Quantum nondemolition measurement of a photon number using electron interferometers of semiconductor microstructures

Akira Shimizu*
Canon Research Center, 5-1 Morinosato-Wakamiya, Atsugi, Kanagawa 243-01, Japan
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We discuss effects of *quantized* electromagnetic fields upon the electronic conduction in interferometers of semiconductor microstructures. An optical field in an arbitrary quantum state is assumed to hit the interferometer, and the time evolution of the coupled photon-electron system is evaluated. It is found that the interferometer works as a quantum nondemolition photodetector if the interferometer is designed such that the interaction is switched adiabatically.

Electronic conduction in microstructures of metals or semiconductors is a subject of growing interest.[1] Most previous works discussed effects of static magnetic or electric fields upon the electronic conduction.[1] A few works have appeared recently that discussed the conductance modulation by classical electromagnetic fields.[2] In this paper we discuss effects of *quantized* electromagnetic fields upon the electronic conduction in semiconductor interferometers of a "mesoscopic" size.[3] An optical field in an arbitrary quantum state, such as a number state or a coherent state, is assumed to hit the mesoscopic interferometer, and the time evolution of the coupled photon-electron system is evaluated. Various quantities, such as a quantum-mechanical noise in the electron-interference current, are given in closed forms. Most importantly, it is found that the interferometer works as a quantum nondemolition (QND) photodetector[4–7] if the interferometer is designed such that the interaction is switched adiabatically. That is, one can measure the photon number without "backactions" upon the photon number by measuring the electron-interference current. In contrast, ordinary measuring apparatus contaminate the observable of interest (the photon number) by backactions of the measurements.[4] Due to the specific natures of the electrons in semiconductor microstructures as probe quanta, the operation principle of the present QND scheme is different from any of the previously proposed ones.[4–7]

To contrast the present case with the case of classical optical fields,[2] and also to reduce mathematical complexities, we employ the structure of Fig. 1 as a model system, although experiment may be more easily performed in other structures, as mentioned later. Our structure is basically the one described in Ref. 2, except that the "electron mode converter" is employed and both of the quadrature components of the interference currents, J_+ and J_-, are measured. This improvement has been made to avoid reflections of electrons back to the source region, and also, as explained later, to improve the signal-to-noise ratio (SNR) of the device. Two quantum-well wires (QWW's) of *different well widths,* the narrow (N) and the wide (W) ones, constitute an Aharonov-Bohm (AB) geometry. We assume that the size of the ring is small enough (~ 1 μm typically) for the electrons at the Fermi surface to move *ballistically* from the source to the drain regions. This can be realized in, say, high-quality GaAs/AlAs QWW's at low temperatures. An optical field polarized in the z direction propagates in the $+y$ direction, and is confined in the center region (which we call the interaction region) by an optical waveguide structure, where the optical field interacts with electrons in the QWW's. If the photon energy $\hbar\omega$ is slightly detuned from the intersubband transition energies, or, more rigorously, if the right-hand side (rhs) of Eq. (3) is small enough, the interaction induces only "virtual transitions" so that no photons are absorbed. The virtual transitions

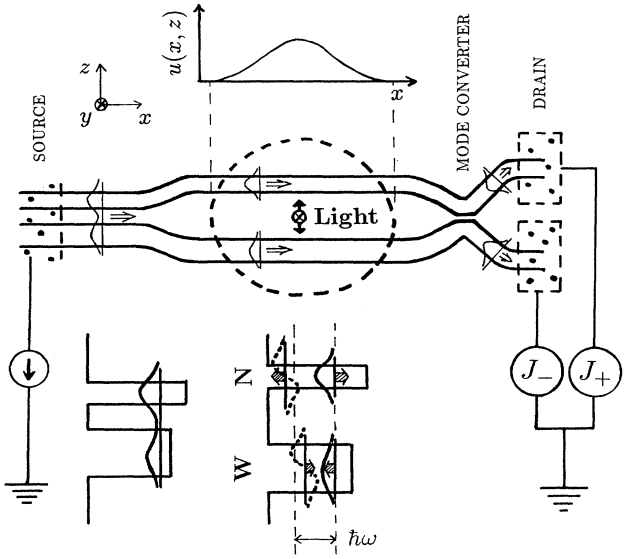

FIG. 1. Schematic structure of the quantum nondemolition photodetector used in the analysis. The optical field propagating in the $+y$ direction is confined in the center dotted region, with the transversal mode function $u(x,z)$, which is schematically shown in the upper side. The band diagrams and subband eigenfunctions in the source and center regions are shown in the lower side, where the arrows indicate the directions of the optical Stark shifts.

