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Quantum-noise properties of a constant-voltage-operated semiconductor laser

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Quantum-noise properties of a semiconductor laser under constant-voltage operation, where the laser-field amplitude is regulated through nonlinear gain, have been studied using quantum-mechanical Langevin equations. In the ideal constant-voltage-drift case, the linewidth enhancement from the gain–refractive-index coupling, as well as the relaxation oscillations, disappear, but for realistic nonlinear gains the amplitude noise will increase. The requirement on the laser's series resistance for constant-voltage operation is, however, very strict, and seems difficult to meet in practice.

I. INTRODUCTION

The suppression of quantum noise in lasers has been the subject of intense research in recent years. Yamamoto *et al.* showed theoretically[1,2] that the external amplitude noise of a laser could be squeezed below the standard quantum limit provided that the noise of the laser's pumping mechanism is suppressed. As was shown in Ref. 2, and later experimentally,[3] this can be achieved by simply feeding a semiconductor laser from a high-impedance current source.

The phase noise, and specifically the linewidth, of a laser oscillator, on the other hand, is dominated by phase-diffusion noise. For semiconductor lasers, the linewidths were found to be further enhanced,[4] which was explained by Henry[5] as being due to changes in laser frequency with gain fluctuations. The gain–refractive-index coupling causing this change in frequency is usually described by the α parameter, being the ratio of changes of the real and imaginary parts of the susceptibility. Ever since, substantial efforts have been made to reduce the laser linewidth, either by improving the Q value using long lasers, or lasers with external resonators, or by reducing α using gain detuning, or by using quantum-well lasers with an intrinsically lower α, see for example, Refs. 5 and 6.

The purpose of this paper is to investigate the consequences of using a low source impedance instead of a high impedance when feeding a laser. Whereas a high source impedance regulates the injected current into the laser, a low source impedance will regulate the junction voltage of the laser. This, in turn, is related to the carrier density, so suppressing junction voltage fluctuations equals suppressing the carrier-density fluctuations responsible for the linewidth enhancement from gain–refractive-index coupling. However, "pinning" the carrier density decouples amplitude fluctuations from being regulated by the gain clamping. To circumvent this, a laser with high nonlinear gain, that is the saturation of laser gain with photon number, can be used to limit amplitude fluctuations. This mode of operation has, to the author's best knowledge, not been studied in the literature so far, and although it may be difficult to realize in practice, it is still of interest since it represents one fundamental limit of noise performance.

The paper is organized as follows. In Sec. II we present the formalism based on quantum-mechanical Langevin equations. In Sec. III we discuss some examples with a complete "pinning" of carrier density, and show that the gain–refractive-index enhancement of the laser linewidth disappears, but that the amplitude noise in general will be above the standard quantum limit, how much is depending on the strength of the nonlinear gain. We also investigate the elimination of the series resistance by the use of a negative resistance coupling, but as will be shown, this will not lead to any improvement with respect to noise performance. In Sec. IV we present some numerical examples, and also state the requirement on the source resistance for constant-voltage operation. Finally, in Sec. V, the results are discussed and summarized.

II. THEORY

The laser can be described by the quantum-mechanical Langevin equations for the internal field and total excited carrier number, which has been derived in Refs. 1, 2, and 7 and can be written as

$$\frac{d\hat{A}(t)}{dt} = -\frac{1}{2}\left[\frac{\omega}{Q} + i2(\omega_r - \omega) - \frac{\omega}{\mu^2}(\tilde{\chi}_i - i\tilde{\chi}_r)\right]\hat{A}(t) + \hat{H}(t) , \tag{1}$$

$$\frac{d\tilde{N}_c(t)}{dt} = p - \frac{\tilde{N}_c(t)}{\tau_{\rm sp}} - (\tilde{E}_{cv} - \tilde{E}_{vc})\hat{n}(t) + \tilde{\Gamma}_{\rm sp}(t) + \tilde{\Gamma}(t) . \tag{2}$$

Here we denote operators for the optical field by a circumflex, and operators for the electron system by a tilde. The total Q value Q of the assumed one-sided laser cavity depends on the output coupling losses Q_e and the internal losses Q_0 according to

$$\frac{1}{Q}=\frac{1}{Q_e}+\frac{1}{Q_0} . \tag{3}$$

The angular resonance frequency of the unpumped cavity is denoted ω_r, μ is the refractive index, and the $\tilde{\chi}$ is the electronic susceptibility operator, whose imaginary part equals the gain:

$$\frac{\omega}{\mu^2}\tilde{\chi}_i=\tilde{E}_{cv}-\tilde{E}_{vc} , \tag{4}$$

where $\tilde{E}_{cv}$ and $\tilde{E}_{vc}$ are the operators of the stimulated emission and absorption rates, respectively. The noise operator $\hat{H}(t)$ stems from the dipole fluctuation mechanism, the internal losses, and the incoming vacuum fluctuation. In the equation describing the electronic system, p is the pumping rate, also including the pump noise, τ_{sp} is the spontaneous lifetime of the electrons, and $\hat{n}(t)\equiv\hat{A}^\dagger(t)\hat{A}(t)$ is the photon number operator. The two last terms in Eq. (2) are fluctuating terms: $\tilde{\Gamma}_{sp}(t)$ is the spontaneous emission noise, and $\tilde{\Gamma}(t)$ is from the dipole moment fluctuation noise.

