

$$T_k=\int_0^\infty t^kC(t)dt . \quad (14)$$

Equation (11) contains information on the short-time behavior of $C(t)$. This expansion is the one involved in the usual continued fraction expansion method. In fact, to a given order, the continued fraction method contains a finite number of exact derivatives of $C(t)$ at $t=0$. In the other limit, the low-frequency expansion (12) is related to the large-time behavior of $C(t)$. The coefficients of this expansion can be interpreted as the "moments" of $C(t)$, so that they contain information about the distribution of area under the curve $C(t)$. The first coefficient is the usual relaxation time T_0. It provides a global characterization in terms of the total area under the curve of

$C(t)$. The relaxation time plays the counterpart role of the effective eigenvalue $\lambda_{\rm eff}=|\dot{C}(0)|=\mu_1$, which gives information on the local behavior of $C(t)$. For systems with essentially only one time scale both characteristic times will be identical. However, when different time scales coexist it will be necessary to take into account more "effective time scales" in the description. These time scales can be obtained with the double-expansion method.

The aim of the double-expansion method is to obtain a systematic approximation for $C(t)$ that contains simultaneously information from the low- and high-frequency expansions of $C(\omega)$. To do so, one uses a Padé approximant to a given order N for the combination of both expansions:

$$\tilde{C}(\omega)=\sum_{n=1}^{N} a_n/(\omega+\lambda_n) \ . \tag{15}$$

Now, the coefficients have to be determined by imposing simultaneously the conditions on the high- and low-frequency limits. From Eq. (15) one can invert the Laplace transform resulting in

$$C(t)=\sum_{n=1}^{N} a_n \exp(-\lambda_n t) \ . \tag{16}$$

To determine the weights and time scales of these exponentials, one asks first for the normalization condition $C(0)=\mu_0=1$. Then, one imposes the conditions on these coefficients for which n derivatives and m relaxation moments are obtained from Eq. (16) in an exact way. We call this approximation "$ndmt$" ($n+m$ has to be odd and $n+m+1=2N$). The conditions are[18,19]

$$\sum_{i=1}^{N} a_i(\lambda_i)^k=(-1)^k\mu_k \quad (k=0,1,\ldots,n) \ , \tag{17}$$

$$\sum_{i=1}^{N} a_i(\lambda_i)^{-k-1}=T_k/k! \quad (k=0,1,\ldots,m-1) \ . \tag{18}$$

The last point of the method is the calculation of μ_k and T_k. In the one-dimensional Markovian case, as in the white-gain-noise model defined by Eq. (1), these parameters can be easily obtained by quadratures.[18,19] The derivatives μ_k $(k\geq 1)$ are given by

$$\mu_k=\frac{1}{\langle I^2\rangle-\langle I\rangle^2}\langle I[L^{\dagger}(I)]^k I\rangle \ , \tag{19}$$

where the Fokker-Planck operator is defined by (6)–(8) and the steady-state moments can be calculated from (9). The relaxation moments are given by

$$T_k=\frac{(-1)^k k!}{\langle I^2\rangle-\langle I\rangle^2}\int_0^{\infty}\frac{G_0(I)G_k(I)}{D(I)P_{\rm st}(I)}dI \ , \tag{20}$$

where

$$G_0(I)=-\int_0^{I}(I'-\langle I\rangle)P_{\rm st}(I')dI' \tag{21}$$

and the functions $G_k(I)$ follow the recurrence

$$G_k(I)=\int_0^{I}P_{\rm st}(I')\left[\int_0^{I'}\frac{G_{k-1}(I'')}{D(I'')P_{\rm st}(I'')}dI'' - \left\langle\int_0^{I}\frac{G_{k-1}(I')}{D(I')P_{\rm st}(I')}dI'\right\rangle\right]dI' \ . \tag{22}$$

IV. INTENSITY CORRELATION FUNCTIONS

To analyze the decay of the intensity correlation function $C(t)$ for the dye laser with white gain noise we use the double-expansion method. In this way we approximate $C(t)$ by a sum of exponentials. We consider the domain of parameters in which the initial slope of $C(t)$ (effective eigenvalue $\lambda_{\rm eff}$) takes very small values. This domain corresponds[15] to the pump parameter α close the gain parameter α_1. The decay of $C(t)$ is analyzed for different values of α/α_1 greater than 0.5. In this region the saturation effects are important and the predictions of the white-gain-noise model are very different from those of the white-loss-noise model.[15] This model corresponds to the limit $\alpha/\alpha_1\to 0$. When only colored noise is present, $\lambda_{\rm eff}$ is strictly zero.[8] This is the case of the colored loss-noise model when spontaneous-emission noise is neglected. As a consequence the small initial slope of $C(t)$ has been interpreted as a signature of colored noise.[7,8,13] However, this effect can also be satisfactorily explained with the white-gain-noise model[15] in the domain of parameters that we consider here. We also note that this domain includes the one in which $P_{\rm st}(I)$ has a relative maximum,[15] which has also been explained as a colored noise effect in the loss-noise model.[13]