cause phase shifts in the electron wave functions, and thus give rise to changes in the electron-interference currents, from which we can deduce the photon number.

We will investigate the time evolution of the initial (i.e., prior to the interaction) photon state of the general form: $|\psi\rangle=\sum_n a_n|n\rangle$, where $|n\rangle$ is the number state, which is defined according to the following quantized optical electric field:[8]

$$\hat{\mathcal{E}}_z(\mathbf{r})=\left[\frac{2\pi\hbar\omega}{\epsilon L_y}\right]^{1/2}[\hat{a}u(x,z)e^{i\beta y}+\text{H.c.}]\ , \tag{1}$$

where ϵ is the dielectric constant, $\hat{a}$ is the photon annihilation operator, $u(x,z)$ is the normalized transversal mode function, β is the propagation constant, and L_y is a normalization length in the y direction. Electrons are supplied from the source region. We note that the wave function of each electron must be a wave packet of some finite length l. Previous experiments on a neutron source[9] and an electron source[10] suggest that the wave packet has a Gaussian-like envelope and that l is close to the length determined by the uncertainty principle. It is thus natural to consider that the electron wave function in our case also has the Gaussian envelope of the length $l\simeq v_F\tau_\phi'$, where v_F is the Fermi velocity and τ_ϕ' is the phase breaking time[1] *in the source region.* (τ_ϕ' would be shorter than that in the *AB* ring region). We can safely employ this assumption because our final results are independent of the length and detailed forms of the wave packet.[11] For the same reason, we will not write explicitly the Gaussian envelope function in the following equations. We will also drop in the equations the eigenenergies and eigenfunctions corresponding to the y-direction confinement, because they play no important roles.[11]

By appropriately designing the widths and alloy compositions of the wells, we can easily make the lowest subband energies of the two QWW's to satisfy $\epsilon_a^N=\epsilon_a^W\equiv\epsilon_a$. An electron wave packet emitted from the source region will be split into two, and its wave function with the Fermi energy of $\varepsilon_F=\varepsilon_a+\hbar^2k^2/2m^*$ becomes of the form[2]

$$|\Phi\rangle=(e^{ikx}|\varphi_a^N\rangle+e^{ikx}|\varphi_a^W\rangle)/\sqrt{2}\ , \tag{2}$$

where $\varphi_a^{N,W}(z)$ is the lowest-subband eigenfunctions of N or W. Hence, the initial state of the total system (electron plus optical field) is $|\Psi\rangle\equiv|\psi\rangle|\Phi\rangle$. We can analytically solve the time-dependent Schrödinger equation to obtain the time evolution of this initial state, using an adiabatic approximation (ADA) to separate the x and z coordinates of the electron, another ADA to treat pulsed optical fields, the rotating-wave approximation (RWA), and the Wentzel-Kramers-Brillouin (WKB) approximation.[11] The total resulting error is estimated to be $\sim1\%$, which is dominated by the RWA. However, the RWA as well as the WKB approximation are irrelevant to the QND property of the device because the neglected terms cause *no* real transitions.[11] It is found that the error resulting from the two ADA's, which is related with destruction of photons, is given by[11]

$$\frac{\|\Psi-\Psi_{\text{ADA}}\|^2}{\|\Psi\|^2}\simeq\left|\frac{\gamma\sqrt{n}}{\Delta}\right|^2\left[\frac{\hbar}{\tau_I\Delta}\right]^2, \tag{3}$$

