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Comparison of the Compton-scattering process from *K* electrons of tin and lead

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The differential cross sections for Compton scattering of 662-keV photons from *K* electrons of tin and lead were measured at scattering angles between 15° and 150°. Both cross sections were found to be similar and to exhibit the qualitative features predicted by relativistic calculations. Additional experiments were carried out with two thick silver and platinum targets, and the influence of second-order scattering processes on the values of the cross sections was estimated.

I. INTRODUCTION

Experimental investigations of the Compton-scattering process from bound electrons are usually carried out by detecting the incoherently scattered photon in coincidence with a characteristic x ray emitted from the filling of a *K*-shell vacancy created by the scattering process. Much work has been done along this line with 662-keV photons on medium- and large-*Z* targets, and results for tin and lead have been reported by several authors.[1–7] However, there exist other processes, besides incoherent scattering from *K* electrons, which may occur in the target and which also lead to the simultaneous emission of a *K*-shell x ray and a γ ray with a continuous energy distribution indistinguishable from a Compton-scattered photon. These processes, which are discussed in detail in a paper by Shimizu *et al.*,[4] involve double scattering and may affect the experimental data in a way difficult to evaluate.

In a previous paper[8] we reported on measurements of the differential cross section for Compton scattering by lead *K* electrons. While our data agreed well with those of other authors for forward scattering angles, a considerable discrepancy was noted at large scattering angles. This discrepancy was attributed to the different ways in which coincident background events were taken into account. However, we were unable to distinguish events truly resulting from Compton scattering by *K* electrons from coincident background events involving the emission of an x-ray.

One possible way of testing the sensitivity of data for Compton scattering by *K* electrons to this kind of background consists of measuring, under identical conditions, elements of medium and large *Z*. As pointed out below, certain processes responsible for coincident background are strongly *Z* dependent, while the cross section for Compton scattering is expected to vary little with *Z*, since the *K*-shell ionization energies are always small in comparison to the energy of 662 keV of the incident photons. This statement is supported by Whittingham's[9] calculations carried out for samarium, lead, and uranium. Experimental data on medium- and large-*Z* targets available in the literature, however, show a considerable increase, with increasing *Z*, of the Compton *K*-shell cross section at backward scattering angles, being approximately 50% larger for lead than for tin. This discrepancy between theory and experiment may, at least partially, be attributable to background coincident events not recognized as such.

Therefore we undertook an investigation of tin and lead under identical conditions. The *K*-shell binding energy for tin is 29.2 keV, only a third of the corresponding value of 88.0 keV for lead. As a consequence the photoelectric cross section for 662-keV photons is roughly six times larger for lead than it is for tin. Since photoelectric absorption of the incident photon by a *K* electron and subsequent x-ray emission, when accompanied by the emission of a bremsstrahlung photon from the recoiling photoelectron, may be a major source of coincident background events, and since the probability of bremsstrahlung emission also increases with *Z*, this double scattering process is expected to affect more seriously the data for lead than for tin.

Another process which may contribute to background coincident events involving x-ray emission, but which has a different *Z* dependence, is started with the incident photon being incoherently scattered by any target electron. The recoiling Compton electron may subsequently collide with a *K* electron of another atom in the target, producing a *K*-shell vacancy in that atom which in turn leads to the emission of an x ray. The probability for the first step in this double scattering process, i.e., Compton scattering from any target electron, is approximately proportional to the number of electrons in the target atom, that is, increases approximately linearly with *Z*. However, the second step, i.e., *K*-shell ionization of a target atom by an incident Compton electron with energies up to 570 keV, has a cross section[10,11] which decreases with increasing *Z* faster than $1/Z$. This double scattering process is therefore expected to be more important in medium-*Z* targets.

With the possible presence of two double scattering processes which can contribute to background coincident events involving x-ray emission, one increasing with the atomic number and the other decreasing, it is not clear whether data of cross sections for Compton scattering by *K* electrons of elements of large *Z* are more sensitive to background, and an experimental investigation may help to answer this question.

II. EXPERIMENTAL PROCEDURE

Since the experimental procedure of the present work is the same as that previously employed for lead,[8] only an abbreviated description will be presented. The setup consists of a ^{137}Cs source with an activity of 1 Ci providing the primary 662-keV photon beam, and of a GeHp (where Hp represents hyperpure) x-ray detector and a Ge(Li) γ-ray detector wired in coincidence. The x-ray detector is mounted at a fixed angle of 125° with respect to the incident beam, while the position of the γ-ray detector may vary between 15° and 150°.

