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# Additional vacuum-field Rabi splittings in cavity QED

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I show how coherence on a long-time scale between two dressed states of the same doublet can be produced. Such a coherence leads to additional vacuum-field Rabi splittings at new frequencies in the cavity transmission and in sideways fluorescence. These additional vacuum-field Rabi splittings are shown to be analogous to the dephasing-induced coherences in nonlinear optics.

Vacuum-field Rabi splittings arise in the cavity QED whenever the total excitation in the cavity is unity and when the coupling $g$ of the atoms with the cavity mode is strong compared to the photon leakage rate so that the periodic exchange of energy between the atom and the cavity mode is possible. The spontaneous emission by an excited atom in the cavity leads to vacuum-field Rabi splittings.[1,2] We can also consider ground-state atoms in the cavity and apply weak external fields so that the atoms can absorb, at most, one photon from the external field of frequency $\omega_l$. The details of the absorption process depend on the interaction of the atom with the cavity mode. The absorption spectra[3] exhibit doublet structure with resonances at $\omega_l = \omega_c \pm g\sqrt{N}$ if the cavity is on resonance with the atom. Here $N$ is the number of atoms in the cavity. Such vacuum-field Rabi splittings have been seen in a number of experiments on absorption spectra.[4–6] These splittings can be understood in a number of ways. One can, for example, consider the transition to the dressed states corresponding to either one excited atom or one photon in the cavity, i.e., to the dressed states[7]

$$|\psi_\pm\rangle = (1/\sqrt{2})\left(\left|-\frac{N}{2}+1,0\right\rangle \pm \left|-\frac{N}{2},1\right\rangle\right), \qquad E_\pm = \pm g\sqrt{N}. \tag{1}$$

The ground state of the combined atom-cavity mode system is $|\psi_0\rangle \equiv |-(N/2),0\rangle$. One has two absorption channels $|\psi_0\rangle \rightarrow |\psi_\pm\rangle$. The classical interpretations for vacuum-field Rabi splittings also have been given.[2,5]

In this Rapid Communication I demonstrate the existence of yet other types of vacuum-field Rabi splittings in cavity transmission or fluorescence spectra. These additional vacuum-field Rabi splittings arise at the frequencies $\omega_l = \omega_0$, $\omega_0 \pm 2g\sqrt{N}$. The resonances at $\omega_0 \pm 2g\sqrt{N}$ are particularly attractive since one may have a situation where it is difficult to see the resonance at $\pm g\sqrt{N}$. I further demonstrate that the additional vacuum-field Rabi splittings arise from the coherence between the two dressed states. I show how a long-time coherence[8] between two dressed states $\psi_+$ and $\psi_-$ can be induced. I use the terminology additional splittings analogous to extra resonances due to dephasing in the context of four wave mixing experiments.[9–11] These additional resonances do not arise from any new property of the vacuum of the radiation field.

We consider the interaction of $N$ two-level atoms of frequency $\omega_0$ with the cavity mode on resonance and with the external field $\epsilon$ of frequency $\omega_l$. The interaction is given by

$$H = \sum_j [\hbar g(S_j^+ a + S_j^- a^\dagger) + \hbar G(t)(S_j^+ e^{-i\omega_l t - i\phi(t)} + \text{c.c.})] + \hbar\omega_0\left(\sum_j S_j^z + a^\dagger a\right). \tag{2}$$

Here $2G(t)$ is equal to the Rabi frequency of the external field and $\phi(t)$ is the instantaneous phase fluctuation of the external field. This phase fluctuation is responsible for the finite temporal width of the external field. The density matrix for the combined system consisting of atom and cavity mode obeys the equation

$$\dot\rho = -\frac{i}{\hbar}[H,\rho] - \kappa(a^\dagger a\rho - 2a\rho a^\dagger + \rho a^\dagger a) - \gamma\sum_j (S_j^+ S_j^- \rho - 2S_j^- \rho S_j^+ + \rho S_j^+ S_j^-). \tag{3}$$

Here $2\kappa$ $(2\gamma)$ is the rate at which field (atoms) decays. We have the case of optical transitions and hence, the spontaneous emission is generally quite significant. In fact, in a good cavity one has $\gamma \gg \kappa$. We next make a canonical transformation

$$\tilde\rho = U^\dagger \rho U; \qquad U = \exp\left[-i\int(\omega_l + \dot\phi)dt\left(\sum_j S_j^z + a^\dagger a\right)\right]. \tag{4}$$