where τ_I is the characteristic time that it takes for the photon-electron interaction to switch, γ is the maximum value of $\gamma_q(x)$ [see Eq. (7)], and $|\Delta|\equiv\min(|\Delta_N|,|\Delta_W|)$ [see Eq. (6)]. This equation is the standard expression for the validity of the ADA in time-dependent problems, and thus demonstrates that our QND device relies upon the adiabatic switching of the interaction. τ_I is of the order of the shorter one of τ_t, the transit time of the electron through the interaction region, and the optical-pulse duration τ_p. Owing to the smooth profiles of the optical fields as a function of x and y, as described by $u(x,z)$ (see Fig. 1) and the optical-pulse shape, the collision of the electron moving in the $+x$ direction with the optical field becomes adiabatic with the switching time of τ_I. This should be contrasted with the situation of the Jaynes-Comming model,[8] where the collision occurs abruptly, resulting in the quantum Rabi flopping.[8] In the numerical example discussed below, $\tau_t\sim1$ μm$/v_F\sim10$ ps for $v_F\sim10^7$ cm/s, so that, if we assume $\tau_p\geq10$ ps, the error due to the ADA's is estimated to be as small as 10^{-6}.

Because the full analysis is quite lengthy and complicated,[11] we will describe essential points only. As the electron proceeds to the interaction region, the electron and the photons will be coupled by the interaction, $H_I=-ez\hat{\mathcal{E}}_z(\mathbf{r})$. Since this interaction does not commute with $\hat{n}$, the photon number is *not* conserved *during the interaction.*[12] Each component $e^{ikx}|\varphi_a^q\rangle|n\rangle$ in $|\Psi\rangle$, where $q=N$ or W, evolves into the dressed state[8] of the form

$$\sqrt{k/K_k^q}\exp\left[i\int_0^x dx'K_k^q(x')\right]\times(\cos\theta_n^q|\varphi_a^q\rangle|n\rangle+\sin\theta_n^q|\varphi_b^q\rangle|n-1\rangle)\ , \tag{4}$$

where

$$\tan\theta_n^q(x)\approx\gamma_q(x)\sqrt{n}\,/\Delta_q\ , \tag{5}$$

$$\Delta_q\equiv(\varepsilon_b^q-\varepsilon_a^q)-\hbar\omega\ , \tag{6}$$

$$\gamma_q(x)\equiv\left[\frac{2\pi\hbar\omega}{\epsilon L_y}\right]^{1/2}u(x,z_q)\langle\varphi_b^q|-ez|\varphi_a^q\rangle\ . \tag{7}$$

Here, z_q denotes the z coordinate of the QWW q, and $K_k^q(x)$ is the local wave number, which is shifted from k due to the local (i.e., x-dependent) optical Stark shift.[8]

$$K_k^q(x)\approx k\left[1+\frac{\gamma_q(x)^2n/2\Delta_q}{\hbar^2k^2/2m^*}\right]. \tag{8}$$

When the electron further proceeds off the interaction region, the interaction is over and each component of $|\Psi\rangle$ *adiabatically* returns to its original form, but with a phase shift

$$|\Psi'\rangle=\sum_n a_ne^{-in\omega t}|n\rangle(e^{i\theta_n^N}|\varphi_a^N\rangle+e^{i\theta_n^W}|\varphi_a^W\rangle)/\sqrt{2}\ , \tag{9}$$

where we have dropped the common factor $\exp(ikx-i\epsilon_Ft/\hbar)$, and the phase shifts are given by

$\theta_n^q \approx \zeta_q n$ plus *terms independent of n,* where ζ_q is the *effective coupling constant* given by

$$\zeta_q = \int_{-\infty}^{\infty} dx \frac{\gamma_q(x)^2/\Delta_q}{\hbar^2 k^2/2m^*} \frac{k}{2} . \tag{10}$$

The electron then enters the mode converter, which consists of the crossed QWW's and the thin barrier layer of 50% transmittance (see Fig. 1). Just as the 50% beam splitter for optical fields, the mode converter transforms the electron wave function in Eq. (9) into

$$|\Psi''\rangle = \sum_n a_n e^{-in\omega t}|n\rangle(C_{n+}|\varphi_+\rangle + C_{n-}|\varphi_-\rangle) , \tag{11}$$

where φ_+ and φ_- are eigenfunctions of the final two output channels, and, for $\sigma = \pm 1$,

$$C_{n\sigma} \equiv (e^{i(\theta_n^N + \Delta\theta_0/2)} + \sigma e^{i(\theta_n^W - \Delta\theta_0/2)})/2 . \tag{12}$$

Here, $\Delta\theta_0$ is a phase that is determined by the structures of the mode converter. We finally measure the currents of the two output channels: $\hat{J}_\sigma = j_0|\varphi_\sigma\rangle\langle\varphi_\sigma|$, where j_0 is a constant.

