Near-threshold photodetachment of H^- in parallel and crossed electric and magnetic fields

I. I. Fabrikant

Department of Physics and Astronomy, University of Nebraska, Lincoln, Nebraska 68588-0111

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Photodetachment of negative H^- ions in parallel and crossed static electric and magnetic fields is considered for the case of photon polarization parallel to the electric field. The final-state interaction between the electron and the H atom is included in the framework of the zero-range-potential approximation combined with the frame transformation theory. The cross section shows modulations caused by the electric field that are enhanced by the magnetic field. Singular behavior of the cross section at Landau thresholds due to the magnetic field can be strongly suppressed by a weak electric field. Final-state interaction effects in the case under consideration are significant for very strong magnetic fields $B \geq 60$ T.

I. INTRODUCTION

The one-photon photodetachment of negative ions in external static electric and magnetic fields is studied very intensively both theoretically and experimentally. Many interesting effects associated with this process have been observed. Some of them are due to the influence of the external field on the initial state (e.g., Zeeman splitting) and some are due to the interaction of the electron with the external field in the final state. Effects of the second type were observed for both magnetic and electric fields. In the case of a magnetic field the observed structure[1–4] is caused by the Landau resonances. The theory was given by Blumberg, Itano, and Larson,[5] Clark,[6] Gurvich and Zil'bermints,[7] Larson and Stoneman,[2] Greene,[8] and Crawford.[9] In the case of a pure electric field a "ripple-type" structure in the photodetachment of H^- was observed[10] which is due to the interference between the electron wave reflected by the potential barrier formed by the electric field and the unreflected wave.[11] The quantitative theory of this effect was given by Slonim and Dalidchik,[12] Reinhardt (see Ref. 10), Rau and Wong,[13] and Du and Delos.[14]

Another important effect in the case of a pure electric field is rescattering of the electron by the atomic residue due to reflection of the electron by the potential barrier. It was shown[12,15] that observation of this effect may be a good tool for the spectroscopic determination of scattering parameters, such as the scattering length and phase shifts. The first calculation[16] of this effect for the photodetachment of the H^- ion showed that it is noticeable for high electric fields, $F \geq 1$ MV/cm. This is due to the p-wave symmetry of the final-state wave function near the origin, which is not affected by the short-range potential of the residue. Thus, strong l mixing due to the external field is necessary to obtain rescattering.

Both interference and rescattering effects may be enhanced by the presence of an external magnetic field. For example, if the external magnetic field is parallel to the electric field the problem becomes more one dimensional and in this case both interference and rescattering effects may be more pronounced. The process of photodetachment in parallel fields was recently considered by Du,[17] who followed the transition from the case of a pure electric field ($B=0$) to that of a pure magnetic field ($F=0$). The final-state interaction of the electron with the atomic residue was not included.

The case of perpendicular fields was considered by Blumberg, Itano, and Larson.[5] They were interested in relatively small electric fields randomly distributed in both direction and absolute value arising from the thermal motion of the negative ions in the presence of the magnetic field in the laboratory frame. This motional electric field together with the Doppler effect led to a broadening of the peaks in the photodetachment cross sections.

If the negative ion velocity is very large, i.e., comparable with the velocity of light as in the Los Alamos experiment,[10] the motional electric field dominates the process and the magnetic field in the ion frame may be neglected.[10]

In the present paper we will be mostly interested in the case when the dynamical structures due to both the electric and the magnetic fields are important. We will not be interested in the influence of the external fields on the initial state and consider therefore H^- ions which do not experience the Zeeman splitting in the ground state. The diamagnetic shift of the H^- ground state can be estimated[18] in the zero-range-potential approximation and turns out to be negligible for all magnetic-field strengths considered below. We will consider photodetachment of H^- ions in parallel and perpendicular fields using the ideas of the frame transformation theory.[19] We will study in detail the influence of the magnetic field on the interference and rescattering effects. We will consider the case of photon polarization parallel to the electric field since the effects are more pronounced in this case.[15,16]

II. PHOTODETACHMENT IN PARALLEL FIELDS

A. Theory

Let us consider photodetachment of an s electron in parallel fields for the photon polarization parallel to the fields. We will follow closely the approach suggested by Du,[17] but include the final-state interaction in the same way as was done for the case of a pure electric field.[16]

The photodetachment cross section can be written as (in a.u.)

