Calculation of a linear-Stark effect in D-line absorption in rubidium

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An external electric field can induce $M1$ and $E2$ amplitudes in the D lines of alkali-metal atoms, which interfere with the normal $E1$ amplitudes of these lines, causing the absorption to depend linearly upon the external field and upon the initial spin polarization of the atoms. We discuss this effect and calculate its size for the D lines of rubidium.

An interesting and well-known kind of Stark interference occurs when an $M1$ (magnetic dipole) optical transition between levels of the same parity, always very weak, acquires an $E1$ (electric dipole) component in the presence of an electric field.[1-4] Here we discuss the converse effect in which an electric field induces $M1$ and $E2$ (electric quadrupole) amplitudes in an allowed $E1$ transition. In this case the interference is a tiny fractional effect, but it still should be observable in precise optical experiments with alkali-metal atoms, and could be significant in experiments designed to search for a permanent atomic electric dipole moment.[5,6] If the effect can be measured precisely in alkali-metal atoms, it will provide another quantitative test of the atomic theory of these atoms.

As we will show later, the fractional change in the absorptivity α turns out to be of the form

$$\frac{\Delta\alpha}{\alpha}=(a_{E2}+a_{M1})(\hat{\boldsymbol{\epsilon}}\cdot\mathbf{E}_s)(\hat{k}\times\hat{\boldsymbol{\epsilon}})\cdot\hat{\boldsymbol{\sigma}}\ , \tag{1}$$

where $\mathbf{E}_s$ is an external static electric field, $\hat{\boldsymbol{\epsilon}}$ and $\hat{k}$ are the polarization and propagation directions of the incident light, and $\hat{\boldsymbol{\sigma}}$ is the initial atomic spin direction. In this paper we calculate the coefficients a_{E2} and a_{M1} for the D_1 absorption line in rubidium (i.e., the $5s_{1/2}\rightarrow 5p_{1/2}$ transition), using a central field atomic potential. Here we quote our results:

$$a_{E2}=-4.57\times10^{-8}\ (\mathrm{kV/cm})^{-1}\ ,$$
$$a_{M1}=+2.45\times10^{-8}\ (\mathrm{kV/cm})^{-1}\ . \tag{2}$$

If the D_1 and D_2 lines are not resolved, the net effect is zero for a_{M1} and a_{E2}.

To keep the physical origin of the effect as clear as possible, we begin by using one specific orientation of the vectors in Eq. (1) to set up the calculation. We then integrate over the angular parts of the atomic wave functions, and find the correct radial equations to be solved. Next we describe how we have performed the radial part of the calculation that leads to the results in Eq. (2). Finally, we generalize these results to arbitrary geometry by using irreducible tensor methods to derive Eq. (1).

The Rb atoms are assumed to be initially in the $5s_{1/2}$ state with electronic spin oriented along the positive z axis ($m_s=m_j=+\frac{1}{2}$). These atoms are illuminated with D_1 resonance radiation propagating along the x axis with wave vector $\mathbf{k}=k\hat{\mathbf{x}}$ and electric field $\mathbf{E}$ plane polarized along the y axis.

$$\mathbf{E}=\hat{\boldsymbol{\epsilon}}E(x,t)=\hat{\mathbf{y}}\frac{E}{2}(e^{i(kx-\omega t)}+e^{-i(kx-\omega t)})\ . \tag{3}$$

The applied static electric field $\mathbf{E}_s=\hat{\mathbf{y}}E_s$ also points along the y axis. This geometry maximizes the effect in Eq. (1).

In the absence of external fields the Rb valence electron is described by a Hamiltonian $H_0=p^2/2m+V(r)$, with $V(r)$ the assumed central potential. For a state of energy W_{nl}, the equation for the radial wave function $R_{nl}(r)\equiv U_{nl}/r$ may be written as

$$\left[\frac{d^2}{d\rho^2}+2(W_{nl}-V)-\frac{l(l+1)}{\rho^2}\right]U_{nl}=0\ , \tag{4}$$

where we have introduced $\rho=r/a_0$, and use atomic units $me^4/\hbar^2$ (2 Ry) for energy.