The fast timing signals from both detectors are fed into a time to pulse height converter (TPHC). A data acquisition system with two gated analog to digital converters (ADC's) is used to generate timing spectra from the TPHC output. The criterion for acceptance of a TPHC signal is established separately for each ADC by a slow coincidence circuit. For this purpose narrow x-ray windows of equal width are placed on the target's $K\alpha$ x-ray line and on a region of smooth background located slightly above the target's $K\beta$ x-ray line. On the other hand, all events from the γ-ray detector exceeding a threshold of 100 keV are accepted. Ideally the γ-ray detector should be allowed to detect all photons down to very low energies, since photons which are Compton scattered from bound electrons possess a continuous energy distribution,[9] which according to some authors[7,12] may even diverge at very low energies. In practice, however, a lower detection limit located slightly above the characteristic x rays of lead must be imposed, as a considerable amount of coincident events involving lead x rays in both detectors was observed in trial experiments. In any case the detection efficiency of the Ge(Li) γ-ray detector used in this work is very low for energies below 60 keV, so it is not possible to detect Compton-scattered photons of very low energy.

Since the purpose of the experiment is to perform a critical comparison of the Compton K-shell cross sections for tin and lead, both elements were measured under identical conditions. The lead target was the same 6.8-mg/cm^2 metal foil previously used.[8] The tin target was a 5.1-mg/cm^2 foil of the same size (5 cm by 5 cm) as the lead target. For both thin targets a complete angular distribution was obtained. Additional measurements were performed on two thick targets, a 108-mg/cm^2 platinum foil containing 16.8 times more atoms than the lead target, and a 52.5-mg/cm^2 silver foil containing 11.3 times more atoms than the tin target. All other dimensions were the same. Coincidence counting rates for these targets were observed at angles of 15° and 125°.

III. RESULTS

The experimental raw data consisted of timing spectra produced by the time to pulse height converter for each x-ray window. One such spectrum, obtained from the tin target at 125° with the $K\alpha$ window, is shown in Fig. 1(a). The coincidence peak is sitting on a constant background of chance events. One notes the poor statistics due to the exceedingly small coincidence counting rate, which makes a proper evaluation of the peak's net area more difficult. The following method was adopted to obtain this area. First the numbers of counts contained in the intervals ranging from channel numbers 0–10, 0–20, . . . , 0–1000 were extracted from the spectra, that is, experimental distribution functions were generated. This procedure gives rise to graphs like that shown in Fig. 1(b), which gives the number of counts as a function of interval width. The coincidence peak appears as a step flanked on both sides by smoothly rising curves representing the constant background. The height of this step is equal to the net area of the coincidence peak. By carefully fitting straight lines to the background on both sides of the step it is possible to determine the peak's net area with a precision limited essentially by the statistical error associated with the number of counts contained in that peak.

TABLE I. Experimental cross-section ratios $d\sigma_K/d\sigma_{\text{free}}$.

θ (deg)	Sn	Pb
15	0.30±0.03	0.21±0.03
40	0.76±0.06	0.63±0.07
60	1.01±0.07	0.86±0.10
75	1.05±0.10	1.12±0.11
100	1.00±0.13	0.92±0.14
125	0.88±0.08	0.89±0.09
150	0.77±0.12	0.86±0.17

The net area of the coincidence peak of the TPHC spectrum generated by the background x-ray window was subtracted from the area of the coincidence peak generated by the $K\alpha$ x-ray window. This difference was then corrected for the unobserved $K\beta$ transitions by multiplication with a factor $1+I_\beta/I_\alpha$, yielding the true coincident counting rate $(\Delta N/\Delta t)_{CK}$ for Compton scattering by K electrons. The $K\beta$ to $K\alpha$ intensity ratios I_β/I_α were taken from Khan and Karimi.[13] Their values are 0.222 for tin and 0.277 for lead. By dividing $(\Delta N/\Delta t)_{CK}$ by the counting rate, $(\Delta N/\Delta t)_\gamma$, observed with the γ-ray detector and correcting for the x-ray detector efficiency ϵ_X,[8] the x-ray fluorescence yield ω_K,[14] for x-ray absorption in the target a_K (Ref. 15) and for the incoherent scattering form factor $S(\theta)$,[16] one finally arrives at the ratio of Compton scattering by K electrons to Compton scattering by free electrons $d\sigma_K/d\sigma_{\text{free}}$:

$$\frac{d\sigma_K}{d\sigma_{\text{free}}}=S(\theta)\frac{Z}{2}\frac{(\Delta N/\Delta t)_{CK}}{(\Delta N/\Delta t)_\gamma}\frac{1}{\epsilon_X a_K \omega_K}\ . \qquad (1)$$