The transformed density matrix obeys (3) with $H$ replaced by

$$\tilde H = \sum_j [\hbar g(S_j^+ a + S_j^- a^\dagger) + \hbar G(t)(S_j^\dagger + S_j^-)] + \hbar(\omega_0 - \omega_l - \dot\phi)\left(\sum_j S_j^z + a^\dagger a\right). \tag{5}$$

From now onwards, we omit the tildes. The physical quantities that we examine are the mean number of photons $\langle a^\dagger a\rangle$ and mean atomic population $\langle S^z\rangle$ which can be calculated using either $\rho$ or $\tilde\rho$.

For the purpose of calculating the physical effects in weak external fields it is sufficient to work in the space spanned by the states $|\psi_0\rangle$ and $|\psi_\pm\rangle$. Using the secular approximation[7] one can prove that

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$$\dot{\rho}_{\pm 0} = -\{\gamma_0 + i[\delta(t) \pm g\sqrt{N}]\}\rho_{\pm 0} - iG(t)\left[\frac{N}{2}\right]^{1/2}(\rho_{00} - \rho_{\pm +} - \rho_{\pm -}) , \tag{6a}$$

$$\dot{\rho}_{+-} = -2(\gamma_0 + ig\sqrt{N})\rho_{+-} - iG(t)\left[\frac{N}{2}\right]^{1/2}(\rho_{0-} - \rho_{+0}) , \tag{6b}$$

$$\dot{\rho}_{\pm\pm} = -2\gamma_0\rho_{\pm\pm} - iG(t)\left[\frac{N}{2}\right]^{1/2}(\rho_{0\pm} - \rho_{\pm 0}), \quad \rho_{00} + \rho_{++} + \rho_{--} = 1 , \tag{6c}$$

where

$$\delta(t) = \omega - \omega_l - \dot{\phi}(t) = \delta - \dot{\phi}(t), \quad \gamma_0 = \frac{\gamma + \kappa}{2} . \tag{7}$$

We can now average over the phase fluctuations. Assuming $\dot{\phi}(t)$ to be Gaussian and $\delta$ correlated with a strength $2\Gamma$, i.e.,

$$\langle \dot{\phi}(t)\dot{\phi}(t')\rangle = 2\Gamma\delta(t - t') , \tag{8}$$

we find that (6a) is replaced by

$$\dot{\rho}_{\pm 0} = -[\gamma_0 + \Gamma + i(\delta \pm g\sqrt{N})]\rho_{\pm 0} - iG(t)\left[\frac{N}{2}\right]^{1/2}(\rho_{00} - \rho_{\pm +} - \rho_{\pm -}) . \tag{9}$$

Note that the mean number of photons $\langle a^\dagger a\rangle$ and the total atomic excitation are obtained from dressed state density-matrix elements as follows:

$$\langle a^\dagger a\rangle = \frac{N}{2}(\rho_{++} + \rho_{--} - \rho_{+-} - \rho_{-+}) = I_C ,$$
$$\sum_i \langle S_i^+ S_i^-\rangle = \frac{N}{2}(\rho_{++} + \rho_{--} + \rho_{+-} + \rho_{-+}) = I_F . \tag{10}$$

We next assume that the external field has a frequency $\omega_0$ but is modulated at the frequency $\Omega$, i.e.,

$$G(t) = G(1 + 2m\cos\Omega t), \quad \delta = \omega_0 - \omega_l = 0 . \tag{11}$$

The signal can now be studied as a function of the modulation frequency. To order $G^2$, we find the results

$$\rho_{++} + \rho_{--} = (2\gamma_0 - i\Omega)^{-1}G^2Nm\{[\gamma_0 + \Gamma + i(g\sqrt{N} - \Omega)]^{-1} + (\gamma_0 + \Gamma - ig\sqrt{N})^{-1} + (g \to -g)\}e^{-i\Omega t} + \text{c.c.}, \tag{12}$$

where $(g \to -g)$ represents preceding terms with $g$ replaced by $-g$,

$$\rho_{+-}(t) = \rho_{+-}(\Omega)e^{-i\Omega t} + \rho_{+-}(-\Omega)e^{+i\Omega t} ,$$
$$\rho_{+-}(\pm\Omega) = G^2Nm(\mp i\Omega + 2\gamma_0 + 2ig\sqrt{N})^{-1}[(\gamma_0 + \Gamma + ig\sqrt{N})^{-1} + (\gamma_0 + \Gamma + ig\sqrt{N} \mp i\Omega)^{-1}] . \tag{13}$$