Based upon the above results for the system of optical field plus single electron, we next consider the actual case when many electrons at the Fermi surface travel from the source to the drain regions. To do this, we note that (i) there is no coherence between different electron wave packets emitted from the source region, and (ii) they do not overlap each other for a small source-drain current, such as the one we are treating. In this case, the calculations on the system of optical field plus N traveling electrons become straightforward.[11] Here, N is *not* the total number of electrons in the QWW's, but the number of electrons detected as interference currents during the measurement [see Eq. (17)]. Let us first consider how we can deduce the photon number from the measurements of $J_\pm$. In order to get the maximum sensitivity at $n=0$, let us design the device such that $\Delta\theta_0 = -\pi/2$. In this case, we can define the *readout variable* $\hat{n}_r$ by

$$\sin(g\hat{n}_r) \equiv \frac{\hat{J}_+ - \hat{J}_-}{\hat{J}_+ + \hat{J}_-} , \tag{13}$$

where $g \equiv \zeta_N - \zeta_W$ is the overall effective coupling constant of the device, and the rhs is well defined because $[\hat{J}_+, \hat{J}_-] = 0$. It can be shown that

$$\langle \sin(g\hat{n}_r)\rangle = \langle \sin(g\hat{n})\rangle_0 , \tag{14}$$

$$\langle [\delta \sin(g\hat{n}_r)]^2\rangle = \langle [\delta \sin(g\hat{n})]^2\rangle_0 + \langle \cos^2(g\hat{n})\rangle_0/N , \tag{15}$$

where $\langle\ \rangle_0 \equiv \langle\psi|\ |\psi\rangle$ denotes the average over the initial photon-number distribution. Equation (14) ensures that the device works as a photon-number counter. In particular, if $g\langle\hat{n}\rangle \ll 1$, it is reduced to $\langle\hat{n}_r\rangle = \langle\hat{n}\rangle_0$. On the other hand, Eq. (15) is reduced to $\langle\delta\hat{n}_r^2\rangle = \langle\delta\hat{n}^2\rangle_0 + \delta n_{\rm err}^2$, where $\delta n_{\rm err}^2$ is the *measurement error*[4-6] (or *quantum noise* in the measurement), which is given by

$$\delta n_{\rm err}^2 = 1/g^2 N . \tag{16}$$

Note that a similar expression for $\delta n_{\rm err}^2$ was obtained for the QND measurement based upon the optical Kerr effect,[5] although the operation principle of such a QND photodetector and that of the present one are very different (for example, in the former one n is conserved *throughout* the measurement[5]). This is because Eq. (16) reflects the general principle of quantum mechanics that many particles in the same state are needed to measure the phase of the particle's wave function. In our case, N is determined by the quantized conductance[1] $e^2/\pi\hbar$ and thus is given by

$$N \simeq e\tau_p N_{\rm dev} V_{\rm SD}/\pi\hbar , \tag{17}$$

where τ_p is the optical-pulse duration or the detection period (in the case of cw optical fields), $V_{\rm SD}$ is the source-drain voltage, and we have assumed that an array of $N_{\rm dev}$ devices are used as a single photodetector. We can make N arbitrarily large by increasing $N_{\rm dev}$ or by confining the optical field in a cavity and thereby making τ_p long, as discussed later. Hence, $\delta n_{\rm err}^2$ can be made arbitrarily small: for example, we can realize $\delta n_{\rm err}^2 < \langle\delta\hat{n}^2\rangle_0$ when $N > 1/g^2\langle\delta\hat{n}^2\rangle_0$. Note also that N itself fluctuates from measurement to measurement at a microscopic level, because the electron emissions from the source region are random processes. This fluctuation, however, is irrelevant to the above results because N is canceled between the numerator and the denominator on the rhs of Eq. (13). Hence, the SNR is better for the present structure than that of Ref. 2.

