Nonlinear atomic homodyne detection: A technique to detect macroscopic superpositions in a micromaser

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We propose a measurement scheme to detect the multiphoton coherences inherent in the macroscopic quantum superpositions that can be generated in a superconducting micromaser cavity. The scheme is a nonlinear version of a single-atom homodyne detector. It is shown that the ionization probability of a single test atom is given in terms of the Wigner characteristic function of the field.

I. INTRODUCTION

With the recent refinement of experimental techniques in quantum optics, it now appears possible to generate macroscopic quantum superpositions (Schrödinger cats[1]) under laboratory conditions.[2–7] A number of potential nonlinear optical schemes to achieve this goal have recently been analyzed. In particular, it has been pointed out[2] that a coherent state propagating through an optical Kerr medium may evolve into a superposition of two macroscopically distinct coherent states 180° out of phase. Another proposal[3] uses nondegenerate parametric amplification and mixing of two modes via rotating of their polarization to generate optical Schrödinger cats. Most recently, the possibility to generate superpositions of macroscopically distinct number states in the field of a micromaser cavity has been discussed.[4–7] In this scheme, a superconducting micromaser cavity is pumped by a stream of polarized two-level atoms. For weak enough cavity damping, the stationary state of the field inside the cavity is almost pure[5] and is very close to a coherent superposition of number states, called "cotangent state."[4]

This latter scheme is particularly attractive in that it produces macroscopic quantum superpositions in steady state, a result that might appear surprising as such superpositions are notoriously fragile in the presence of dissipation.[8] Indeed, it is generally believed that the coupling of a Schrödinger cat to the environment erases the macroscopic coherences quasi-instantaneously, leaving one with a incoherent mixture of states.[9] The reason why the micromaser cotangent states are quite insensitive to dissipation[5,6] is that the micromaser is an open system: the polarized atoms successively injected inside the cavity rebuild the quantum coherence of the field at a rate that precisely balances the loss of coherence due to dissipation.

Although a number of formidable technical challenges need to be met to generate such a macroscopic superposition, it appears that none of them is fundamental and that this goal can be achieved with available technology. Indeed, the major difficulty seems to reside not so much in the generation of such states as in their detection. This is due in large part to the lack of appropriate photocounters operating in the microwave regime, and hence to the impossibility to directly measure the micromaser field. Micromaser experiments provide only an indirect access to the state of the field, which is inferred from the state of the atoms as they exit the cavity, as measured by state-selective field ionization.[10] The goal of this paper is to propose and analyze a measurement scheme that respects this basic constraint while at the same time providing a direct measurement of the Wigner characteristic function of the field, and hence of the field itself.

This paper is organized as follows. Section II discusses the basic difficulty in inferring the complete state of the micromaser field from simple atomic ionization rates. This discussion suggests using nonlinear atomic homodyne detection as a solution to the problem. A simple model for this measurement scheme is developed. Section III analyzes this scheme and finds that the resulting state-selective atomic ionization rate is a direct measure of the Wigner characteristic function of the field. Hence it contains all possible information on its state. Finally, Sec. IV is a summary and conclusion.

II. NONLINEAR ATOMIC HOMODYNE DETECTION

In micromasers, the injected atoms play a dual role of pumps and measurement devices: they generate the field inside the cavity while at the same time acting as single-atom photon detectors. They interact with the cavity mode via the Jaynes-Cummings Hamiltonian,[11] so that an absorption process is always followed by an emission process. In other words, their state depends only on a very restricted class of combinations of field annihilation and creation operators that are sums of terms of the forms $(a^{\dagger}a)^n$, $(a^{\dagger}a)^n a^{\dagger}$, or $(a^{\dagger}a)^n a$, where n is an integer. Physically, this means that the atoms only probe coherences between neighboring Fock states of the field. However, to detect macroscopic superpositions one needs a mea-

surement scheme sensitive to coherences between vastly different Fock states, and what we would need instead is an atomic response that is sensitive to more general products of field creation and annihilation operators, of the generic form $a^\dagger a^\dagger a^\dagger a^\dagger \cdots a^\dagger aaa \cdots a$.

We have just seen that this is not possible in the dipole and rotating-wave approximation if the atom interacts with a single mode of the electromagnetic field. However, the situation changes drastically if two modes of the field are present: although it is still true that an absorption process must be followed by an emission process, one can well imagine situations where, say, absorption from the first mode is followed by emission into the second mode, then again by absorption from the first mode, etc. In such situations, the atomic response would indeed be sensitive to coherences between distant Fock states of the field mode under investigation.