In order to analyze the noise behavior of Eqs. (1) and (2), we expand them in mean and small fluctuating parts according to

$$\tilde{N}_c(t)=N_{c0}+\Delta\tilde{N}_c(t) , \tag{5}$$

$$\hat{A}(t)=[A_0+\Delta\hat{A}(t)]e^{-i[\omega t+\Delta\hat{\phi}(t)]} , \tag{6}$$

$$\hat{n}(t)=\hat{A}^\dagger(t)\hat{A}(t)\approx[A_0+\Delta\hat{A}(t)]^2$$
$$\approx A_0^2+2A_0\Delta\hat{A}(t)=n_0+2A_0\Delta\hat{A}(t) , \tag{7}$$

$$\tilde{\chi}_i=\langle\tilde{\chi}_i\rangle+\frac{d\langle\tilde{\chi}_i\rangle}{dN_{c0}}\Delta\tilde{N}_c+\frac{d\langle\tilde{\chi}_i\rangle}{dn_0}2A_0\Delta\hat{A}(t) , \tag{8}$$

$$\tilde{\chi}_r=\langle\tilde{\chi}_r\rangle+\frac{d\langle\tilde{\chi}_r\rangle}{dN_{c0}}\Delta\tilde{N}_c+\frac{d\langle\tilde{\chi}_r\rangle}{dn_0}2A_0\Delta\hat{A}(t) . \tag{9}$$

Here N_{c0}, A_0, and n_0 are the average excited electron number, field amplitude, and photon number (c numbers). $\Delta\tilde{N}_c$, $\Delta\hat{A}$, and $\Delta\hat{\phi}$ are the Hermitian excited electron number, field amplitude, and phase operators, the phase operator being a valid approximation for large photon numbers. In (8) and (9), we also have included nonlinear gain. The dispersive part of the nonlinearity [the third term in Eq. (9)] can in many cases be neglected,[8] and has therefore not been used in the following.

We also introduce the notations

$$\frac{1}{\tau_{st}}=\frac{\omega}{\mu^2}A_0^2\frac{d\langle\tilde{\chi}_i\rangle}{dN_{c0}} , \tag{10}$$

$$\alpha=\frac{d\langle\tilde{\chi}_r\rangle}{dN_{c0}}\bigg/\frac{d\langle\tilde{\chi}_i\rangle}{dN_{c0}} , \tag{11}$$

$$n_{sp}=\frac{\langle\tilde{E}_{cv}\rangle}{\langle\tilde{E}_{cv}\rangle-\langle\tilde{E}_{vc}\rangle} , \tag{12}$$

$$g=\frac{\omega}{\mu^2}\langle\tilde{\chi}_i\rangle , \tag{13}$$

and

$$s_P=\frac{n_0}{g}\frac{\partial g}{\partial n_0} , \tag{14}$$

where τ_{st} is the electron lifetime due to stimulated emission, α is the linewidth enhancement factor,[5] n_{sp} is the population inversion factor, g is the average of the stimulated emission gain, and s_P the "logarithmic" nonlinear gain. We have assumed that the nonlinear gain mechanism does not introduce any additional primary noise. We also need an equation describing the laser coupled to the electrical bias circuitry, which we model as an ideal voltage source, E, in series with the laser's series resistance R_s:

$$E-R_sI+v_s=U_f(N) . \tag{15}$$

Here v_s is the thermal noise voltage of the laser's series resistance R_s, I the laser current, and $U_f(N)$ is the junction voltage, which can be uniquely related to the injected carrier number.[2] A small signal expansion of (1), (2), and (15), using (4)–(14) gives

$$\frac{d}{dt}\Delta\tilde{N}_c(t)=\Delta p(t)-\left[\frac{1}{\tau_{sp}}+\frac{1}{\tau_{st}}\right]\Delta\tilde{N}_c(t)$$
$$-2g(1+s_P)A_0\Delta\hat{A}(t)+\tilde{\Gamma}_{sp}(t)+\tilde{\Gamma}(t) , \tag{16}$$

$$\frac{d}{dt}\Delta\hat{A}(t)=\frac{1}{2A_0\tau_{st}}\Delta\tilde{N}_c(t)+gs_P\Delta\hat{A}(t)+\hat{H}_r(t) , \tag{17}$$

$$\frac{d}{dt}\Delta\hat{\phi}(t)=\frac{\alpha}{2A_0^2\tau_{st}}\Delta\tilde{N}_c(t)-\frac{1}{A_0}\hat{H}_i(t) , \tag{18}$$

$$\Delta p(t)=\frac{v_s(t)}{qR_s}-\frac{dU_f}{dN}\frac{1}{qR_s}\Delta\tilde{N}_c(t) . \tag{19}$$