$$\sigma=\frac{4\pi^2\omega}{c}\sum_n\int d\frac{q^2}{2}|(\mathbf{e}\cdot\mathbf{r})_{ni}|^2\delta\left[\varepsilon_n+\frac{q^2}{2}-E\right] \quad (1)$$

$$=\frac{4\pi^2\omega}{c}\sum_n|(\mathbf{e}\cdot\mathbf{r})_{ni}|^2_{q=q_n}\ , \quad (2)$$

where ω,c are the light frequency and velocity, $q^2/2$ is the energy of the z component of the detached electron, $q_n^2=2(E-\varepsilon_n)$, and

$$\varepsilon_n=\omega_B(n+\tfrac{1}{2}) \quad (3)$$

is the energy of Landau level; E is the total energy of the system, which is equal to the electron energy in the final state, and $(\mathbf{e}\cdot\mathbf{r})_{ni}$ is the dipole matrix element between the initial negative-ion state and the final electron state $\chi_n^{(-)}$ with incoming-wave boundary conditions. We assume that the interaction between the electron and the atom can be described by the zero-range potential. Then, as in the case of a pure electric field,[16] we have

$$\chi_n^{(-)}=\chi_n^{(0)}-2\pi a s_n G_E^{(-)}(\mathbf{r},0)\ , \quad (4)$$

where $G_E^{(-)}$ is the Green's function of the electron in the parallel fields, and

$$s_n=\frac{\chi_n^{(0)}(0)}{1+ay^*}\ , \quad (5)$$

$$y=2\pi\frac{\partial}{\partial r'}[r'G_E^{(+)}(\mathbf{r}',0)]_{r'=0}\ , \quad (6)$$

where $G^{(+)}=(G^{(-)})^*$, and a is the scattering length.

As in Refs. 8, 13, 16, and 17 we will use the frame transformation theory to evaluate the dipole matrix element. We assume that the wave functions near the origin, where integration has to be carried out, are not affected by the external fields. Then for the initial state we have

$$\psi_i=C\frac{e^{-r/a}}{r}\ . \quad (7)$$

In the case of photodetachment of an s electron and π polarization of the laser, the z component of the electron's angular momentum in the final state is 0. The function $\chi^{(0)}$ in cylindrical coordinates ρ,z can thus be written in the form

$$\chi_n^{(0)}=\frac{2^{1/3}}{(2\pi)^{1/2}F^{1/6}}\mathrm{Ai}(\xi^n)\omega_B^{1/2}e^{-\omega_B\rho^2/4}L_n\left[\frac{\omega_B\rho^2}{2}\right]\ , \quad (8)$$

where

$$\xi^n=-(2F)^{1/3}\left[z+\frac{q_n^2}{2F}\right]\ . \quad (9)$$

Here Ai is the regular Airy function and L_n is a Laguerre polynomial. We can complete the frame transformation for this function using the results of Rau and Wong[13] for Ai and that of Greene[8] for L_n. Since we are interested in the case of π polarization, we consider that part of $\chi_n^{(0)}$ which has odd parity under the transformation $z\to -z$:

$$\chi_n^{(\mathrm{odd})}\approx -3\left[\frac{\omega_B}{2\pi}\right]^{1/2}\frac{2^{2/3}F^{1/6}}{k}\mathrm{Ai}'(\xi_o^n)j_l(kr)\cos\theta\ , \quad (10)$$

where $\xi_0^n=-q_n^2/(2F)^{2/3}$, $k^2=2E$. The prime means the derivative with respect to ξ_0^n, and j_1 is a spherical Bessel function.

In order to complete the frame transformation for $G^{(-)}$ let us consider the spectral representation for $G^{(+)}$:

$$G^{(+)}(\rho,z)=\frac{\omega_B}{(2F)^{1/3}}\sum_n e^{-\omega_B\rho^2/4}L_n\left[\frac{\omega_B\rho^2}{2}\right]\mathrm{Ai}(\xi_>^n)\times\mathrm{Ci}(\xi_<^n)\ , \quad (11)$$

where $\xi_>^n=\max(\xi^n,\xi_0^n)$, $\xi_<^n=\min(\xi^n,\xi_0^n)$, and $\mathrm{Ci}(\xi)$ is the irregular Airy function with the asymptotic form of an outgoing wave.

Near the origin the z-dependent term in Eq. (11) has the form

$$\mathrm{Ai}(\xi_>^n)\mathrm{Ci}(\xi_<^n)\approx a^{\pm}\sin q_n z+b\cos q_n z\ , \quad (12)$$

where the coefficients $a^{\pm},b$ can be obtained from the matching conditions. We obtain

$$a^+=-\frac{(2F)^{1/3}}{q_n}\mathrm{Ai}(\xi_0^n)\mathrm{Ci}'(\xi_0^n)\ , \quad (13)$$

$$a^-=-\frac{(2F)^{1/3}}{q_n}\mathrm{Ai}'(\xi_0^n)\mathrm{Ci}(\xi_0^n)\ , \quad (14)$$

where we employ a^+ for $z>0$ and a^- for $z<0$, and

$$b=\mathrm{Ai}(\xi_0^n)\mathrm{Ci}(\xi_0^n)\ . \quad (15)$$

For the ρ-dependent term we have the same expression near the origin as Greene[8] and Du,[17]

$$e^{-\omega_B\rho^2/4}L_n\left[\frac{\omega_B\rho^2}{2}\right]\approx J_0((2\varepsilon_n)^{1/2}\rho)\ . \quad (16)$$

Turning now to spherical coordinates and extracting the p-wave part from the Green's function $G^{(+)}$, we have

$$G^{(+)}(\rho,z)=-\frac{3\omega_B}{2k}j_1(kr)\cos\theta\times\sum_n\frac{d}{d\xi_0^n}[\mathrm{Ai}(\xi_0^n)\mathrm{Ci}(\xi_0^n)] \quad (17)$$

for the p-wave part.