This observation suggests an alternative detection scheme that is essentially a nonlinear version of a single-atom homodyne detector.[12] To measure the macroscopic superpositions that have been generated in mode a (the "signal"), a second mode b is excited (the "local oscillator") bringing the cavity into the state $\rho^{\text{field}}=\rho^a\rho^b$. A test atom is then injected into the cavity, where it interacts with both modes. After the atom has left the cavity, the state of the atom is measured. We show in the following that this measurement indeed provides full information on the state of the field in mode a.

There are some practical problems associated with this scheme, the most important one being that the local oscillator mode should not interact with the atoms used to prepare the macroscopic superposition to be detected. This problem can be solved by using different atoms for the preparation and the measurement stages, so that the "preparation atoms" have selection rules such that they are coupled to the first mode only, while the "measurement" atoms are coupled to both modes. This could be achieved by considering, e.g., two cavity modes of orthogonal propagation directions and orthogonal polarizations.

Let us assume then that the micromaser a mode has been prepared in a macroscopic quantum superposition, and consider a test atom injected inside the cavity in the state described by the density matrix

$$\rho^{\text{test}}=\rho_\uparrow|\uparrow\rangle\langle\uparrow|+\rho_\downarrow|\downarrow\rangle\langle\downarrow| , \tag{1}$$

where $|\uparrow\rangle$ and $|\downarrow\rangle$ stand for its upper and lower state, respectively. The atom-field interaction energy is given by the generalized Jaynes-Cummings Hamiltonian

$$H=\hbar\kappa[\sigma_+(a+b)+(a^\dagger+b^\dagger)\sigma_-]=\sqrt{2}\hbar\kappa(\sigma_+ A+A^\dagger\sigma_-) , \tag{2}$$

where κ measures the atom-field interaction strength. (For the sake of simplicity, we assume that the transition $|\uparrow\rangle\leftrightarrow|\downarrow\rangle$ is in resonance with the both fields. Doppler-effect corrections can be trivially corrected for by an appropriate detuning of the local oscillator field.) The signal and local oscillator modes are described by the pairs of boson operators $a,a^\dagger$ and $b,b^\dagger$, respectively, where $[a,a^\dagger]=[b,b^\dagger]=1$. The atomic polarization operators are represented by the fermion operators σ_+ and σ_-, where $\{\sigma_+,\sigma_-\}=1$. In Eq. (2) we have also introduced the annihilation and creation operators A and $A^\dagger$ of the composite mode, with

$$A=\frac{1}{\sqrt{2}}(a+b) , \tag{3}$$

and $[A,A^\dagger]=1$.[13]

The Hamiltonian (2) implies the conservation law

$$[H,K+\sigma_+\sigma_-]=0 , \tag{4}$$

where

$$K=A^\dagger A=\tfrac{1}{2}(b^\dagger b+a^\dagger a+a^\dagger b+b^\dagger a) \tag{5}$$

denotes the number operator of the A photons. The physical meaning of the conservation law (4) is that the number of A photons can change during the interaction with the test atom by at most 1, the "1" being contained in the atom operator $\sigma_+\sigma_-$. However, the number of a photons may change individually by more than 1, opening up the possibility to trace the multiphoton coherences in the a mode.

When the atom leaves the cavity after an interaction time τ, the combined atom-field density matrix has evolved towards $\rho(\tau)=U\rho^{\text{test}}\rho^{\text{field}}U^\dagger$, where

$$\begin{aligned}U&\equiv e^{-(i/\hbar)H\tau}\\&=\cos(\sqrt{2}\kappa\tau\sqrt{K+1})\sigma_+\sigma_-+\cos(\sqrt{2}\kappa\tau\sqrt{K})\sigma_-\sigma_+\\&\quad-i\frac{\sin(\sqrt{2}\kappa\tau\sqrt{K})}{\sqrt{K}}A^\dagger\sigma_--i\frac{\sin(\sqrt{2}\kappa\tau\sqrt{K+1})}{\sqrt{K+1}}A\sigma_+\end{aligned} \tag{6}$$

is the propagator of the Hamiltonian (3) in the interaction picture.