The ratios of the cross sections obtained in this way for tin and lead are listed in Table I. Table II contains the true coincident counting rates for both thin and thick tar-

TABLE II. Thin and thick target coincident Compton K counting rates normalized for equal target thickness.

	$(\Delta N/\Delta t)_{CK}/a_K\omega_K N_A$ (10^{-25} counts/sec atoms)			
θ (deg)	Ag	Sn	Pt	Pb
15	2.1±0.2	1.9±0.2	2.7±0.2	1.6±0.3
125	11.6±0.4	6.3±0.6	20.6±1.0	6.8±0.7

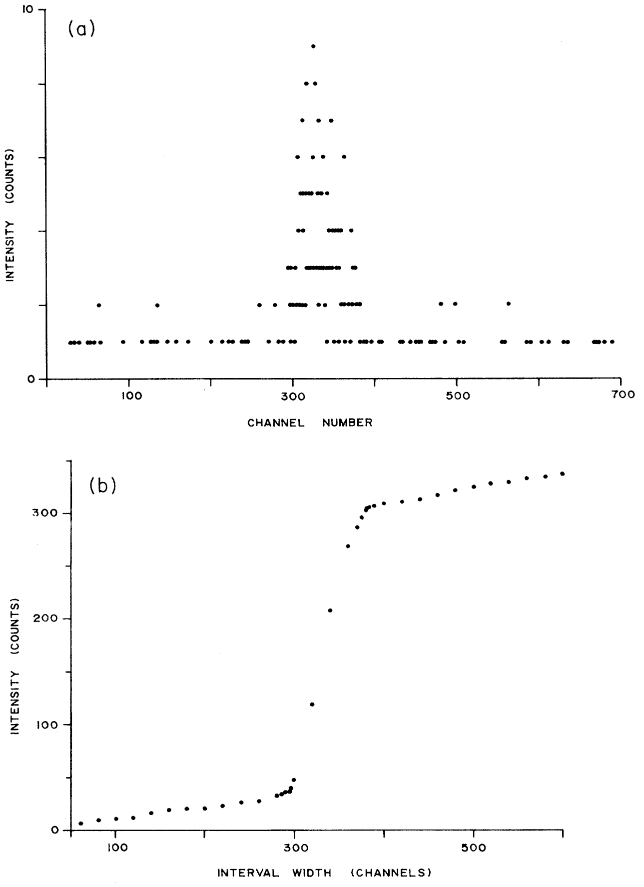

FIG. 1. (a) Time to pulse height converter spectrum obtained from the tin target at 125° with the $K\alpha$ x-ray window. The channel width is 0.8 nsec. (b) Experimental distribution function generated from the spectrum of (a) as described in the text.

gets $(\Delta N/\Delta t)_{CK}$ as defined above but corrected for x-ray absorption in the target and x-ray fluorescence yield, and divided by the number of target atoms N_A. These coincidence counting rates normalized for equal target thickness exhibit, at 125°, a considerable difference between thin and thick targets, indicating the presence of thick target effects that are discussed below.

Due to the extremely low coincidence counting rates observed in this experiment, the differences of the net areas of the $K\alpha$ and background coincidence peaks exhibit large statistical uncertainties, which vary between $\pm 10\%$ and $\pm 20\%$, depending on target and scattering angle. In addition the coincidence counting rates are afflicted with a systematical error due to the presence of coincident background events, which will be discussed and estimated below. All other quantities in Eq. (1) are known to a higher degree of accuracy than the differences of the coincidence counting rates, so their uncertainties have not been taken into account. Accordingly the errors indicated in the two tables were solely determined from the statistical uncertainties of the coincidence peaks' net areas.