Substituting now (5), (10), and (17) into (4), we have the following expression:

$$\chi_n^{(-)}\approx\left[\frac{\omega_B}{2\pi}\right]^{1/2}\frac{3\times 2^{2/3}F^{1/6}}{k}\times[-\mathrm{Ai}'(\xi_0^n)+g^*\mathrm{Ai}(\xi_0^n)]j_1(kr)\cos\theta \quad (18)$$

for the p-wave part, where

$$g=\frac{\pi a\omega_B}{(2F)^{1/3}(1+ay)}\sum_{n'}\frac{d}{d\xi_0^{n'}}[\mathrm{Ai}(\xi_0^{n'})\mathrm{Ci}(\xi_0^{n'})]\ . \tag{19}$$

The dipole matrix element evaluated with this final-state wave function has the form

$$(\mathbf{e}\cdot\mathbf{r})_{ni}=4C(2\pi\omega_B)^{1/2}\frac{2^{2/3}F^{1/6}}{(a^{-2}+k^2)^2}\times[-\mathrm{Ai}'(\xi_0^n)+g^*\mathrm{Ai}(\xi_0^n)]\ . \tag{20}$$

Substituting this matrix element into the expression (2) for the cross section, the result is

$$\sigma=\sigma_0\frac{6\pi\omega_B(2F)^{1/3}}{k^3}\sum_{n=0}^{\infty}|-\mathrm{Ai}'(\xi_0^n)+g\ \mathrm{Ai}(\xi_0^n)|^2\ , \tag{21}$$

where σ_0 is the cross section in the absence of the fields.

In order to complete calculations according to Eq. (21), we have to evaluate y defined by the expression (6). As in the case of a pure electric field,[16] we start from the Feynman expression[20] for the propagator (the time-dependent Green's function) and get

$$y=ik+(2\pi i)^{-1/2}\int_0^{\infty}\left\{\frac{\omega_B\exp[i(Et-\frac{1}{24}F^2t^3)]}{2t^{1/2}\sin(\omega_B t/2)}-\frac{\exp(iEt)}{t^{3/2}}\right\}dt\ , \tag{22}$$

where the principal value of the integral should be taken.

Then we deform the path of integration in order to have it pass through the saddle point,

$$t_s=\frac{2k}{F}\ , \tag{23}$$

in the direction of steepest descent,

$$\arg(t-t_s)=-\frac{\pi}{4}\ . \tag{24}$$

After this deformation of the path we have to add the contributions of the poles of the integrand lying on the real axis

$$t_l=\frac{2\pi l}{\omega_B},\quad l=1,2,\ldots\ . \tag{25}$$

The contribution of each pole is defined by the expression

$$y_l=\pm\pi i(-1)^l\frac{\exp[i(Et_l-\frac{1}{24}F^2t_l^3)]}{(2\pi i t_l)^{1/2}}\ , \tag{26}$$

and we have a plus sign for $t_l<t_s$ and a minus sign for $t_l>t_s$. In order to eliminate the singularity $t^{-1/2}$ near the origin it is useful to perform an integration by parts as in the case $B=0$.[16] Then the integration can be carried out easily with ordinary computer precision.

B. Discussion and calculations

Analysis of the expressions (19) and (21) allows one to make some conclusions about the relative importance of the magnetic field, the electric field, and rescattering effects. If the parameter

$$\gamma=\frac{\omega_B}{(2F)^{2/3}} \tag{27}$$

is small, ξ_0^n in the sums of Eqs. (19) and (21) may be considered as a continuous variable. Making the substitution

$$\sum_n\rightarrow\int_0^{\infty}dn=\frac{1}{\omega_B}\int_{-\infty}^{k^2/2}d\frac{q^2}{2}\ , \tag{28}$$

we obtain exactly the same result as in the $B=0$ case.[16] This limit was obtained by Du[17] for the case $g=0$.

Thus the case of small γ is essentially equivalent to the case of a pure electric field, i.e., the cross section has small ripples, which can be described by a periodic function of $\beta=2k^3/3F$. Therefore we can define the characteristic scale of the electric-field-induced oscillations near threshold as

$$E_{\mathrm{osc}}=\tfrac{1}{2}(3\pi F)^{2/3}\ . \tag{29}$$

If $\gamma\geq 1$ we can expect that the modulations become deep, especially between the first and second Landau thresholds. Physically it means that due to the presence of the magnetic field the problem becomes effectively one dimensional. Figures 1 and 2 illustrate this point.