We first analyze the domain of parameters in which the decay of the intensity correlation function can be described by only one exponential. In this case, $\lambda_{\rm eff}$ and the inverse of the relaxation time, T_0^{-1}, are very close. Then, $\epsilon=(\lambda_{\rm eff}-T_0^{-1})/(\lambda_{\rm eff}+T_0^{-1})$ must be small. These characteristic times can be obtained from (19)–(22). We plot ϵ in Fig. 1 as a function of the mean intensity $\langle I\rangle$ for different values of α/α_1 in the region where $\lambda_{\rm eff}$ takes values much smaller than the ones for the white-loss-noise model. We also include for comparison the case $\alpha/\alpha_1=0$ that corresponds to the white-loss-noise model. Since we have neglected the spontaneous-emission noise, we only consider large values for the mean intensity: $\langle I\rangle>1$. We note that in this region the intensity fluctuations are rather insensitive[15] to the value of the gain parameter α_1. Then, for a given intensity $\langle I\rangle$ we can analyze the behavior of $C(t)$ for different values of α/α_1 with no significant changes in the intensity fluctuations. Our results in Fig. 1 indicate that ϵ decreases when both α/α_1 and $\langle I\rangle$ increase. Both characteristic times are nearly equal ($\epsilon<0.05$) whenever $\alpha/\alpha_1\geq 0.7$ and $\langle I\rangle>2$. Therefore, when the initial slope of $C(t)$ is small, that is for α close to α_1, only one time scale is involved in the decay of $C(t)$. This domain of parameters corresponds to the region where saturation effects are important.

To analyze in more detail the decay of the correlation function we approximate $C(t)$ by the sum of two exponentials:

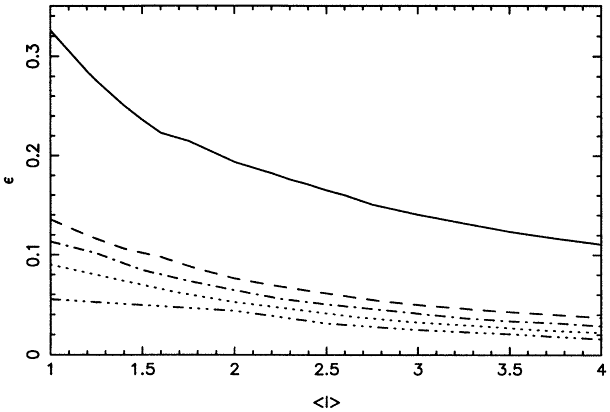

FIG. 1. The parameter $\epsilon=(\lambda_{\rm eff}-T_0^{-1})/(\lambda_{\rm eff}+T_0^{-1})$ vs the mean intensity $\langle I\rangle$ for different values of α/α_1: ——, 0; — — —, 0.5; —·—·—·, 0.6; · · · ·, 0.7; — ··· — ··· —, 0.8. The parameter α/α_1 measures the importance of saturation effects. When α/α_1 is close to 1, saturation effects are important. The case $\alpha/\alpha_1=0$ corresponds to the loss-noise model (no saturation). Note that when ϵ is small the intensity correlation function can be approximated by one exponential.

$$C(t)=a_1\exp(-\lambda_1 t)+(1-a_1)\exp(-\lambda_2 t)\ , \qquad (23)$$

where the parameters a_1,λ_1,λ_2 are determined from (17) and (18) in the $1d2t$ approximation, that is from the knowledge of $\lambda_{\rm eff}$, T_0, and T_1. These parameters are obtained from (19)–(22). The results are shown in Figs. 2–4 for different values of $\alpha/\alpha_1\geq 0.5$. We also include the results for the white-loss-noise model ($\alpha/\alpha_1=0$). When α/α_1 and $\langle I\rangle$ increase, a_1 tends to 1 and there is only one time scale in agreement with the results found for ϵ.