IV. DISCUSSION

The cross-section ratios for tin and lead of Table I are shown in Fig. 2, together with theoretical predictions for lead and samarium by Whittingham[9] and for tin by Talukdar *et al.*[6] Both targets produced similar results, the difference between the two sets of data being almost always well within experimental error bars. Tin gave somewhat larger cross-section values under forward scattering angles, while lead showed somewhat larger results for backward angles. Both cross-section ratios rise toward a slight maximum for $\theta \simeq 70°$ and then drop below unity. These qualitative features are also present in Whittingham's calculations, which show little difference between samarium and lead, exhibit a maximum which is rather more pronounced, and drop to values below the experimental data at large angles. However, it will be shown in the subsequent discussion of second-order scattering effects that we estimate our cross-section values at backward angles to be systematically too large by approximately 10% for tin and by 15% for lead. On the other hand, no agreement whatsoever exists between the data and the calculations of Talukdar *et al.*

In Fig. 3 we compare the tin data of the present work with data from Motz and Missoni,[2] Shimizu *et al.*,[4] and Reddy *et al.*[5] While there is reasonable overall agreement at forward scattering angles, the present data are systematically smaller for angles greater than 100°. This is essentially the same situation already encountered in the case of lead. The reason is that at large scattering angles the TPHC spectra produced by the background x-ray window contain considerable amounts of coincident

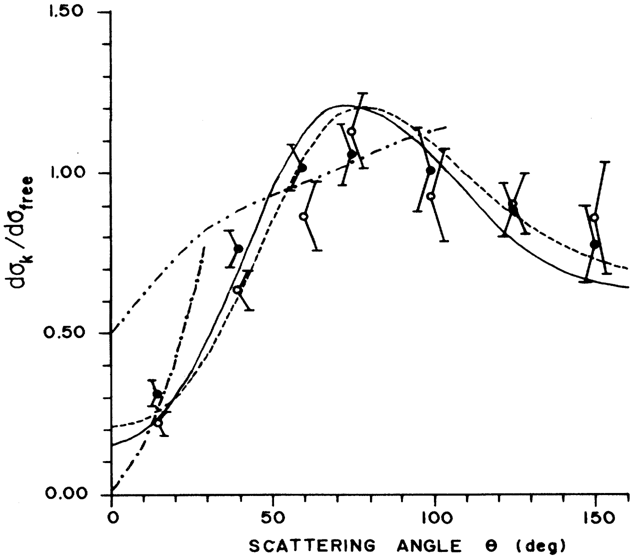

FIG. 2. Comparison of the present experimental results for Sn and Pb with theoretical predictions. ●, result for Sn target; ○, result for Pb target; —·—·—·, Talukdar nonrelativistic calculations for Sn; –··–, Talukdar relativistic calculations for Sn; ——, Whittingham exact relativistic calculation for Sm; — — —, Whittingham exact relativistic calculation for Pb.

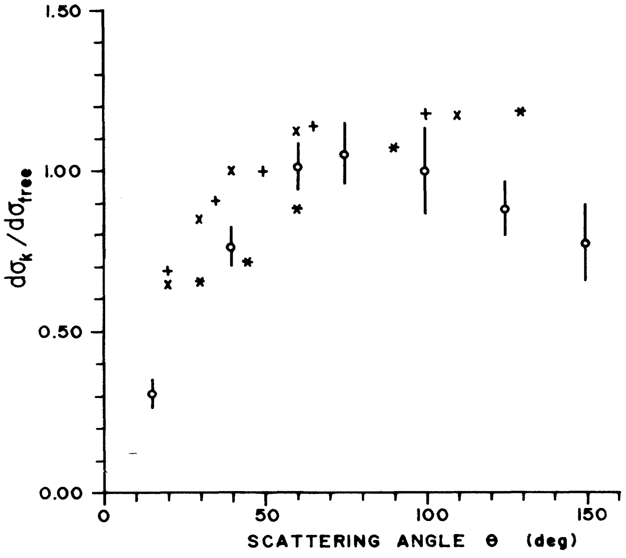

FIG. 3. Comparison of experimental results for tin. ○, this work; +, Shimizu *et al.*; ×, Motz and Missoni; $*$, Reddy *et al.*

TABLE III. Target thickness coefficients of Eq. (2).