The ordinary $E1$ amplitude for the transition of interest is proportional to the matrix element

$$\mathcal{E}_1(-\tfrac{1}{2},+\tfrac{1}{2})=\langle 5p_{1/2},-\tfrac{1}{2}|-e\mathbf{r}\cdot\mathbf{E}|5s_{1/2},+\tfrac{1}{2}\rangle\ , \tag{5}$$

where m_j must change as indicated since $\mathbf{E}$ is perpendicular to the z axis. (Here and throughout, e is algebraic, $e=-|e|$ for the electron.) The initial and final states may be factored into radial, angular, and spin parts

$$\begin{aligned}|5s_{1/2},+\tfrac{1}{2}\rangle&=R_{5s}Y_{00}X_{+1/2}\ ,\\ |5p_{1/2},-\tfrac{1}{2}\rangle&=R_{5p}(\sqrt{2/3}\,Y_{1,-1}X_{+1/2}-\sqrt{1/3}\,Y_{1,0}X_{-1/2})\ .\end{aligned} \tag{6}$$

After performing the angular integration, the matrix element becomes

$$\mathcal{E}_1(-\tfrac{1}{2},+\tfrac{1}{2})=-\frac{ieE}{6}\int_0^\infty U_{5p}\rho U_{5s}\,d\rho\ . \tag{7}$$

The amplitude of the magnetic dipole transition between the same two states is given by

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$$\mathcal{M}_1(-\tfrac{1}{2},+\tfrac{1}{2})=\sum_{m_l',m_s'}\left[\frac{\langle 5p_{1/2},-\tfrac{1}{2}|-e\mathbf{E}_s\cdot\mathbf{r}|5s,m_l',m_s'\rangle\langle 5s,m_l',m_s'|-\mu_B(\sigma+l)\cdot\mathbf{B}|5s_{1/2},+\tfrac{1}{2}\rangle}{W_{5p}-W_{5s}}+\frac{\langle 5p_{1/2},-\tfrac{1}{2}|-\mu_B(\sigma+l)\cdot\mathbf{B}|5p,m_l',m_s'\rangle\langle 5p,m_l'm_s'|-e\mathbf{E}_s\cdot\mathbf{r}|5s_{1/2},+\tfrac{1}{2}\rangle}{W_{5s}-W_{5p}}\right], \tag{8}$$

where only $n=5$ contributes to the sum over intermediate states because the $M1$ operator cannot change n. In the sums over intermediate states we indicate the orbital and spin projections m_l and m_s, which amounts to assuming that the spin-orbit splittings are small compared with the energy denominators. The field $\mathbf{B}$ in the light wave is [compare to Eq. (3)]

$$\mathbf{B}=\hat{\mathbf{z}}\frac{B}{2}(e^{i(kx-\omega t)}+e^{-i(kx-\omega t)}) . \tag{9}$$

In our (CGSE) units, we have $B=E$.

Expanding the wave functions in the intermediate states in terms of spherical harmonics and integrating over angles, we can simplify Eq. (8) to yield

$$\mathcal{M}_1(-\tfrac{1}{2},+\tfrac{1}{2})=\frac{e\mu_B E_s B}{2(W_{5p}-W_{5s})}\langle 5p_{1/2},-\tfrac{1}{2}|y|5s_{1/2},\tfrac{1}{2}\rangle=\frac{-\mu_B E_s}{(W_{5p}-W_{5s})}\mathcal{E}_1(-\tfrac{1}{2},+\tfrac{1}{2}) , \tag{10}$$

where in the second line, we have used Eq. (5). The transition probability is proportional to

$$|\mathcal{E}_1+\mathcal{M}_1+\cdots|^2=|\mathcal{E}_1|^2+2\,\mathrm{Re}(\mathcal{M}_1^*\mathcal{E}_1)+\cdots , \tag{11}$$

where $\mathcal{E}_1$ and $\mathcal{M}_1$ are given by Eqs. (7) and (10), respectively. Thus the change in transition probability due to the small $M1$ interaction is

$$\left[\frac{\Delta\alpha}{\alpha}\right]_{M1}\cong\frac{2\,\mathrm{Re}(\mathcal{M}_1^*\mathcal{E}_1)}{|\mathcal{E}_1|^2}=\frac{2|\mu_B|E_s}{W_{5p}-W_{5s}} , \tag{12}$$

which is readily calculable from well-known energies and moments. Using a_{M1} as defined in Eq. (1), we find

$$a_{M1}=\frac{2|\mu_B|}{W_{5p}-W_{5s}}=2.45\times10^{-8}\ (\mathrm{kV/cm})^{-1} \tag{13}$$

as quoted in Eq. (2).