To check the $1d2t$ approximation we have performed numerical simulations of the steady-state intensity correlation function $\lambda(t)$ defined as

$$\lambda(t)=\frac{\langle I(t+s)I(s)\rangle-\langle I\rangle^2}{\langle I\rangle^2}\ . \qquad (24)$$

In Figs. 5–8 we compare simulation results[20] for $\lambda(t)$

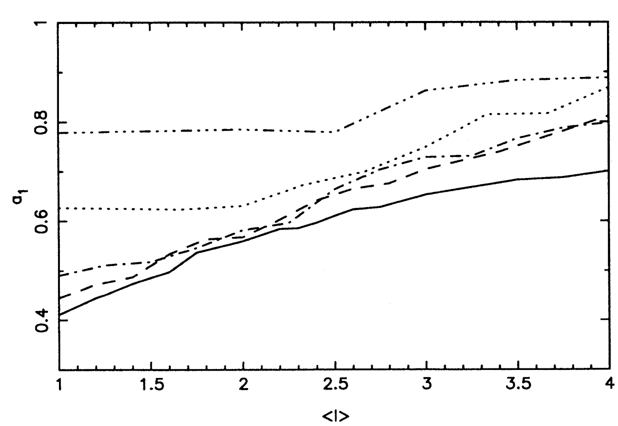

FIG. 2. The amplitude a_1 of the first exponential used to approximate the intensity correlation function vs the mean intensity $\langle I\rangle$. The amplitude of the second exponential is $(1-a_1)$. Same parameters as in Fig. 1.

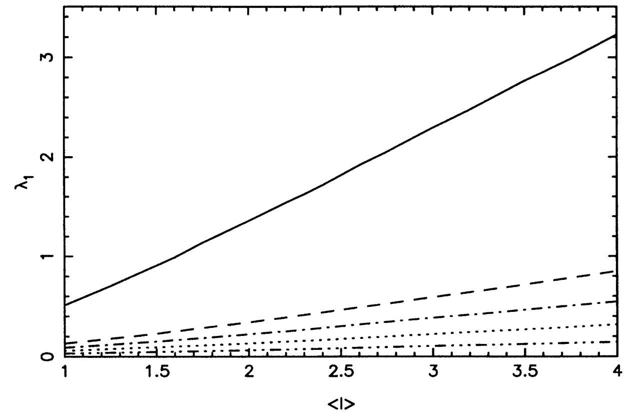

FIG. 3. Time scale λ_1 of the first exponential used to approximate the intensity correlation function vs the mean intensity $\langle I\rangle$. Same parameters as in Fig. 1.

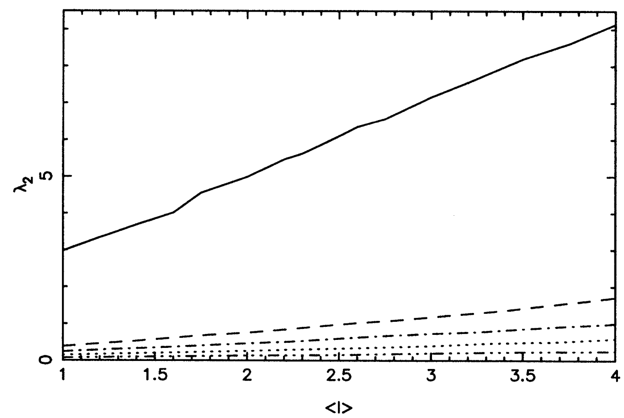

FIG. 4. Time scale λ_2 of the second exponential used to approximate the intensity correlation function vs the mean intensity $\langle I\rangle$. Same parameters as in Fig. 1.

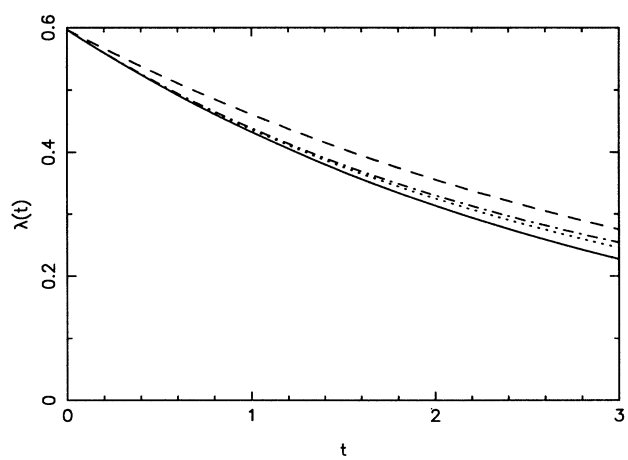

FIG. 5. Intensity correlation function $\lambda(t)$ for the white-gain-noise model for $\alpha/\alpha_1=0.53$, $\langle I\rangle=1.4$, $\lambda_{\rm eff}=0.32$. ——, $1d0t$ approximation; — — —, $0d1t$ approximation; —·—·—·, $1d2t$ approximation; · · · ·, simulation.