Z	$\theta=15°$		$\theta=125°$	
	a[a] (10^{-25})	b[a] (10^{-48})	a (10^{-25})	b (10^{-47})
47–50	1.9	2.5	6.0	7.8
78–82	1.6	13.0	5.9	17.5

[a]Units: a: (counts/sec atoms) and b: counts/sec atoms^2.

events, which must be subtracted from those of the $K\alpha$ window. At 150° this background amounted in the case of tin to 36% and in the case of lead to 46% of the respective $K\alpha$ counting rate.

The sources for coincident background events have been discussed thoroughly by Shimizu *et al.*[4] and include several double scattering processes within the target. The two x-ray windows technique employed here takes care of the following processes: (a) double Compton scattering and (b) Compton scattering from any target electron followed by emission of bremsstrahlung produced during subsequent collisions of the recoiling electron. However, there are other processes which escape identification. These are (c) Compton scattering of the incident photon by any target electron followed by K-shell ionization of another target atom by the recoiling electron and (d) photoelectric absorption of the incident photon by a K electron and emission of a bremsstrahlung photon during subsequent collisions of the recoiling photoelectron. Both processes (c) and (d) result in the simultaneous emission of a K-shell x-ray and of a γ ray with a continuous energy distribution, and both are Z dependent. However, as has been pointed out in the Introduction, the probability for process (c) to occur decreases with increasing Z, while process (d) tends to increase.

In order to arrive at a more quantitative measure of the importance of the second-order processes (c) and (d), we turn to a comparison between the normalized thin and thick target coincidence counting rates of Table II. At a scattering angle of 15° the ratio of the coincidence counting rates for Compton scattering by K electrons between silver and tin was observed to be 1.1 and the ratio between platinum and lead to be 1.7. At 125° these same ratios are 1.8 and 3.0, respectively. Considering the similarity in binding energies, silver and tin should give very similar results, as should platinum and lead. The actually observed differences are due to thick target effects, i.e., processes involving double scattering and the emission of an x ray. This effect is seen to be much more pronounced for the targets of large Z, especially at 125°.

For the following discussion we assume that the normalized coincident Compton K counting rates of Table II still depend on the number of atoms in the target N_A through a linear term,

$$\frac{(\Delta N/\Delta t)_{CK}}{a_K\omega_K N_A}=a+bN_A\ . \tag{2}$$

For our present purpose we further assume equal Compton K cross sections for platinum and lead and the same for silver and tin. The coefficients a and b can then be determined for the $47\leq Z\leq 50$ and $78\leq Z\leq 82$ target combinations from the respective data of Table II, and the results are listed in Table III. The coefficient a, which represents the contributions of single scattering processes, i.e., Compton scattering by K electrons, assumes almost equal values for the two target combinations. This fact reinforces the aforementioned observation that elements of medium and large Z have very similar cross sections for Compton scattering of 662-keV photons by K electrons. However, since only two data points were available for the determination of this coefficient, no fit was possible. Therefore we do not think that a should be used to extrapolate the values of the Compton K cross sections to zero target thickness.

The coefficient b which represents the contributions of double scattering processes exhibits a pronounced dependence on both the scattering angle and the atomic number. The strong increase of b at 125° reflects the fact, already noted, that coincident background events contribute much more at large scattering angles. Its strong increase with Z gives evidence that cross sections for elements of large Z are much more sensitive to coincident background events.

Although a and b were not determined by a fit procedure, because of the lack of targets of varying thicknesses, they nevertheless can be used to obtain an order-of-magnitude estimate of the systematic error introduced into the final cross-section data by background coincident events involving the emission of an x ray. This is simply done by calculating the ratio bN_A/a which depends on the atomic number Z, on the target thickness, and on the scattering angle. For the tin target the estimated excess in the coincidence counting rate amounts to 1% at 15° and to 10% at 125°, while for the lead target the respective numbers are 5% and 15%.

V. CONCLUSIONS

It was observed that Compton scattering of 662-keV photons by K electrons exhibits similar cross sections for medium and large atomic number elements, as predicted by the relativistic calculations of Whittingham.[9] The qualitative features of these calculations are reproduced, and there is reasonable quantitative agreement, especially when target thickness corrections are taken into account. The influence of the target thickness on the observed coincidence counting rates was also investigated and it turned out that these are much more seriously affected, for the energy studied in this work, by background coincident events at large scattering angles and large Z. This probably explains why in the past experimentally determined ratios of Compton scattering of 662-keV photons by K electrons to Compton scattering by free electrons were systematically found to above unity for large-Z targets at backward scattering angles.

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