Turning now to the electric quadrupole transition, the amplitude is given by

$$\mathcal{E}_2(-\tfrac{1}{2},+\tfrac{1}{2})=\sum_{n,m_l',m_s'}\left[\frac{\langle 5p_{1/2},-\tfrac{1}{2}|-e\mathbf{E}_s\cdot\mathbf{r}|nd,m_l'm_s'\rangle\langle nd,m_l'm_s'|-\frac{1}{6}\sum_{i,j}Q_{i,j}\frac{\partial E_i}{\partial x_j}|5s_{1/2},+\tfrac{1}{2}\rangle}{W_{5p}-W_{nd}}+\frac{\langle 5p_{1/2},-\tfrac{1}{2}|-\frac{1}{6}\sum_{i,j}Q_{i,j}\frac{\partial E}{\partial x_j}|np,m_l'm_s'\rangle\langle np,m_l'm_s'|-e\mathbf{E}_s\cdot\mathbf{r}|5s_{1/2},+\tfrac{1}{2}\rangle}{W_{5s}-W_{np}}\right], \tag{14}$$

where the electric quadrupole operator is $Q_{i,j}=e(3x_ix_j-\delta_{ij}x_i^2)$. The first term in Eq. (14) gives the Stark mixing of the final state by the electric field $\mathbf{E}_s$, which we call the "d channel." The second term corresponds to the Stark mixing of the initial state, which we call the "p channel."

We now need to reduce the two terms to a calculable form. We will outline the procedure for the d channel; that for the p channel should then be evident. Using the geometry specified initially, the first term in Eq. (14) becomes

$$\mathcal{E}_2(d\,\text{channel})=+i\frac{e^2E_sEk}{4}\sum_{n,m_l',m_s'}\frac{\langle 5p_{1/2},-\tfrac{1}{2}|y|nd,m_l'm_s'\rangle\langle nd,m_l'm_s'|xy|5s_{1/2},+\tfrac{1}{2}\rangle}{W_{5p}-W_{nd}} . \tag{15}$$

By a standard technique[7] we can transform the infinite sum of Eq. (15) into an inhomogeneous differential equation that defines a new function Ψ_d:

$$(H-W_{5p})|\Psi_d\rangle=-y|5p_{1/2},-\tfrac{1}{2}\rangle . \tag{16}$$

We can separate this into radial and angular parts by using the angular dependence of $|5p_{1/2},-\frac{1}{2}\rangle$ from Eq. (6) to set

$$|\Psi_d\rangle=2\frac{D(\rho)}{\rho}\sum_{m_l,m_s}Y_{2,m_l}(\theta,\phi)X_{m_s}\int d\Omega' Y_{2,m_l}^*(\theta',\phi')X_{m_s}^\dagger\sin\theta'\sin\phi'\times[\sqrt{2/3}Y_{1,-1}(\theta'\phi')X_{+1/2}-\sqrt{1/3}Y_{1,0}(\theta',\phi')X_{-1/2}] . \tag{17}$$

Thus $D(\rho)$ satisfies the equation

$$\left[\frac{d^2}{d\rho^2}-\frac{6}{\rho^2}+2(W_{5p}-V)\right]D(\rho)=\rho U_{5p} \ . \tag{18}$$

Substituting Eqs. (6) and (17) into Eq. (15), we find that

$$\mathcal{E}_2(d \text{ channel})=+i\frac{e^2E_sEk}{4}\langle \Psi_d|xy|5s_{1/2},+\tfrac{1}{2}\rangle \ , \tag{19}$$

where

$$\langle \Psi_d|xy|5s_{1/2},+\tfrac{1}{2}\rangle = F\int_0^\infty D(\rho)\rho^2U_{5s}(\rho)d\rho \ , \tag{20}$$

and the angular factor F is found from the angular integration of Eq. (17), using Eq. (6), to be $F=\frac{2}{15}$.