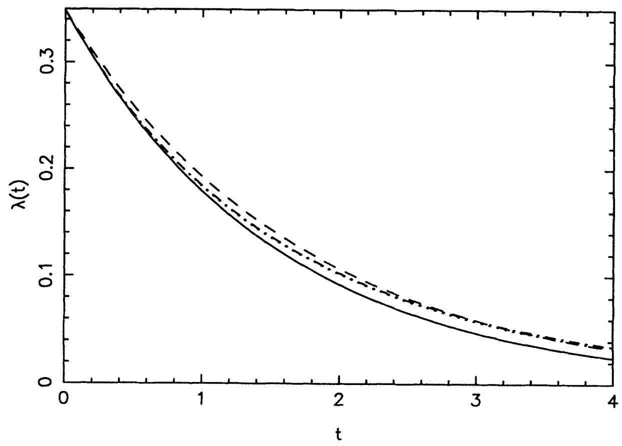

FIG. 6. Intensity correlation function $\lambda(t)$ for the white-gain-noise model for $\alpha/\alpha_1=0.5$, $\langle I\rangle=2.6$, $\lambda_{\text{eff}}=0.66$. ——, $1d0t$ approximation; — — —, $0d1t$ approximation; —·—·—·, $1d2t$ approximation; · · · ·, simulation.

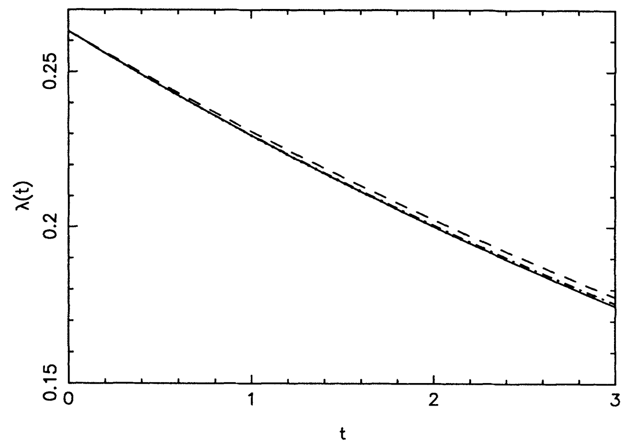

FIG. 8. Intensity correlation function $\lambda(t)$ for the white-gain-noise model for $\alpha/\alpha_1=0.8$. $\langle I\rangle=3.5$, $\lambda_{\text{eff}}=0.136$. ——, $1d0t$ approximation; — — —, $0d1t$ approximation; —·—·—·, $1d2t$ approximation; · · · ·, simulation.

with the $1d0t$ approximation (one exponential with time scale given by $\lambda_{\text{eff}}^{-1}$), the $0d1t$ approximation (one exponential with time scale given by T_0),and the $1d2t$ approximation given by (23). The values of α/α_1 and $\langle I\rangle$ are chosen in such a way that we can compare the decay of $\lambda(t)$ for similar values of $\langle I\rangle$ and $\lambda(0)$ (Figs. 5 and 7) and for similar values of α/α_1 (Figs. 5–8). Figure 5 corresponds to a situation ($\alpha/\alpha_1=0.53$, $\langle I\rangle=1.4$) where the stationary intensity distribution $P_{\text{st}}(I)$ given by (9) diverges at $I=0$, decreasing monotonically with the intensity. In this case the parameter ϵ is not very small. Then the $1d0t$ and $0d1t$ approximations behave differently. The $1d0t$ approximation is only valid for very short times. When two exponentials are considered, as in the $1d2t$ approximation, a better description of the decay of $\lambda(t)$ is obtained. Figure 6 corresponds to a greater value of the mean intensity ($\alpha/\alpha_1=0.5$, $\langle I\rangle=2.6$). In this case, $P_{\text{st}}(I)$ has a single maximum at a nonzero value of I. The decay of the intensity correlation function is faster than in Fig. 5. The $1d0t$ and $0d1t$ approximations are valid for short and long times, respectively. The agreement of the $1d2t$ approximation with the simulation in all time regimes is excellent. In Fig. 7 we consider values of $\lambda(0)$ and $\langle I\rangle$ similar to those of the case in Fig. 5, but with a greater value of $\alpha/\alpha_1=0.76$. The decay of $\lambda(t)$ is slower than in Fig. 5. Then the $1d0t$ and $1d2t$ approximations are valid for a large time interval. In this case $P_{\text{st}}(I)$ diverges at $I=0$ and a relative maximum and minimum exist. When we increase the mean intensity as in Fig. 8, the parameter ϵ decreases and all the approximations give a good description of the early time decay of $\lambda(t)$. The shape of $P_{\text{st}}(I)$ is the same as in Fig. 7.