The operators $\hat{H}_r$ and $\hat{H}_i$ are the Hermitian quadrature noise operators:

$$\hat{H}_r=\tfrac{1}{2}(\hat{H}e^{i+\Delta\hat{\phi}(t)}+\hat{H}^\dagger e^{-i+\Delta\hat{\phi}(t)}) , \tag{20}$$

$$\hat{H}_i=\frac{1}{2i}(\hat{H}e^{i+\Delta\hat{\phi}(t)}-\hat{H}^\dagger e^{-i+\Delta\hat{\phi}(t)}) . \tag{21}$$

These equations are identical to those of Ref. 2, apart from the inclusion of nonlinear gain, which was made in Ref. 7. dU_f/dN is calculated using the approximation of Joyce and Dixon;[9] an often used form is $dU_f/dN=mV_T/N_c$, where V_T is the thermal voltage ≈ 0.026 V, N_c the carrier number, and m a dimensionless number. Equations (16)–(19) can be Fourier transformed, and with the help of the Wiener-Khinchine theorem the resulting noise spectra can be calculated.

Introducing

$$\epsilon_c=\left[\frac{1}{\tau_{sp}}+\frac{dU_f}{dN}\frac{1}{qR_s}\right]\bigg/\frac{1}{\tau_{st}} , \tag{22}$$

we can Fourier transform (16) and (17), finding for the amplitude fluctuation

$$\left[j\Omega+g\frac{1-(\epsilon_c+j\Omega\tau_{st})s_P}{1+\epsilon_c+j\Omega\tau_{st}}\right]\Delta\hat{A}(\Omega)=\frac{1}{2A_0(1+\epsilon_c+j\Omega\tau_{st})}\left[\frac{v_s(\Omega)}{qR_s}+\tilde{\Gamma}_{sp}(\Omega)+\tilde{\Gamma}(\Omega)\right]+\hat{H}_r(\Omega)\ . \tag{23}$$

Calculating the junction voltage fluctuation $\Delta v_N(\Omega)$, we can use (23) and (17) finding

$$\Delta v_N(\Omega)=\frac{dU_f}{dN}\Delta\tilde{N}_c(\Omega)=\left[\frac{dU_f}{dN}\right]\left[\frac{(j\Omega-gs_p)\tau_{st}}{(1+\epsilon_c+j\Omega\tau_{st})j\Omega+g[1-(\epsilon_c+j\Omega\tau_{st})s_P]}\left[\frac{v_s(\Omega)}{qR_s}+\tilde{\Gamma}_{sp}(\Omega)+\tilde{\Gamma}(\Omega)\right]-\frac{2A_0g(1+s_P)\tau_{st}}{(1+\epsilon_c+j\Omega\tau_{st})j\Omega+g[1-(\epsilon_c+j\Omega\tau_{st})s_P]}\hat{H}_r(\Omega)\right]\ . \tag{24}$$

With

$$\alpha_{eff}=\alpha\frac{g(1+s_P)}{(1+\epsilon_c+j\Omega\tau_{st})j\Omega+g[1-(\epsilon_c+j\Omega\tau_{st})s_P]}\ , \tag{25}$$

one obtains for the phase fluctuation

$$j\Omega\Delta\hat{\phi}(\Omega)=\frac{\alpha-\alpha_{eff}}{2A_0^2(1+\epsilon_c+j\Omega\tau_{st})}\times\left[\frac{v_s(\Omega)}{qR_s}+\tilde{\Gamma}_{sp}(\Omega)+\tilde{\Gamma}(\Omega)\right]-\frac{1}{A_0}[\hat{H}_r(\Omega)\alpha_{eff}+\hat{H}_i(\Omega)]\ . \tag{26}$$

As we see in Eq. (26), α_{eff} can for low frequencies be interpreted as an effective α parameter, also the difference $\alpha-\alpha_{eff}$ will for low frequencies be zero if there is no nonlinear gain. Equation (23) gives $\Delta\hat{A}(\Omega)$, the Fourier component for the internal field fluctuation. The external field $\hat{r}(t)$, normalized so that $\hat{r}^\dagger\hat{r}$ is the photon flux in photons/s, can be found by using[1]

$$\hat{r}(t)=-\hat{f}(t)+\sqrt{\omega/Q_e}\,\hat{A}(t)\ . \tag{27}$$

Here $\hat{f}(t)$ represents a reflected vacuum fluctuation. The correlation functions of the noise sources are[1,2]

$$\langle\hat{H}_r(t)\hat{H}_r(u)\rangle=\langle\hat{H}_i(t)\hat{H}_i(u)\rangle=\frac{1}{4}\left[\frac{\omega}{Q_e}+\frac{\omega}{Q_0}+\langle\tilde{E}_{cv}\rangle+\langle\tilde{E}_{vc}\rangle\right]\delta(t-u)\ , \tag{28}$$