Following the same procedure for the p channel, we introduce $P(\rho)$, which is the solution of

$$\left[\frac{d^2}{d\rho^2}-\frac{2}{\rho^2}+2(W_{5s}-V)\right]P(\rho)=\rho U_{5s} \ . \tag{21}$$

This leads to the expression for the total $E2$ amplitude

$$\mathcal{E}_2(-\tfrac{1}{2},+\tfrac{1}{2}) = +i\frac{e^2E_sEk}{4}F\int_0^\infty [D(\rho)\rho^2U_{5s}+U_{5p}\rho^2P(\rho)]d\rho \ . \tag{22}$$

The angular factor F is the same ($\frac{2}{15}$) for both channels. As in Eq. (12), we find

$$\left[\frac{\Delta\alpha}{\alpha}\right]_{E2} \cong \frac{2\,\mathrm{Re}(\mathcal{E}_2^*\mathcal{E}_1)}{|\mathcal{E}_1|^2} = 2\frac{|e|E_sk}{5}\frac{\langle D(\rho)|\rho^2|U_{5s}\rangle+\langle U_{5p}|\rho^2|P(\rho)\rangle}{\langle U_{5p}|\rho|U_{5s}\rangle} = a_{E2}E_s \ , \tag{23}$$

where in the last step we used Eq. (1). Thus we find

$$a_{E2}=+\frac{2|e|k}{5}\left[\int_0^\infty [D(\rho)\rho^2U_{5s}+U_{5p}\rho^2P(\rho)]d\rho\right]\Big/\left[\int_0^\infty U_{5p}\rho U_{5s}d\rho\right] \tag{24}$$

which we next evaluate using a central field model of the Rb atom.

For a first calculation we have chosen to use the central field potential published by Herman and Skillman,[8] based on Hartree-Fock-Slater calculations. With this potential, the computation proceeds in three stages. First, we solve the Schrödinger equation [Eq. (4)] for the $5s$ and $5p$ wave functions. Then the differential equations [Eqs. (18) and (21)] are solved for the functions $D(\rho)$ and $P(\rho)$. Finally, the integrations in Eq. (24) are performed.

We make here only the briefest of remarks about the numerical techniques we employed. We used the Numerov method[9] to solve the differential equations by iteration, integrating the solutions outward from $\rho=0$ and inward from ∞, and matching at a meeting point in the exponentially dying region of the wave functions by the "shooting method."[10] We solve Eq. (4) by varying the energy to match the logarithmic derivatives of the inward and outward solutions at the meeting point, while the inhomogeneous equations [Eqs. (18) and (21)] are solved by varying the amplitudes of the inward and outward solutions so that the wave function and its first derivative match at the meeting point. The integrations in Eq. (24) were performed using a cubic spline interpolation polynomial.[11] A number of self-consistency checks were devised to ensure against numerical errors.

In Table I we list our calculated energies for several states, and compare with the experimentally known energies found in Moore.[12] Although the calculated energies agree with the experimental energies to within 2%, the energy difference between the ground state and the $4d$ state is in error by almost 5%. In Table II we list several calculated matrix elements, which agree within 10% with the experimentally determined values when the latter are available.

The results of some of the numerical checks, and the final calculations appear in Table III. From our definitions of $P(\rho)$ and $D(\rho)$, it follows that rows $a1$ and $a2$ of Table III should be equal, as should rows $b1$ and $b2$. We see that the numerical agreement is good (using the calculated energies), which provides a check on the consistency of the calculation. In Table III rows $c1$ and $c2$, and $d1$ and $d2$, the contributions to the interference from the nearest levels alone are compared to the sum over all states. We see that the $4d$ and $5p$ states dominate the d channel and p channel, respectively. Because the calculated energy denominator in row $c2$ is in error by roughly 5%, we expect that our result for a_{E2} below will have an uncertainty at least this large. Given the simple single-particle potential employed, however, a much larger error in a_{E2} is possible. Using Table II, Table III, and Eq. (24), we find that:

$$a_{E2}=-4.57\times10^{-8}\ (\mathrm{kV/cm})^{-1} \ . \tag{25}$$

To complete the discussion of the calculation, we now derive the full angular dependence assumed in Eq. (1).

TABLE I. Rubidium energy levels in units of 2 Ry. The experimental values are taken from Moore (Ref. 12).

nl	Exp. energy	Calc. energy
$5s$	−0.1535	−0.1509
$5p$	−0.0956	−0.0933
$4d$	−0.0653	−0.0665

TABLE II. Rubidium matrix elements in atomic units.