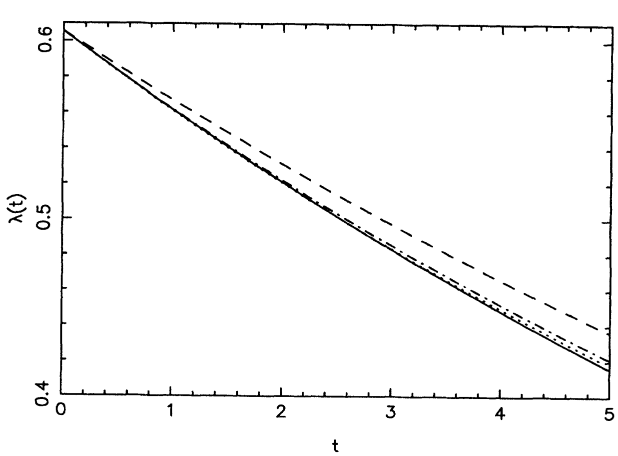

FIG. 7. Intensity correlation function $\lambda(t)$ for the white-gain-noise model for $\alpha/\alpha_1=0.76$, $\langle I\rangle=1.35$, $\lambda_{\text{eff}}=0.075$. ——, $1d0t$ approximation; — — —, $0d1t$ approximation; —·—·—·, $1d2t$ approximation; · · · ·, simulation.

From the analysis of Figs. 5–8 we can conclude that the $1d2t$ approximation gives a good description of the decay of the intensity correlation function. The results in Figs. 3 and 4 for the time scales λ_1 and λ_2 show that with $\langle I\rangle$ fixed the decay is slower when the pump parameter approaches the gain parameter. This is in agreement with the values found[15] for λ_{eff}. As noted above, when α/α_1 and $\langle I\rangle$ increase, the decay of the intensity correlation function can be described with only one time scale given by λ_1. The results for λ_1 can be fitted very well by a straight line. The slope of this line is given by the one obtained[15] by linearizing Eq. (1): $(1-\alpha/\alpha_1)^2$, with an error smaller than 10%. The results for λ_2 can also be fitted by a straight line. When $\alpha/\alpha_1\geq 0.5$, the slope is approximately given by $1.6(1-\alpha/\alpha_1)^2$. Moreover, when $\alpha/\alpha_1\geq 0.5$ and $\langle I\rangle\geq 2$, the time scales λ_1 and λ_2 are such that $\lambda_2/\lambda_1<2$. Then they are not clearly separated as they appear in the experiments.[13] We conclude that whenever the gain parameter is close to the pump parameter the relaxation of $C(t)$ involving two time scales, that is observed experimentally, cannot be described by the white-gain-noise model. This is the domain of parameters in which, in contrast with the white-loss-noise model, the gain-noise model is able to reproduce[15] the slow initial decay of $C(t)$. In this domain saturation effects are important and consequently there are important differences between the loss-noise and the gain-noise model in the white-noise limit.

Therefore, a colored noise model is required to describe both the small initial slope of the intensity correlation function $C(t)$ and the two time scales involved in the decay of $C(t)$.

V. CONCLUSIONS

We have analyzed the decay of the intensity correlation function $C(t)$ for a single-mode dye laser incorporating pump white noise through gain-parameter fluctuations. This model includes a fluctuating saturation term. A method based on the combination of the low- and high-frequency expansions of the Laplace transform of $C(t)$ is used to approximate the correlation function by a superposition of two decaying exponentials. Numerical simulations show that this approximation is accurate.

When saturation effects are important, the white-gain-noise model is able to reproduce the small initial slope of the intensity correlation function. The situation corresponds to a domain of parameters such that the gain parameter is close to the pump parameter. Our results show that in this domain of parameters the two exponentials used to approximate the decay of the intensity correlation function have time scales that are not clearly separated as they appear in the experiments.

The analysis of the relaxation of the intensity correlation function leads then to the conclusion that fluctuations in dye lasers must be modeled by a colored noise, either in a loss-noise or in a gain-noise model.

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