$$\langle\tilde{\Gamma}_{sp}(t)\tilde{\Gamma}_{sp}(u)\rangle=\frac{N_{c0}}{\tau_{sp}}\delta(t-u)\ , \tag{29}$$

$$\langle\tilde{\Gamma}(t)\tilde{\Gamma}(u)\rangle=A_0^2(\langle\tilde{E}_{cv}\rangle+\langle\tilde{E}_{vc}\rangle)\delta(t-u)\ , \tag{30}$$

$$\langle\tilde{\Gamma}(t)\hat{H}_r(u)\rangle=\langle\hat{H}_r(t)\tilde{\Gamma}(u)\rangle=-\frac{A_0}{2}(\langle\tilde{E}_{cv}\rangle+\langle\tilde{E}_{vc}\rangle)\delta(t-u)\ , \tag{31}$$

$$\langle\hat{f}_r(t)\hat{f}_r(u)\rangle=\langle\hat{f}_i(t)\hat{f}_i(u)\rangle=\tfrac{1}{4}\delta(t-u)\ , \tag{32}$$

$$\langle\hat{H}_r(t)\hat{f}_r(u)\rangle=\langle\hat{f}_r(t)\hat{H}_r(u)\rangle=\langle\hat{H}_i(t)\hat{f}_i(u)\rangle=\langle\hat{f}_i(t)\hat{H}_i(u)\rangle=\tfrac{1}{4}\sqrt{\omega/Q_e}\,\delta(t-u)\ , \tag{33}$$

$$\langle v_s(t)v_s(u)\rangle=2k_BTR_s\delta(t-u)\ . \tag{34}$$

All other correlations are zero. The spectral density can then be found by using Wiener-Khinchine's theorem as

$$S_f(\Omega)\equiv\langle|f(\Omega)|^2\rangle=2\int_{-\infty}^{\infty}\langle f(\tau)f(0)\rangle e^{-j\Omega\tau}d\tau\ . \tag{35}$$

Equations (23)–(35) form the basis for the calculation of amplitude and frequency noise, but before we show some numerical examples, we look at some ideal cases.

III. SOME IDEAL CONSTANT-VOLTAGE CASES

We first treat the ideal case $R_s\to0$, then $\epsilon_c\to\infty$, $\alpha_{eff}\to0$. Using Eqs. (23), (26), and (27), and that gain equals losses, we find for low frequencies that

$$S_{\Delta\hat{r}}=\frac{1}{2}+\left[\frac{\omega}{Q_e}\bigg/\frac{\omega}{Q}\right]\left[\frac{n_{sp}}{s_P^2}+\frac{1}{s_P}\right]\ , \tag{36}$$

$$S_{\Delta\hat{\Phi}}=\frac{1}{\Omega^2A_0^2}n_{sp}\frac{\omega}{Q}\ , \tag{37}$$

implying that the external amplitude noise will be determined by the strength of the nonlinear gain together with the interference from the reflected vacuum fluctuation. The laser linewidth[5] $\Delta\nu$ is found from Eq. (37) as $\Delta\nu=(1/4\pi)S_{\Delta\hat{\Phi}}$; apparently the ordinary linewidth enhancement factor $1+\alpha^2$ is absent for a constant-voltage-driven semiconductor laser. Also the relaxation resonance will be absent, since it is damped out by the fast decay time due to R_s.

This ideal case assumes that both the laser internal resistance and the voltage fluctuation approach zero. In a more realistic case, one could speculate over what the effect would be to compensate for a finite laser series resistance through the use of a negative resistance coupling. That case is also easily investigated by letting the voltage fluctuation $v_s(t)$ be given by the sum of the fluctuation from the laser's series resistance and the noise from the negative resistance, which we for simplicity can assume to follow Nyquist's formula with the same noise temperature as the laser's series resistance. Letting the total series resistance approach zero, but retaining the voltage fluctuation we find from Eq. (19) that the fluctuation in carrier density is directly given from the fluctuations in voltage of the negative resistance and the laser's series resistance. Using this in the Fourier-transformed Eqs. (17) and (18), we find the low-frequency amplitude and phase noise as

$$S_{\Delta\hat{r}}=\frac{1}{2}+\frac{n_{\rm sp}}{s_P^2}\left[\frac{\omega}{Q_e}\Big/\frac{\omega}{Q}\right]\times\left[\left[\frac{R_{si}+|R_{\rm neg}|}{R_{\rm st}}\right]\left[\frac{V_T}{N_c}\Big/\frac{dU_f}{dN}\right]+1\right]+\frac{1}{s_P}\left[\frac{\omega}{Q_e}\Big/\frac{\omega}{Q}\right], \tag{38}$$

$$S_{\Delta\hat{\Phi}}=\frac{1}{\Omega^2A_0^2}n_{\rm sp}\frac{\omega}{Q}\left[\left[\frac{R_{si}+|R_{\rm neg}|}{R_{\rm st}}\right]\times\left[\frac{V_T}{N_c}\Big/\frac{dU_f}{dN}\right]\alpha^2+1\right], \tag{39}$$

where $R_{\rm st}$ is the differential resistance from stimulated emission [defined by $R_{\rm st}=(\tau_{\rm st}/q)(dU_f/dN)$, compare also with Eq. (47) later], $R_{\rm si}$ is the series resistance of the laser, and $R_{\rm neg}$ is the negative resistance. Equations (38) and (39) show that it is the laser's intrinsic series resistance that has to be decreased. From (39) we also see that requirements on the series resistance are stricter for the phase noise due to the appearance of the α parameter.