Matrix element	Calc. value
$\langle U_{5p}\|P(\rho)\rangle$	+46.86
$\langle U_{4d}\|D(\rho)\rangle$	−127.33
$\langle U_{5p}\|\rho\|U_{5s}\rangle$	−5.395
$\langle U_{4d}\|\rho\|U_{5p}\rangle$	+6.855
$\langle D(\rho)\|\rho^2\|U_{5s}\rangle$	+4792.4
$\langle U_{5p}\|\rho^2\|P(\rho)\rangle$	+2714.2
$\langle U_{4d}\|\rho^2\|U_{5s}\rangle$	−35.03
$\langle U_{5p}\|\rho^2\|U_{5p}\rangle$	61.09

We begin with the $M1$ term, rewriting Eq. (8) for arbitrary directions of $\hat{\boldsymbol{\epsilon}}$, $\hat{\mathbf{k}}$, and $\mathbf{E}_s$ $(=E_s\hat{\boldsymbol{\epsilon}}^s)$:

$$\mathcal{M}_1(m_j',m_j)=\langle 5p_{1/2},m_j'|\underline{\mathcal{M}_1}|5s_{1/2},m_j\rangle \ , \tag{26}$$

where

$$\underline{\mathcal{M}_1}=\frac{e\mu_B E_s}{W_{5p}-W_{5s}}[\hat{\boldsymbol{\epsilon}}^s\cdot\mathbf{r},(\boldsymbol{\sigma}+\boldsymbol{l})\cdot\hat{\mathbf{B}}]$$
$$=-\frac{e\mu_B E_s}{W_{5p}-W_{5s}}i[(\hat{\boldsymbol{\epsilon}}^s\cdot\hat{\mathbf{k}})(\hat{\boldsymbol{\epsilon}}\cdot\mathbf{r})-(\hat{\boldsymbol{\epsilon}}^s\cdot\hat{\boldsymbol{\epsilon}})(\hat{\mathbf{k}}\cdot\mathbf{r})] \ . \tag{27}$$

The interference term in Eq. (11) may be written

$$2\,\mathrm{Re}(\mathcal{E}_1^*\mathcal{M}_1)=\sum_{m_j'}\langle 5s_{1/2},m_j|-e\mathbf{r}\cdot\mathbf{E}|5p_{1/2},m_j'\rangle$$
$$\times\langle 5p_{1/2},m_j'|\underline{\mathcal{M}_1}|5s_{1/2},m_j\rangle \tag{28}$$

We can set matrix elements of $\mathbf{r}$ proportional to corresponding matrix elements of $\mathbf{j}$, the angular momentum, since j $(=\frac{1}{2})$ is the same for all states. After summing over m_j' and introducing $\boldsymbol{\sigma}=2\langle m_j|\mathbf{j}|m_j\rangle$, we find (using vector component notation with repeated indices *abcd* summed)

$$2\,\mathrm{Re}(\mathcal{E}_1^*\mathcal{M}_1)\propto\epsilon_a^s(\epsilon_a k_b+\epsilon_b k_a)$$
$$\times\epsilon_c\langle m_j|i(j_c j_b-j_b j_c)|m_j\rangle$$
$$=\epsilon_a^s(\epsilon_a k_b+\epsilon_b k_a)\epsilon_c\epsilon_{dcb}\sigma_d/2$$
$$=(\hat{\boldsymbol{\epsilon}}\cdot\hat{\boldsymbol{\epsilon}}^s)\boldsymbol{\sigma}\cdot(\hat{\mathbf{k}}\times\hat{\boldsymbol{\epsilon}})/2 \ , \tag{29}$$

which verifies the angular dependence of Eq. (1). Note that only the $\hat{\boldsymbol{\epsilon}}^s\cdot\hat{\boldsymbol{\epsilon}}$ term from Eq. (27) gives a contribution.

To treat the $E2$ term, we rewrite Eq. (14) for arbitrary geometry

$$\mathcal{E}_2(m_j',m_j)=\langle 5p_{1/2},m_j'|\underline{\mathcal{E}_2}|5s_{1/2},m_j\rangle \ , \tag{30}$$

where

$$\underline{\mathcal{E}_2}=iE_sEk\left[(\hat{\boldsymbol{\epsilon}}^s\cdot\mathbf{r})\frac{1}{W_{5p}-H}(\hat{\boldsymbol{\epsilon}}\cdot\mathbf{r})(\hat{\mathbf{k}}\cdot\mathbf{r})\right.$$
$$\left.+(\hat{\boldsymbol{\epsilon}}\cdot\mathbf{r})(\mathbf{k}\cdot\mathbf{r})\frac{1}{W_{5s}-H}(\epsilon^s\cdot\mathbf{r})\right]$$
$$=2iE_sEkQ_{abc}T_{abc} \ , \tag{31}$$

with

$$T_{abc}=r_a\frac{1}{W_{5p}-H}r_b r_c+r_c r_b\frac{1}{W_{5s}-H}r_a \ ,$$
$$Q_{abc}=\epsilon_a^s(\epsilon_b k_c+k_b\epsilon_c) \ . \tag{32}$$