The second-order contribution (13) can also be represented in terms of the double-sided diagrams which are normally used to derive nonlinear optical susceptibilities.[9] For the present case the atomic system is replaced by the interacting system consisting of the atom and the cavity mode. Thus as basis states we use dressed states and we need to consider the damping of the dressed states rather than bare states. The two double-sided diagrams contributing to $\rho_{+-}(\Omega)$ are shown in Fig. 1. From Eqs. (10), (12), and (13) it is clear that the modulated signal will have resonances at

$$\Omega = \begin{cases} 0, \text{ width } 2\gamma_0 , \\ \pm g\sqrt{N}, \text{ width } \gamma_0 + \Gamma , \\ \pm 2g\sqrt{N}, \text{ width } 2\gamma_0 . \end{cases} \tag{14}$$

Note that for $\Gamma = 0$, the resonances at $\Omega = 0$, $\pm 2g\sqrt{N}$ do not appear because of the very interesting cancellation effect. As a matter of fact, (12) and (13) can be written in the instructive form

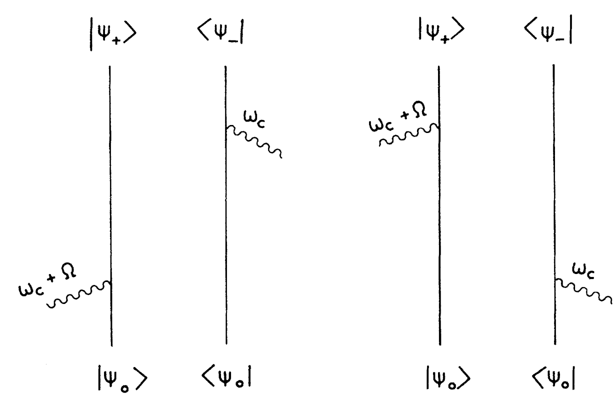


FIG. 1. The two-sided diagrams leading to the coherence between the dressed states $|\Psi_+\rangle$ and $|\Psi_-\rangle$.

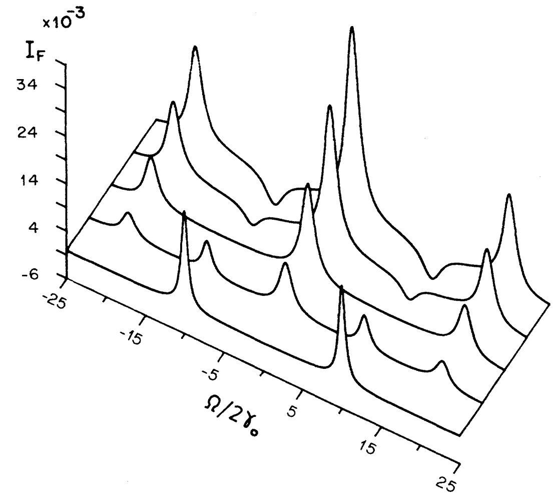


FIG. 2. The cosine component of the signal $I_F$ as a function of $\Omega/2\gamma_0$ and for $g\sqrt{N}/2\gamma_0=10$. The $\Gamma/\gamma_0$ values for curves from bottom to top are 0.0, 0.5, 1.0, 1.5, and 2.

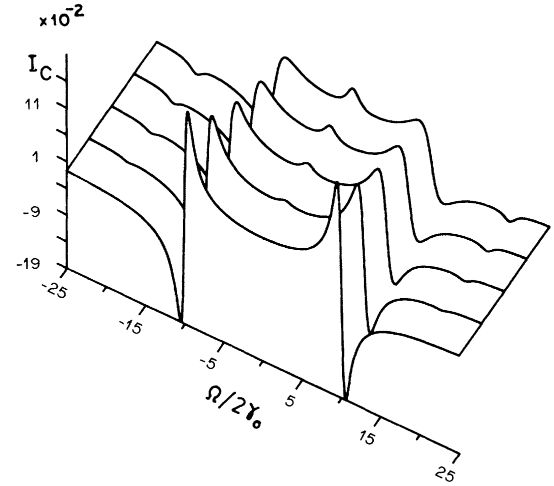


FIG. 3. The cosine component of the signal $I_C$ as a function of $\Omega/2\gamma_0$ for the same parameters as in Fig. 2.