IV. NUMERICAL EXAMPLES

For numerical results, we must first consider the steady-state solution. It can be obtained after some preliminary assumptions of the nature of the nonlinear gain. Here we assume that the carrier number required for transparency remains constant when the photon number increases. A prime question then is the functional form of the gain nonlinearity. For this application where we must use hard pumping in order to have a large value of the gain nonlinearity s_P, the functional form is crucial. Using a form derived by Agrawal[8] we have

$$g(N_{c0},n_0)=A(N_{c0}-N_0)/(1+\epsilon_{\rm NL}n_0)^{1/2}\ . \tag{40}$$

Here A is the differential gain parameter, and N_0 is the carrier number required for transparency. This model yields

$$s_P=-\frac{1}{2}\frac{\epsilon_{\rm NL}n_0}{1+\epsilon_{\rm NL}n_0}\ . \tag{41}$$

The amount of noise increase, or decrease, depends, as seen in (36), on the strength of the nonlinear gain. With the chosen form of nonlinear gain we see from Eq. (41) that the minimum value of $s_P=-\frac{1}{2}$. This will not give any squeezing of the outgoing amplitude. One could perhaps also argue that in this case, where the nonlinearity is part of the gain itself, $s_P<-1$ seems unlikely since it corresponds to a situation where the dissipated power through stimulated emission decreases when the photon number increases.

As independent parameters for the calculations we have chosen relative pump level R_p, defined as $R_p\equiv I/I_{\rm th}-1$, the spontaneous lifetime $\tau_{\rm sp}$, the carrier number required for transparency N_0, the carrier number required to reach threshold $N_{\rm th}$, external Q value Q_e, internal Q value Q_0, and finally the nonlinear gain parameter $\epsilon_{\rm NL}$. With nonlinear gain included we can keep the threshold current value, $I_{\rm th}=qN_{\rm th}/\tau_{\rm sp}$, and then solve the steady-state quantities from

$$R_p=\frac{I-I_{\rm th}}{I_{\rm th}}=\frac{N_{\rm th}-N_0}{N_{\rm th}}[(1+\epsilon_{\rm NL}n_0)^{1/2}-1]+\frac{\omega}{Q}\frac{\tau_{\rm sp}}{N_{\rm th}}n_0\ , \tag{42}$$

$$N_{c0}=N_0+(N_{\rm th}-N_0)(1+\epsilon_{\rm NL}n_0)^{1/2}\ , \tag{43}$$

$$n_{\rm sp}=\frac{N_{c0}}{N_{c0}-N_0}\ , \tag{44}$$

$$\frac{1}{\tau_{\rm st}}=\frac{\omega}{Q}\frac{1}{N_{\rm th}-N_0}\frac{n_0}{(1+\epsilon_{\rm NL}n_0)^{1/2}}\ , \tag{45}$$

$$\alpha=\alpha_0(1+\epsilon_{\rm NL}n_0)^{1/2}\ . \tag{46}$$

The gain, i.e., the difference between stimulated emission and stimulated absorption, is constant, fixed by cavity losses. If nonlinear gain is present, the carrier number is required to increase with increasing photon number to maintain the gain. We have assumed that the relative amount of stimulated emission to stimulated absorption increases, or equivalently that the population inversion factor $n_{\rm sp}$ decreases, as described by Eq. (44). Strictly this should be justified by a rigorous calculation of the gain coefficient in the presence of nonlinear gain. This is, however, beyond the scope of the present paper, also, adapting another model, for instance with a constant $n_{\rm sp}$, will not modify the results of the paper considerably.

The numerical values are for the Q values $\omega/Q_e=5\times10^{11}$ s^{-1}, $\omega/Q_0=0$, so that we study a case with no internal losses, further, we take $\tau_{\rm sp}=3$ ns, $\alpha_0=5$, $N_{\rm th}=2\times10^{10}$, and $N_0=1\times10^{10}$ which is calculated assuming an active volume, $V=10^{-10}$ cm^3. These parameter values give that a fraction 1.5×10^{-5} of the spontaneous emission is coupled into the lasing mode. The numerical value chosen for the nonlinear gain is $\epsilon_{\rm NL}=5\times10^{-7}$, which is in reasonable agreement with experimental results. For the effective density of states to be used in the calculations of the junction voltage derivate,[9] we have chosen $N_v=16.7\times10^{18}$ cm^{-3} for the valence band, and $N_c=0.53\times10^{18}$ cm^{-3} for the conduction band.