The importance of the rescattering effect according to expression (19) can be estimated by evaluating the parameter

$$\delta=\frac{a\omega_B}{(2F)^{1/3}}\ . \tag{30}$$

In order to have an essential influence of the magnetic field on the rescattering effect, we should have $\delta\geq 1$ or

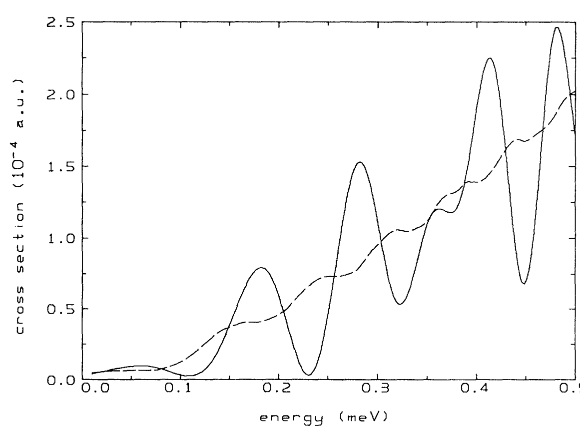

FIG. 1. Photodetachment cross section for H^- in parallel fields of $B=1$ T and $F=20$ V/cm. The corresponding cyclotron frequency is $\omega_B=0.117$ meV/$\hbar$ and the scale of electric-field-induced oscillations defined by Eq. (29) is $E_{\mathrm{osc}}=0.072$ meV. Dashed curve, the case of a pure electric field of 20 V/cm. The photon polarization is parallel to the fields.

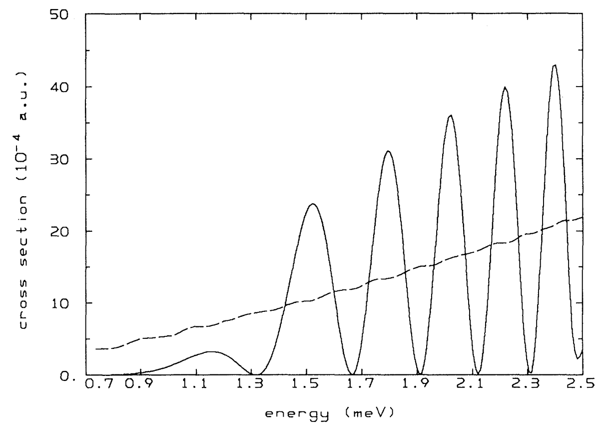

FIG. 2. The same as in Fig. 1 for $B=20$ T, $F=100$ V/cm ($\omega_B=2.3$ meV/$\hbar$, $E_{\rm osc}=0.21$ meV).

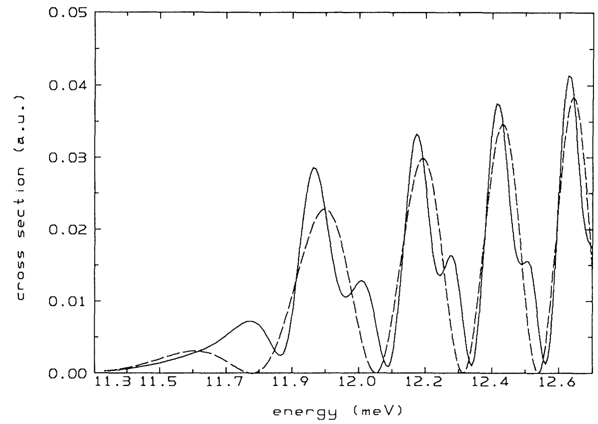

FIG. 4. The same as in Fig. 3 for $B=200$ T, $F=100$ V/cm.

$$\omega_B \simeq \frac{(2F)^{1/3}}{a} \ . \tag{31}$$

Earlier we obtained[16] that at $B=0$ the rescattering effect is noticeable for hydrogen if $F \geq 1$ MV/cm. According to (31) this leads to $B \approx 4\times 10^3$ T. Since such a high field is unrealistic for laboratory experiments, we will consider here smaller electric fields. However, the electric field should not be too small since the oscillation factor β should not be too large. Otherwise, finite experimental energy resolution would not allow the observation of the electric-field effects.