Let us next consider the final photon states. The final density operator traced over the electron coordinates is evaluated to be

$$\hat{\rho}_{\rm ph}'' = \sum_{n,m} a_m a_n^* e^{i(n-m)\omega t}|m\rangle\langle n| \times (\tfrac{1}{2}e^{i\zeta_N(m-n)} + \tfrac{1}{2}e^{i\zeta_W(m-n)})^N . \tag{18}$$

If the initial photon state is a number state ($a_n = \delta_{n,n_0}$), we can see that the photon wave function is unchanged by the measurement, which means an absence of backactions. For other initial states, the photon wave function must be reduced by measurement (even when it is a QND measurement).[5] However, Eq. (18) tells us that the final photon-number distribution *for the statistical ensemble* is $|a_n|^2$, which is just that of the initial photon state. In particular, $\langle\hat{n}\rangle = \langle\hat{n}\rangle_0$, $\langle\delta\hat{n}^2\rangle = \langle\delta\hat{n}^2\rangle_0$. This invariance of the distribution, together with the fact that $\delta n_{\rm err}^2$ can be made sufficiently small, are just those required for general QND measurements.[4-6] On the other hand, the phase of the photon state is randomized through the measurement.[4-7] To see this, we assume a coherent state for the initial state, and evaluate the fluctuation of the cosine operator.[5,11] Then, for $\langle n\rangle \gg 1$ and for $g^2 N \ll 1$, the phase fluctuation of the final photon state is evaluated to be $\langle\delta\phi^2\rangle = \langle\delta\phi^2\rangle_0 + \delta\phi_{\rm BA}^2$, where $\delta\phi_{\rm BA}$ is the *backaction phase noise*[1-4] given by

$$\delta\phi_{\rm BA}^2 \approx Ng^2/4 . \tag{19}$$

Note that the minimum uncertainty relationship[4–6] holds: $\delta n_{\rm err}^2 \delta\phi_{\rm BA}^2 \approx \frac{1}{4}$.

We finally consider a numerical example. For the same structural parameters as in Ref. 2, the readout [Eq. (14)], which increases with the light intensity I, reaches its maximum value 1 when $I = I_{\rm peak} \simeq 2$ MW/cm^2. If $\tau_p = 10$ ps, this corresponds to 10^6 photons. The resolution is determined by the measurement error, and is given by $I_{\rm peak}/\sqrt{N} \simeq 0.7$ kW/cm^2 for, say, $N = 10^7$. This corresponds to 300 photons, which is smaller than the quantum noise $\sqrt{n}$ of $n = 10^5$ coherent state. To realize $N = 10^7$, we may confine the optical field and thereby make τ_p long, or we may increase $N_{\rm dev}$. For the latter case, for example, we can estimate $N_{\rm dev}$ by noting that $V_{\rm SD}$ should be kept small.[13] When $V_{\rm SD} = 0.1$ mV, we obtain, from Eq. (17), $N_{\rm dev} = 2 \times 10^7$. If we assume a 20×10^6 array of the devices with a 200-Å period, the length of the array in the direction of light propagation is 2 cm, which is much shorter than the interaction length of QND photodetectors based on optical nonlinearities due to virtual transitions.[5,6] In actual experiments, one may measure a highly stabilized optical field in an open cavity,[14] where τ_p can be made large and thus the above large scale integration is unnecessary. Also, the electron interferometer of Fig. 1 can be replaced with other ones such as those proposed in Ref. 15 and Fig. 3 of Ref. 3, which are much easier to fabricate.