$$\rho_{++}+\rho_{--}=G^2Nme^{-i\Omega t}\left[[\gamma_0+\Gamma+i(g\sqrt{N}-\Omega)]^{-1}(\gamma_0+\Gamma-ig\sqrt{N})^{-1}\left(1+\frac{2\Gamma}{2\gamma_0-i\Omega}\right)+(g\rightarrow -g)\right]+\text{c.c.}\,, \tag{15}$$

$$\rho_{+-}(\pm\Omega)=G^2Nm(\gamma_0+\Gamma+ig\sqrt{N})^{-1}(\gamma_0+\Gamma+ig\sqrt{N}\mp i\Omega)^{-1}\left(1+\frac{2\Gamma}{\mp i\Omega+2\gamma_0+2ig\sqrt{N}}\right). \tag{16}$$

We have thus shown that a nonzero $\Gamma$ leads to additional vacuum-field Rabi splittings at $\Omega=0$, $\pm 2g\sqrt{N}$. Note further that the widths of these additional Rabi splittings do not depend on $\Gamma$. The peak values compared to the background values are of the order

$$1+\frac{\Gamma}{\gamma_0}=1+\frac{2\Gamma}{\gamma+\kappa}\sim 1+\left(\frac{2\Gamma}{\gamma}\right), \text{ if } \gamma\gg\kappa\,. \tag{17}$$

The background is of the order of $G^2mN/g^2N$. Thus these are easily observable. Note further that for the Na experiment $2\Gamma/\gamma\sim 1$ if the phase noise is of the order of 5 MHz.

The analysis shows that we have produced a coherence between the dressed states $|\psi_+\rangle$ and $|\psi_-\rangle$. Note that the dressed states $|\psi_+\rangle$ and $|\psi_-\rangle$ are connected neither by the atomic dipole moment operator, nor by the mode operators $a$ and $a^\dagger$. We thus find the *dephasing induced phenomena*[10] *in cavity QED.* The mathematical similarity of our fundamental equations (6) and (10) to the usual Hanle system corresponding to the transition $j=0$ to $j=1$ may be noted. The dressed states $|\psi_\pm\rangle$ can be thought of as the analog of the Zeeman states. The role of the magnetic field is played by the coupling between the atom and the cavity mode. This formal analogy enables us to understand the origin of the additional vacuum-field Rabi splittings in the same way[11-13] as collision- or fluctuation-induced resonances. One can consider the dressed states of the system consisting of the atoms, cavity mode, and the modes of the external field. Thus one has to introduce the dressing of the states $|\psi_\pm\rangle$ and $|\psi_0\rangle$ (which are already dressed states of the atom cavity system) by the external fields. One can then show that the stochastic fluctuation $\phi(t)$ leads to the creation of coherence among the states formed by the dressing due to external fields. These coherences lead to additional vacuum-field Rabi splittings at $\Omega=\pm 2g\sqrt{N}$. Similarly one can show that the resonance at $\Omega=0$ arises from the fluctuation-induced population transfer. We show the actual signals, i.e., the cosine component of the modulated signals $I_F$ and $I_C$ in Figs. 2 and 3. Figure 2 shows how the spectral features of the signal change with the addition of phase noise on the pump. For $\Gamma=0$, we have the usual vacuum-field Rabi splittings. As $\Gamma$ increases, the new features start appearing at $\Omega=0$, $\pm 2g\sqrt{N}$ which become quite pronounced with further increase in $\Gamma$. The cosine component of the signal $I_C$ exhibits dispersionlike character. The additional resonance in the cosine component of $I_C$ is also clearly seen. These results can be generalized to treat the off resonant situations.

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[13]In this work I have explained the additional splittings by using the methods as used in the context of dephasing induced phenomena (Refs. 10 and 11). Note that vacuum-field Rabi splittings can be understood in terms of the coupled oscillator model [Ref. 5 and H. J. Carmichael, Phys. Rev. A **33**, 3262 (1986)] though historically these were introduced (Refs. 1 and 2) using QED methods. A colleague has shown me how the results of this paper can be understood in terms of a coupled oscillator system driven by a stochastic field.