Some examples are presented in Figs. 3 and 4. The dashed curves in the figures do not include the rescattering effect and represent the results of the theory developed by Du.[17] We see that the rescattering effect becomes noticeable at very high magnetic fields of about

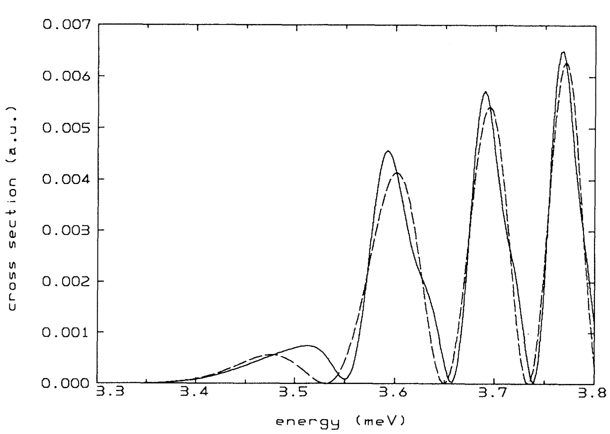

FIG. 3. Photodetachment cross section in parallel fields of $B=60$ T and $F=30$ V/cm. Solid curve and dashed curve, with and without the inclusion of the final-state interaction between the electron and the atom, respectively.

60 T. For lower fields the results of our theory and the theory of Du are essentially the same.

The region between the first and second Landau thresholds is of special interest. From Eq. (21) we see that at an energy defined by the equation

$$\mathrm{Ai}'(\xi_0^0)=0 \tag{32}$$

the cross section turns out to be zero if we do not take into account the rescattering term. (The contribution of the terms with $n>0$ is negligible well below the second Landau threshold.) The inclusion of the rescattering effect makes the cross section nonzero at any energy since $-\mathrm{Ai}'+g\,\mathrm{Ai}$ is a complex function of the real argument E. The cross section at the energy defined by Eq. (32) is proportional to the rescattering factor $|g|^2$. Note that the denominator in the expression (19) for g strongly reduces this factor and it is not so large as we would expect by putting $y=0$ in Eq. (19).

III. PHOTODETACHMENT IN PERPENDICULAR FIELDS

A. Theory

The essential feature of this case is that the Schrödinger equation is not separable in the cylindrical coordinates. However, the variables can be separated in Cartesian coordinates and this property was used by Blumberg, Itano, an Larson[5] in order to take into account the effect of the motional Stark field for the process of photodetachment of S$^-$ ions in a magnetic field.

We start here from the same approach and consider first a free electron in perpendicular fields. Choosing the vector potential in the form[21]

$$A_x=-B_y, \quad A_y=A_z=0 \ , \tag{33}$$

we obtain the magnetic field in the z direction. Choosing then the scalar potential in the form

$$\phi=-Fy \ , \tag{34}$$

we obtain the electric field in the y direction.

A solution of the Schrödinger equation

$$\frac{1}{2}\left[\left[\omega_B y-i\frac{\partial}{\partial x}\right]^2-2Fy-\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial z^2}\right]\chi^{(0)}=E\chi^{(0)} \quad (35)$$

can be written now in the form

$$\chi_n^{(0)}=\frac{e^{ip_x x}}{(2\pi)^{1/2}}\frac{e^{ip_z z}}{(2\pi p_z)^{1/2}}f_n(y)\ , \quad (36)$$

$$f_n(y)=\left[\frac{\omega_B}{\pi}\right]^{1/4}\frac{1}{(2^n n!)^{1/2}}\exp\left[-\frac{\omega_B}{2}(y-y_0)^2\right]\times H_n[\omega_B^{1/2}(y-y_0)]\ , \quad (37)$$

where H_n is a Hermite polynomial and

$$y_0=-\frac{p_x}{\omega_B}+\frac{F}{\omega_B^2}\ . \quad (38)$$

The x-dependent part of the wave function is normalized to $\delta(p_x-p_x')$ and the z-dependent part to $\delta(p_z^2/2-p_z'^2/2)$. With this normalization the total photodetachment cross section can be written in the form

$$\sigma=\frac{4\pi^2\omega}{c}2\sum_n\int dp_x|(\mathbf{e}\cdot\mathbf{r})_{fi}|^2\ , \quad (39)$$

where the matrix element is taken at

$$\frac{p_z^2}{2}=E-\varepsilon_n-\frac{p_xF}{\omega_B}+\frac{F^2}{2\omega_B^2}>0\ . \quad (40)$$

The relation (40) defines the upper limit of the integration over p_x for a given n. The additional factor 2 in the expression (39) corresponds to the two possible signs of p_z in the final state.

As in the case of parallel fields, let us complete first the frame transformation for $\chi^{(0)}$. We will be interested in the case when the photon polarization is parallel to the electric field. In this case the final-state wave function should be odd under the transformation $y\rightarrow -y$, and we get near the origin

$$\chi_n^{(\mathrm{odd})}\approx\frac{f_n'(0)}{p_y}\sin p_y y\frac{e^{ip_x x}}{(2\pi)^{1/2}}\frac{e^{ip_z z}}{(2\pi p_z)^{1/2}}\ , \quad (41)$$

where

$$p_y^2=2\varepsilon_n+\frac{2p_xF}{\omega_B}-\frac{F^2}{\omega^2}-p_x^2\ . \quad (42)$$

With the inclusion of the final-state interaction of the electron with the atomic residue, we have instead of (36)

$$\chi_n^{(-)}=\chi_n^{(0)}-\frac{2\pi a\chi_n^{(0)}(0)}{1+au^*}G^{(-)}(\mathbf{r},0)\ , \quad (43)$$

where u is defined by the same expression as y in the case of parallel fields.