In Figs. 1(a) and 1(b), the external amplitude and frequency noise spectra are shown. The noise levels for the lowest R_s values are nearly the ideal constant-voltage cases. The level of the amplitude noise spectra for the constant-voltage case is given by Eq. (36). The decrease of series resistance will "pin" the carrier-density fluctuations, and reduce the low-frequency part of the frequency noise spectrum, in accordance with Eq. (37).

In Figs. 2(a) and 2(b), the low-frequency amplitude and frequency noise spectral density are shown as functions of the series resistance for various pump levels. The frequency noise is normalized so that ideally $n_{\rm sp}=1$, $\alpha=0$ gives a unity noise level. For the amplitude noise we see that, for vanishing R_s, increasing pumping makes the nonlinearity s_P stronger and the amplitude noise de-

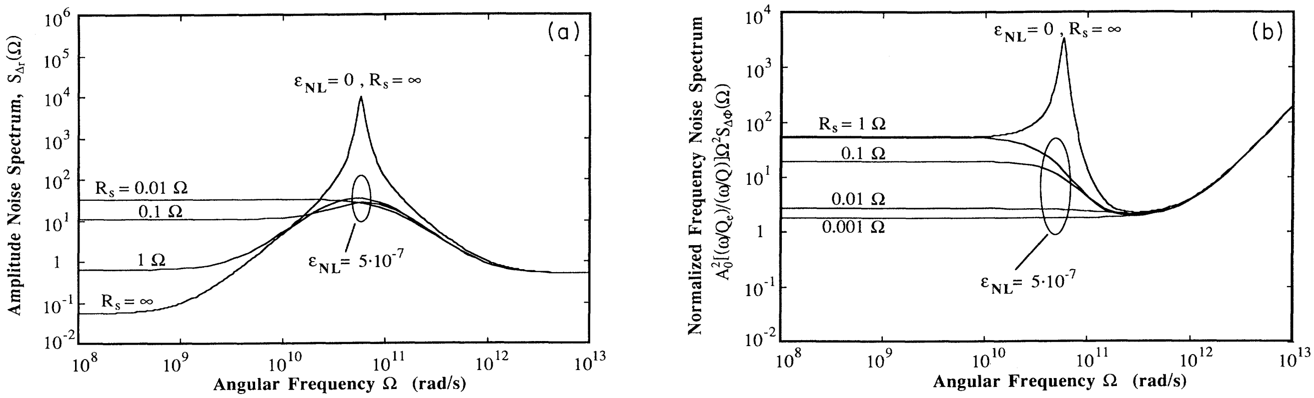

FIG. 1. (a) External amplitude noise spectra $S_{\Delta\hat{r}}(\Omega)$ (standard quantum limit of $\frac{1}{2}$) vs series resistance R_s as a function of angular frequency. The relative pump level is $R_p = 10$. (b) Normalized frequency noise spectra $A_0^2[(\omega/Q_e)/(\omega/Q)]\Omega^2 S_{\Delta\hat{\phi}}(\Omega)$ vs series resistance R_s as a function of angular frequency. The ideal case, $\alpha=0$, $n_{sp}=1$, yields a unity noise level for low frequencies. The relative pump level is $R_p = 10$.

creases; however, the nonlinearity saturates with high pumping. One notices that for high R_s the frequency noise increases with pumping due to nonlinear gain. This yields a linewidth floor for very high power levels, but not any rebroadening of linewidth. However, another nonlinear gain model could give a different result.[7] The level of the normalized frequency noise spectral density for low series resistances is equal to n_{sp} and decreases with increasing pumping, as described by Eq. (44). The dashed lines in Figs. 2(a) and 2(b) are noise level lines, with relative pump level as a parameter, where the laser's series resistance equals the laser's differential resistance, as given by

$$R = \frac{1}{q}\frac{dU_f}{dN}\left[\frac{1}{\tau_{st}} + \frac{1}{\tau_{sp}}\right]^{-1}, \tag{47}$$

which is the same as given by the equivalent electrical circuit analysis in Ref. 2. Note that this resistance is not a differential resistance as given from a simple voltage-current curve, since the lasing mechanism normally clamps the junction voltage. As can be inferred from Figs. 2(a) and 2(b), the series resistance of the laser must be lower than the differential resistance of Eq. (47) for constant-voltage operation. Note that for moderate and high pumping, R is mainly determined from the stimulated recombination contribution.