We have written Q_{abc} as symmetric in its b and c indices because $T_{abc}=T_{acb}$. The tensor product can be written in terms of irreducible components

$$Q_{abc}T_{abc}=\sum_n Q^{(n)}T^{(n)} \ , \tag{33}$$

where $n=0,1,\ldots$ stands for scalar, vector, etc. parts.

Because we are connecting $j=\frac{1}{2}$ to $j=\frac{1}{2}$, only the scalar and vector parts $T^{(0)}$ and $T^{(1)}$ can contribute. The scalar part $T^{(0)}=\epsilon_{abc}T_{abc}$, is zero because $T_{abc}=T_{acb}$. An arbitrary three index tensor has at most three irreducible vector components given by the contractions $\delta_{ab}T_{abc}$, $\delta_{ac}T_{abc}$, and $\delta_{bc}T_{abc}$. The presence of the (scalar) energy denominator in Eq. (32) only affects the magnitude of the vectors in the matrix element. In our case, $\delta_{bc}T_{abc}=0$ because the electric quadrupole operator $Q_{i,j}$ in Eq. (14) is traceless, which yields $T_c^{(1)}=\delta_{ab}T_{abc}=\delta_{ab}T_{acb}\propto r_c$.

We have, then, that $Q_c^{(1)}=\delta_{ab}Q_{abc}=(\hat{\boldsymbol{\epsilon}}^s\cdot\hat{\boldsymbol{\epsilon}})k_c+(\hat{\boldsymbol{\epsilon}}^s\cdot\hat{\mathbf{k}})\epsilon_c$. Thus

$$\underline{\mathcal{E}_2}\propto i[(\hat{\boldsymbol{\epsilon}}^s\cdot\hat{\boldsymbol{\epsilon}})(\hat{\mathbf{k}}\cdot\mathbf{r})+(\hat{\boldsymbol{\epsilon}}^s\cdot\hat{\mathbf{k}})(\hat{\boldsymbol{\epsilon}}\cdot\mathbf{r})] \ , \tag{34}$$

TABLE III. Checks of the calculation. The energy denominators are the calculated energies.

	Checks and comparisons	Calc. value
*a*1	$\langle U_{5p}\|P(\rho)\rangle$	+46.86
*a*2	$\langle U_{5p}\|\rho\|U_{5s}\rangle/2(E_{5s}-E_{5p})$	+46.85
*b*1	$\langle U_{4d}\|D(\rho)\rangle$	−127.33
*b*2	$\langle U_{4d}\|\rho\|U_{5p}\rangle/2(E_{5p}-E_{4d})$	−127.74
*c*1	$\langle D(\rho)\|\rho^2\|U_{5s}\rangle$	4792.4
*c*2	$\langle U_{5p}\|\rho\|U_{4d}\rangle\langle U_{4d}\|\rho^2\|U_{5s}\rangle/2(E_{5p}-E_{4d})$	4474.2
*d*1	$\langle U_{5p}\|\rho^2\|P(\rho)\rangle$	2714.2
*d*2	$\langle U_{5p}\|\rho^2\|U_{5p}\rangle\langle U_{5p}\|\rho\|U_{5s}\rangle/2(E_{5s}-E_{5p})$	2862.2

which contains the same two terms as $\underline{\mathcal{M}_1}$ does in Eq. (27). The analysis following Eq. (27) can be repeated. Again, the $\hat{\epsilon}^s \cdot \hat{\mathbf{k}}$ term yields zero interference, while the $\hat{\epsilon}^s \cdot \hat{\epsilon}$ term leads to the angular dependence shown in Eq. (29), which completes the derivation of Eq. (1).

An interesting aspect of the results in Eq. (2) is that a_{E2} and a_{M1} have opposite signs and cancel to about 50%. Since a_{M1} is known to better than 1%, a measurement of the total effect $a = a_{E2} + a_{M1}$ yields a precise test of the atomic theory needed for the $E2$ part. Accurate measurements of this Stark interference in Rb and other alkali-metal atoms could be used as an exacting test of a more refined atomic theory.

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