Let us complete now the frame transformation for $G^{(-)}$. As in the case of parallel fields, we start from the spectral representation of $G^{(+)}$:

$$G_E^{(+)}(\mathbf{r},0)=\frac{1}{2\pi}\sum_n\int dp_x f_n(y)f_n(0)e^{ip_x x}g_{E_n(p_x)}(z)\ , \quad (44)$$

where

$$E_n(p_x)=E-\varepsilon_n-\frac{p_xF}{\omega_B}+\frac{F^2}{2\omega_B^2} \quad (45)$$

and

$$g_E(z)=\frac{i}{\sqrt{2E}}\exp(i\sqrt{2E}\,|z|)\ , \quad (46)$$

where for $E<0$ the value of $\sqrt{2E}$ lies on the positive imaginary axis.

Near the origin for the odd part of $G_E^{(+)}$ (with respect to $y\rightarrow -y$) we have

$$G_E^{(\mathrm{odd})}(\mathbf{r},0)=\frac{i}{2\pi}\int dp_x\frac{f_n(0)f_n'(0)}{\sqrt{2E_n(p_x)}}\frac{\sin p_y y}{p_y}\times e^{ip_x+i\sqrt{2E_n(p_x)}z}\ . \quad (47)$$

Let us rewrite the expression (47) in spherical coordinates and extract the p-wave part:

$$G_E^{(p)}(\mathbf{r},0)\approx\frac{3i\sin\theta\sin\varphi}{2\pi k}hj_1(kr)\ , \quad (48)$$

$$h=\sum_n\int_{-\infty}^{\infty}dp_x\frac{f_n(0)f_n'(0)}{\sqrt{2E_n(p_x)}}\ . \quad (49)$$

We combine now the expressions (41), (43), and (48) in order to evaluate the matrix element of the dipole operator for the case when the photon polarization is parallel to the y axis. We obtain

$$(\mathbf{e}\cdot\mathbf{r})_{fi}=\frac{8\pi C}{(\pi p_z)^{1/2}(a^{-2}+k^2)^2}\left[f_n'(0)+\frac{iaf_n(0)h^*}{1+au^*}\right]\ . \quad (50)$$

Substituting this result into (37) we finally obtain

$$\sigma=\frac{3\sigma_0}{k^3}\sum_n\int_{-\infty}^{(\omega_B/F)[E-\varepsilon_n+(F^2/2\omega_B^2)]}dp_x[2E_n(p_x)]^{-1/2}\left|f_n'(0)+ia\frac{f_n(0)h^*}{1+au^*}\right|^2\ . \quad (51)$$

Using the explicit expressions for $f_n(0)$ and $f_n'(0)$ following from (37), it is convenient to define a new variable,

$$\zeta=\omega_B^{1/2}y_0=\omega_B^{1/2}\left[-\frac{p_x}{\omega_B}+\frac{F}{\omega_B^2}\right]\ . \quad (52)$$

Then

$$\sigma=\frac{3\sigma_0}{k^3}\frac{\omega_B^{9/4}}{(2F)^{1/2}}\sum_n\int_{\zeta_n}^{\infty}d\zeta(\zeta-\zeta_n)^{-1/2}\left|\frac{d\bar{f}_n}{d\zeta}-\frac{iah^*}{1+au^*}\bar{f}_n\right|^2 , \tag{53}$$

where

$$\zeta_n=\frac{F}{2\omega_B^{3/2}}-(E-\varepsilon_n)\frac{\omega_B^{1/2}}{F} , \tag{54}$$

$$\bar{f}_n(\zeta)=\pi^{-1/4}(2^n n!)^{-1/2}e^{-\zeta^2/2}H_n(\zeta) . \tag{55}$$

B. Discussion and calculations

The integrand in (53) falls off very rapidly for $|\zeta|$ exceeding the classical turning point $(2n+1)^{1/2}$. Therefore the upper limit of integration is effectively given by

$$\zeta_{\max}=A(2n+1)^{1/2} , \tag{56}$$

where A is some reasonably large number. The upper limit of n values which contribute essentially to the sum in Eq. (53) may be obtained from the equation

$$\zeta_n=A(2n+1)^{1/2} . \tag{57}$$

If $n_{\max}$ obtained from Eq. (57) is not very large ($n_{\max}\leq 10$), the evaluation of the sum of Eq. (53) does not present any difficulties. However, for large n, $\bar{f}_n(\zeta)$ becomes a very rapidly oscillating function and the standard expression for the Hermite polynomials fails to give the correct numerical result. In this case we should use the quasiclassical expression for $\bar{f}_n$:

$$\bar{f}_n=\left[\frac{2}{\pi p(\zeta)}\right]^{1/2}\sin\left[\int_{\zeta}^{\sqrt{2n+1}}p(\zeta')d\zeta'+\frac{\pi}{4}\right] , \tag{58}$$

where

$$p(\zeta)=(2n+1-\zeta^2)^{1/2} . \tag{59}$$

Using the expression (58), the integrand can be averaged over the rapid oscillations. In particular, for the most important integral entering Eq. (53) we have

$$\int_{\zeta_n}^{\infty}d\zeta\frac{|d\bar{f}_n/d\zeta|^2}{(\zeta-\zeta_n)^{1/2}}\approx\frac{1}{\pi}\int_{\max(\zeta_n,-(2n+1)^{1/2})}^{(2n+1)^{1/2}}\left[\frac{2n+1-\zeta^2}{\zeta-\zeta_n}\right]^{1/2}d\zeta \tag{60}$$

and the crossed term with the integrand proportional to $\bar{f}_n(d\bar{f}_n/d\zeta)$, equals 0 in this approximation.

If ζ_n is close to the right turning point, $(2n+1)^{1/2}$, the expression (58) may fail to give the correct result. In this case a more precise representation of $\bar{f}_n$ through the Airy function can be used. However, in general, these particular cases give a small contribution to the sum of Eq. (53) and may be ignored.

As similar approach can be used to evaluate the sum of Eq. (49). It can be rewritten in the form

$$h=\frac{\omega_B^{5/4}}{(2\pi F)^{1/2}}\sum_n\int_{-\infty}^{\infty}d\zeta\frac{\bar{f}_n(d\bar{f}_n/d\zeta)}{(\zeta-\zeta_n)^{1/2}} . \tag{61}$$

We see that large n do not contribute to the sum since the term $\bar{f}_n(d\bar{f}_n/d\zeta)$ averaged over the oscillations is equal to 0. But even for small n the integral in Eq. (49) is non-negligible if only ζ_n is close to 0 since $\bar{f}_n(d\bar{f}_n/d\zeta)$ is an odd function of ζ. So near the first Landau threshold we should have

$$\zeta_0=\left|\frac{F}{2\omega_B^{3/2}}-\left[E-\frac{\omega_B}{2}\right]\frac{\omega_B^{1/2}}{F}\right|\leq 1 . \tag{62}$$

From (51) and (61) we also can see that in order to get a noticeable rescattering effect the parameter

$$\eta=\frac{a\omega_B^{5/4}}{(2\pi F)^{1/2}} \tag{63}$$

should be non-negligible. The latter requirement leads approximately to the same restrictions on the F and B fields as in the case of parallel fields. For example, in the case $F=20$ V/cm, $B=60$ T we have $\eta=0.877$. However, the condition (62), which in this case is equivalent to

$$\left|E-\frac{\omega_B}{2}\right|\leq\frac{F}{\omega_B^{1/2}} , \tag{64}$$

leads to a very small energy region near the Landau threshold where the rescattering effect is noticeable. In the case $F=20$ V/cm, $B=60$ T we have

$$\left|E-\frac{\omega_B}{2}\right|\leq 0.0066 \text{ meV} . \tag{65}$$

So in the case of parallel fields, the rescattering effect is a more universal feature since it appears in a much wider energy range.

We will not discuss the rescattering effect for the case of perpendicular fields any more but turn to another interesting feature observable at Landau thresholds. Let us consider the cross section (53) in the limit of zero electric field. According to (54) we have in this limit

$$\begin{aligned}\zeta_n&=-\infty, \quad E>\varepsilon_n\\ \zeta_n&=\infty, \quad E<\varepsilon_n\end{aligned} \tag{66}$$

and only open Landau channels contribute to the sum of Eq. (53). Hence, in the case $h=0$ we have

$$\sigma=\frac{3\sigma_0\omega_B^2}{k^3}\sum_{n=0}^{n_{\max}}[2(E-\varepsilon_n)]^{-1/2}\int_{-\infty}^{\infty}d\zeta\left|\frac{d\bar{f}_n}{d\zeta}\right|^2 \tag{67}$$

and σ becomes infinite at each Landau threshold, as was shown by Blumberg and co-workers.[1,5] Clark[6] and Crawford[9] pointed out that the cross section becomes finite with the inclusion of the final-state interaction of the electron with the atomic residue. However, in our case this inclusion (making $h \neq 0$) does not make the cross section finite. The point here is that for $F=0$ the system acquires cylindrical symmetry and the z component m of the angular momentum becomes a good quantum number. Since we are considering the case of laser polarization perpendicular to the z axis, $|m|=1$ in our case and the zero-range potential does not affect the final state. In order to make the cross section finite at Landau thresholds we have to include scattering for higher partial waves (with $l \geq 1$). However, from Eq. (53) we see that in the case $F \neq 0$ the cross section is finite everywhere even in the framework of the zero-range-potential approximation. The same effect can be observed in the case of parallel fields when the photon polarization is perpendicular to the fields.[17]