In Figs. 3(a) and 3(b) we have calculated the noise spectra of the junction voltage as a function of angular frequency, in Fig. 3(a) without nonlinear gain, and in Fig. 3(b) with $\epsilon_{NL} = 5\times10^{-7}$. The spectra are normalized to the low-frequency value, being determined by $\hat{H}(t)$, and not by the voltage "pinning" through R_s, this being a consequence of the gain clamping. With decreasing R_s the cutoff for amplitude fluctuations and gain clamping given by $\Omega_c = \omega/Q[1+\tau_{st}(1/\tau_{sp}+1/\tau_{CR})]^{-1}$, decreases, here $\tau_{CR} = R_s q(dU_f/dN)^{-1}$. The noise floor above this frequency is given from spontaneous and stimulated recombination noise, and the thermal noise from R_s. This level decreases with decreasing R_s. The upper cutoff

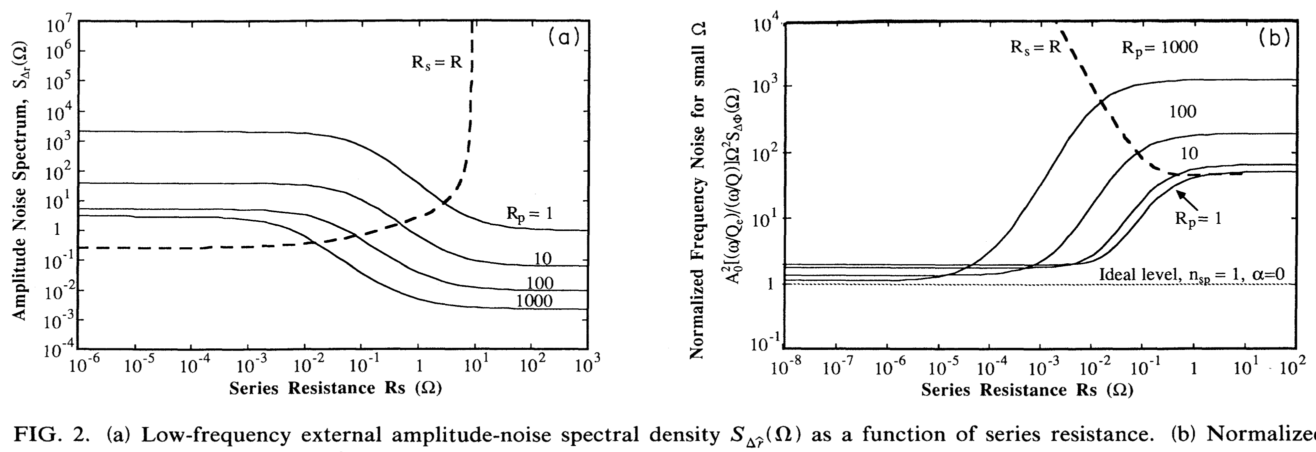

FIG. 2. (a) Low-frequency external amplitude-noise spectral density $S_{\Delta\hat{r}}(\Omega)$ as a function of series resistance. (b) Normalized frequency-noise spectral density $A_0^2[(\omega/Q_e)/(\omega/Q)]\Omega^2 S_{\Delta\hat{\phi}}(\Omega)$ at low frequency as a function of series resistance.

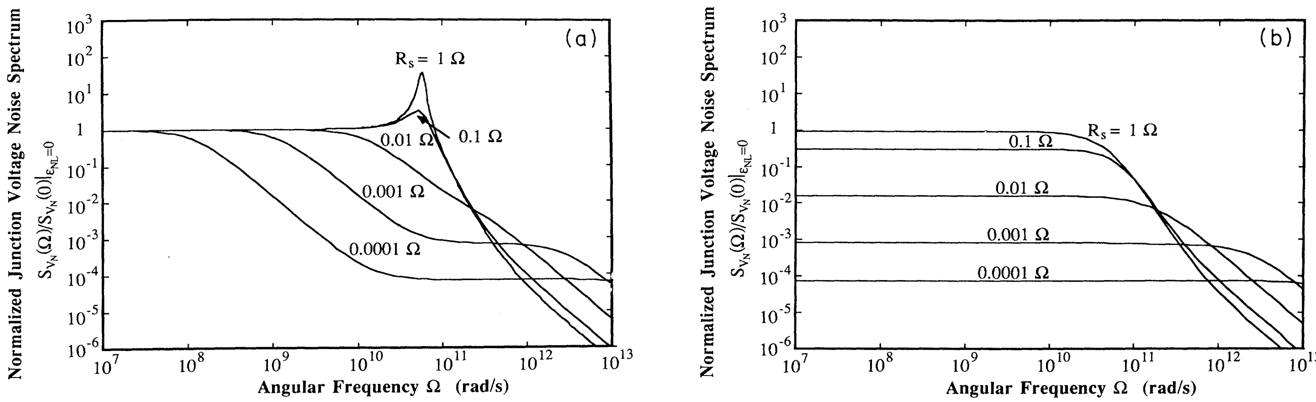

FIG. 3. (a) Junction voltage noise spectra as a function of angular frequency, normalized to the low-frequency value. The nonlinear gain coefficient is zero and the relative pump level is $R_p=10$. (b) Junction voltage noise spectra as a function of angular frequency, normalized to the low-frequency value without nonlinear gain. The nonlinear gain coefficient is $\epsilon_{NI}=5\times10^{-7}$. The relative pump level is $R_p=10$.

frequency is given from the total carrier lifetime. The constant noise level for low frequencies is interesting, since carrier fluctuations only can be suppressed in the sense that the cutoff for this noise level decreases monotonically with decreasing R_s. For the laser linewidth this, heuristically, implies that it will only be affected when the cutoff frequency is pushed below frequency values of linewidth order. This is not the case when nonlinear gain is included; as can be seen in Fig. 3(b), the low-frequency noise now decreases with decreasing R_s.