III. IONIZATION PROBABILITIES

After the atom leaves the cavity, it interacts with a dc electric field weak enough for ionization to be possible from its upper state only (state-selective field ionization).[10] Ensemble averaging the results of a series of experiments started from the same initial conditions yields the upper-state ionization probability

$$\begin{aligned}p&\equiv\mathrm{Tr}(U\rho^a\rho^b\rho^{\text{test}}U^\dagger\sigma_+\sigma_-)\\&=\tfrac{1}{2}+\tfrac{1}{2}\rho_\uparrow\langle\cos(2\sqrt{2}\kappa\tau\sqrt{k+1})\rangle\\&\quad-\tfrac{1}{2}\rho_\downarrow\langle\cos(2\sqrt{2}\kappa\tau\sqrt{K})\rangle,\end{aligned} \tag{7}$$

where the angular brackets denote the expectation value $\langle X\rangle\equiv\mathrm{Tr}(\rho^a\rho^b X)$.

To proceed, we assume that the local oscillator is prepared at the beginning of every measurement in a strongly populated coherent state

$$\rho^b=|\beta\rangle\langle\beta| , \tag{8}$$

with $\beta\gg1$, so that the b mode can be described classically and the quantum-mechanical operators b and $b^\dagger$ can be replaced by their classical amplitudes β and β^*. We fur-

ther assume that the a mode is weakly excited, $\langle a^\dagger a\rangle \ll \langle b^\dagger b\rangle$, which allows us to set $a^\dagger a + b^\dagger b \simeq |\beta|^2 \equiv I$, where I denotes the classical intensity of the b mode. Under these conditions we have

$$\sqrt{K+1} \simeq \sqrt{K} \simeq \frac{1}{\sqrt{2}}\sqrt{I} + \frac{1}{\sqrt{2}}\left[\frac{\beta}{2\sqrt{I}}a^\dagger + \frac{\beta^*}{2\sqrt{I}}a\right] . \tag{9}$$

Inserting this expression into Eq. (7), the ionization simplifies to

$$p = \tfrac{1}{2} + \tfrac{1}{2}(\rho_\uparrow - \rho_\downarrow)\tfrac{1}{2}[e^{i2\kappa\tau\sqrt{I}}\chi(\mu) + \text{c.c.}] , \tag{10}$$

where $\chi(\lambda)$ denotes the Wigner characteristic function[14] of the signal mode

$$\chi(\lambda) = \mathrm{Tr}(\rho^a e^{\lambda a^\dagger - \lambda^* a}) , \tag{11}$$

and μ is given by

$$\mu = i\frac{\kappa\tau\beta}{\sqrt{I}} = \kappa\tau e^{i(\phi_b + \pi/2)} , \tag{12}$$

where we have introduced the phase ϕ_b of the local oscillator.

Equation (10) is the main result of this paper. It shows that for a classical local oscillator, the ionization probability (10) is proportional to the characteristic function (11) of the signal mode. The significance of this result resides in the well-known fact that the characteristic function of a distribution contains all possible information about that distribution. As seen from Eq. (12), the characteristic function can be fully determined by varying the interaction time between the test atoms and the field or, more conveniently, the phase of the local oscillator. Hence the ionization probability $p(\tau,\phi_b)$ reveals all information about the state of the signal mode, a very satisfactory result indeed.[15]

What distinguishes an incoherent mixture from a coherent superposition of number states are the off-diagonal elements ρ_{nm}, $n \neq m$, of the field density matrix. Upon comparing the ionization probabilities p^c and p^i for a coherent superposition and its incoherent counterpart where the off-diagonal density matrix elements ρ_{nm} have been set equal to zero, one observes that the difference $\delta p = p^c - p^i$ is solely determined by the off-diagonal part of the coherent superposition. To illustrate this point, we consider the simple situation of a superposition of two number states $|0\rangle$ and $|N\rangle$ with amplitudes c_0 and c_N, respectively. In this case the difference δp between the ionization probabilities is given by

$$\delta p = \tfrac{1}{4}\bar{\sigma}_z e^{2i\kappa\tau\sqrt{I}} \times \mathrm{Tr}\{(c_0 c_N^* |0\rangle\langle N| + c_0^* c_N |N\rangle\langle 0|) e^{\mu a^\dagger} e^{-\mu^* a}\} + \text{c.c.}, \tag{13}$$

where $\bar{\sigma}_z = \rho_\uparrow - \rho_\downarrow$. The trace over the displaced operator $|O\rangle\langle N|$ is easily calculated as