In summary, we have analyzed the collision between photons in an arbitrary quantum state, and electrons in a semiconductor microstructure, which constitutes an electron interferometer. By evaluating the time evolution of the coupled photon-electron system, we have shown that the interferometer works as a quantum nondemolition photodetector if it is designed such that the interaction is switched adiabatically. The measurement error $\delta n_{\rm err}$ decreases with N, the number of detected electrons as interference currents, in proportion to $1/\sqrt{N}$. A backaction of the measurement occurs as the increased phase noise $\delta\phi_{\rm BA} (\propto \sqrt{N})$ in the photon state. Here, $\delta n_{\rm err}$ and $\delta\phi_{\rm BA}$ satisfy the *minimum* uncertainty relationship; $\delta n_{\rm err}^2 \delta\phi_{\rm BA}^2 \approx \frac{1}{4}$. Owing to the high efficiency of the photon-electron interaction, the required length is typically as small as 2 cm to achieve $\delta n_{\rm err} < \sqrt{n}$ for $n = 10^5$.

Note added. After the submission of the present paper, there appeared two papers on QND photodetectors by M. Brune *et al.* [Phys. Rev. Lett. **65**, 976 (1990)] and by M. D. Levenson [Phys. Rev. A **42**, 2935 (1990)]. Their operation principles are, in a way, analogous to the present one.

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*Present address: Quantum Wave Project, Research Development Corporation of Japan (JRDC), Keyaki House 302, 4-3-24 Komaba, Meguro-ku, Tokyo 153, Japan.

[1]For a review, see R. Webb and S. Washburn, Phys. Today **41** (12), 46 (1988).

[2]A. Shimizu, K. Fujii, M. Okuda, and M. Yamanishi, Phys. Rev. B **42**, 9248 (1990), and references therein.

[3]The basic idea of this work was previously discussed in A. Shimizu, *International Conference on Quantum Electronics: Technical Digest Series 1990* (Optical Society of America, Washington, D.C., 1990), Vol. 8, p. 96; A. Shimizu, *Abstracts of the Spring Meeting of the Physical Society of Japan* (Physical Society of Japan, Tokyo, 1990), paper no. 2aR7.

[4]V. B. Braginsky, Y. I. Vorontsov, and K. S. Thorne, Science **209**, 547 (1980); C. M. Caves, K. S. Thorne, R. W. P. Drever, V. D. Sandberg, and M. Zimmermann, Rev. Mod. Phys. **52**, 341 (1980).

[5]W. G. Unruh, Phys. Rev. D **18**, 1764 (1978); G. J. Milburn and D. F. Walls, Phys. Rev. A **28**, 2065 (1983); N. Imoto, H. A. Haus, Y. Yamamoto, *ibid.* **32**, 2287 (1985); M. D. Levenson, R. M. Shelby, M. Reid, and D. F. Walls, Phys. Rev. Lett. **57**, 2473 (1986).

[6]B. Yurke, J. Opt. Soc. Am. B **2**, 732 (1985); A. La Porta, R. E. Slusher, and B. Yurke, Phys. Rev. Lett. **62**, 28 (1989).

[7]K. Watanabe, H. Nakano, A. Honold, and Y. Yamamoto, Phys. Rev. Lett. **62**, 2257 (1989).

[8]P. Meystre and M. Sargent III, *Elements of Quantum Optics* (Springer-Verlag, Berlin, 1990).

[9]C. Dewdney *et al.*, Phys. Lett. **102A**, 291 (1984).

[10]H. Schmid (unpublished).

[11]A. Shimizu (unpublished).

[12]Although $[H_I, \hat{n}] = 0$ was claimed as a condition for QND measurements in Ref. 4, it is not a necessary condition as demonstrated by Refs. 6 and 7 and by the present work.

[13]The *central* energies of electron wave packets have a finite distribution with the width of $\sim eV_{\rm SD}$, or, at finite temperature, $\sim k_B T$. This distribution smears the interference currents and also causes additional noises in the currents. For this to be irrelevant, the width should be smaller than the difference of the optical Stark shifts in the two QWW's at $I = I_{\rm peak}$. In our example, this condition is well satisfied if $T < 1$ K and if $V_{\rm SD} < 0.1$ mV. Details will be described elsewhere.

[14]Y. Yamamoto *et al.* (unpublished).

[15]M. Okuda, K. Fujii, and A. Shimizu, Appl. Phys. Lett. **57**, 2231 (1990).