What happens then if we have a finite laser series resistance that is eliminated by using a negative resistance coupling? As seen from Eqs. (38) and (39), the noise level should increase if the absolute sum of the series resistance is higher than the laser's differential resistance. This is seen in Figs. 4(a) and 4(b), where the low-frequency amplitude and frequency noise spectral densities are plotted as a function of total series resistance, with the negative resistance R_{neg} defined from $|R_{\text{neg}}|=R_{s,\text{laser}}-R_{s,\text{tot}}$, as a variable parameter. The figures show that the noise performance for both amplitude and frequency noise is worse compared to the high-impedance feeding case, unless the negative resistance and the laser series resistance is smaller than the differential resistance (e.g., R_{st}), being for $R_p=10$, mainly given from the stimulated recombination. Comparing Figs. 4(a) and 4(b), we also see that the demand for a low series resistance of the laser, as stated earlier, is stricter for the frequency noise than for the amplitude noise, due to the α parameter. Notice also that the α parameter in question, i.e., that of Eq. (39), is the material α parameter, which according to Eq. (46) increases with pumping due to nonlinear gain.

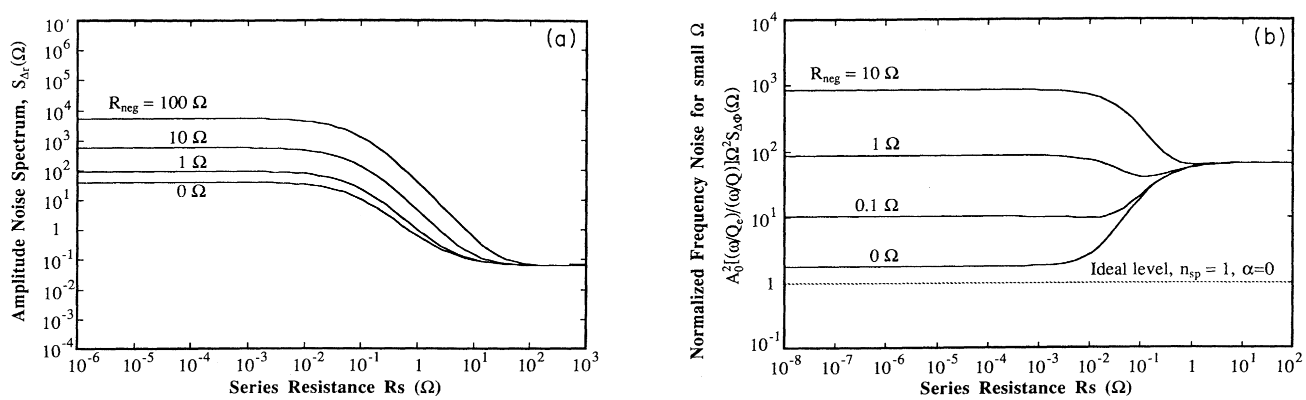

FIG. 4. (a) Low-frequency external amplitude noise spectral density $S_{\Delta\hat{r}}(\Omega)$ as a function of series resistance with the negative resistance as a parameter. (b) Normalized frequency-noise spectral density $A_0^2[(\omega/Q_e)/(\omega/Q)]\Omega^2 S_{\Delta\hat{\phi}}(\Omega)$ at low frequency as a function of series resistance, with the negative resistance as a parameter.

V. DISCUSSION AND CONCLUSIONS

We have shown that constant-voltage operation of a semiconductor laser by means of a low series resistance of the laser leads to a reduction of frequency noise. However, the amplitude noise, which was assumed to be regulated by nonlinear gain, will be increased compared to the free-running case. Active elimination of the laser series resistance by the use of a negative resistance coupling deteriorates the performance, since the voltage fluctuation in that case remains finite despite the fact that the total resistance goes to zero. The requirement that the laser series resistance be lower than the differential resistance is very strict. One can therefore speculate about what could be done to meet this requirement. One way to increase the differential resistance is to decrease the active volume, since dU_f/dN, at least in the Boltzmann approximation, is inversely proportional to the carrier number. One should, of course, be aware that the series resistance also may increase with decreasing volume, and that a lower optical confinement increases the threshold carrier density. An interesting alternative might be low-dimensional structures, for example, quantum-well lasers, which have lower threshold currents compared to ordinary lasers, partly due to smaller active volumes. Also the recently developed "microcavity" lasers are interesting from this viewpoint, since they can have very small active volumes.

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