$$\begin{aligned}\mathrm{Tr}(|O\rangle\langle N| e^{\mu a^\dagger - \mu^* a}) &= e^{-|\mu|^2/2}\mathrm{Tr}(|0\rangle\langle N| e^{\mu a^\dagger} e^{-\mu^* a}) \\ &= e^{-|\mu|^2/2}\langle N| e^{\mu a^\dagger}|0\rangle \\ &= e^{-|\mu|^2/2}\langle N|\mu\rangle = \frac{\mu^N}{\sqrt{N!}} e^{-|\mu|^2/2} ,\end{aligned} \tag{14}$$

where the Baker-Campbell-Hausdorff formula was used to disentangle the exponential operator in the first line, while cyclic invariance of the trace and the identity $a|0\rangle = 0$ was used in the second line. The ket $|\mu\rangle$ in the third line denotes a coherent state with amplitude μ. By inserting the expression (14) into (13) and making use of Eq. (12) one finds

$$\delta p = \bar{\sigma}_z |c_0 c_N| \frac{|\kappa\tau|^N}{\sqrt{N!}} e^{-|\kappa\tau|^2/2} \cos\left[2\kappa\tau\sqrt{I} + N\frac{\pi}{2}\right] \times \cos(N\phi_b + \phi_0 - \phi_N) , \tag{15}$$

where the phases ϕ_0 and ϕ_N of the coherent amplitudes c_0 and c_N have been introduced.

The second cosine function in Eq. (15) can be maximized by a proper choice of the phase of the local oscillator, while the first cosine function becomes maximal by a proper choice of its intensity I. Furthermore, the prefactor is maximum for initially fully inverted test atoms and for equal probability amplitudes c_0 and c_N. Under these conditions we find simply

$$\delta p = \frac{1}{2}\frac{(\kappa\tau)^N}{\sqrt{N!}} \exp[-\tfrac{1}{2}(\kappa\tau)^2] . \tag{16}$$

For fixed N, this expression assumes a maximum

$$\delta p_{\max} = \frac{1}{2}\frac{1}{\sqrt{N\pi}} \exp[-\tfrac{1}{2}N + \tfrac{1}{2}N\ln(N)] \simeq \tfrac{1}{2}(2\pi N)^{-1/4} \tag{17}$$

for an optimum interaction time

$$\tau_{\text{opt}} = \frac{\sqrt{N}}{\kappa} . \tag{18}$$

The approximate equality in Eq. (17) holds up to corrections of relative magnitude $\exp[O(1/N)]$. The weak N dependence of $\delta p_{\max}$, which makes it hard to distinguish a coherent superposition from an incoherent mixture in the large-N limit, is a result of the homodyne detection of the number state of the signal mode. Indeed, by inspection of Eq. (14) one sees that the homodyne detection results in a coherent displacement of the states by an amount μ, where $|\mu| = \sqrt{N}$ if δp is maximal. The $N^{-1/4}$ behavior of $\delta p_{\max}$ simply results from the projection of the coherent state $|\mu = \sqrt{N}\rangle$ onto the number state $|N\rangle$.

Finally, we remark that the result of Eq. (18) can be intuitively understood by noting that the time between successive photon emissions (or absorptions) into the a mode is given by the inverse of its Rabi frequency, $\tau_a \simeq 1/\kappa\sqrt{N}$. The N successive emission (or absorption) acts necessary to detect the coherences therefore need a time $N\tau_a \simeq \sqrt{N}/\kappa \simeq \tau_{\text{opt}}$.

IV. SUMMARY AND CONCLUSION

In this paper, we have proposed a nonlinear atomic homodyne detector scheme that enables us to distinguish macroscopic coherent superpositions from incoherent mixtures. The essential role of the local oscillator is that it provides the possibility of N successive emission acts into the mode to be characterized without interruption by an absorption from this mode. The combined effects of the local oscillator and of the mode under study on the nonlinear response of a test atom reveal then the macroscopic coherences in the signal mode. Specifically, for a classical local oscillator, the ensemble-averaged ionization probability from the upper state of the test atoms is given in terms of the Wigner characteristic function of the signal mode and hence contains all information about its quantum state.

Clearly, this scheme requires one to prepare both the signal and the local oscillator properly before each measurement and to perform ensemble averages in the conventional quantum-mechanical fashion. The reason is that the coupling of the local oscillator to the signal mode through the measurement atom changes the state of the signal mode significantly and may even destroy its macroscopic coherences. The back action imposed by the measurement process on the state of the signal mode is presently under study and will be the subject of future work.

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