It is remarkable that even a very small electric field strongly reduces the threshold singularity. Figure 5 illustrates this point. We have chosen a small electric field of 2V/cm and 7 V/cm. The latter value corresponds to the experimental conditions discussed in Refs. 1 and 5 when the electric field appears due to the thermal motion of the negative ions with the most probable velocity $v_0 = 7 \times 10^4$ cm/s. However, it should be mentioned that the thermal motion leads to a random distribution of the electric field within the limits from 0 to about 10 V/cm corresponding to a random orientation of the velocity vector and Maxwell distribution of v. The calculations of Ref. 5 include the averaging over these distributions and also take into account the photon frequency distribution due to the Doppler effect. Such averaging leads to disappearance of the oscillatory structure due to the electric field. In our case in which the electric field and the photon frequency are fixed, we obtain this structure, which is more pronounced than in the case $B=0$. In Fig. 6 the comparison of two cases $B=1$ T, $F=15$ V/cm and $B=0$, $F=15$ V/cm is presented. The electric-field-induced modulations are deeper in the case $B \neq 0$, although they are not as deep as in the case of parallel fields. For $F=50$ V/cm the magnetic field of 1 T almost does not influence the structure and we again have "ripples" instead of deep modulations.

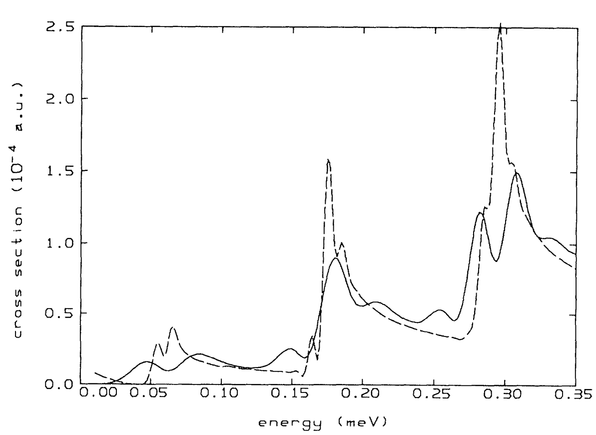

FIG. 5. Photodetachment cross section in perpendicular fields. Solid curve, $B=1$ T and $F=7$ V/cm. Dashed curve, $B=1$ T and $F=2$ V/cm. The photon polarization is parallel to the electric field.

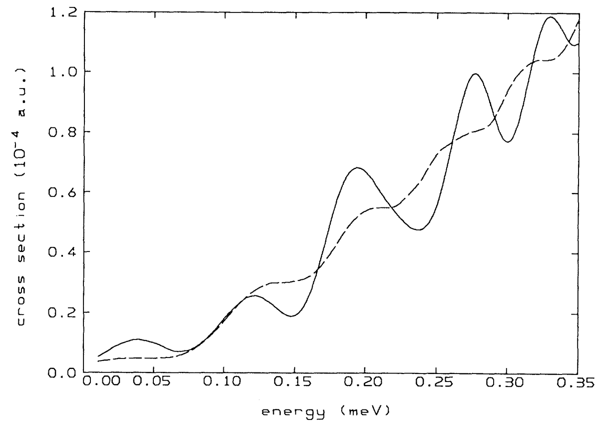

FIG. 6. The same as in Fig. 5 for $F=15$ V/cm. Dashed curve, the case of a pure electric field of 15 V/cm.

It should be mentioned in conclusion that the values of the electric field discussed here could not be obtained as a motional electric field in a beam experiment, since the cyclotron radius of such motion is too small. For example, it equals 1.5×10^{-3} cm for H^- and $v = 1.5 \times 10^5$ cm/s (corresponding to $F=15$ V/cm). Therefore in a real beam experiment one has to obtain a negative-ion beam propagating in the direction of the magnetic field with the external electric field imposed in the perpendicular direction.

Another option is to deal with a relatively fast ion beam moving in the direction perpendicular to the magnetic field. But in this case the motional electric field would be larger than that considered above. For example, Krause[4] deals with a 1.3-keV O^- beam in a magnetic field of $B=0.126$ T. The corresponding motional electric field is 155 V/cm. Krause eliminates this field using the $\mathbf{E} \times \mathbf{B}$ filter. Without using this filter the magnetic-field effects would be completely suppressed by the motional electric field (as in the Los Alamos experiment[10]). In order to observe effects due to both fields, the motional electric field should be strongly reduced but not eliminated completely.

IV. LIMITATIONS OF THE MODEL

We have presented here the results of calculations of photodetachment of H^- in parallel and crossed electric and magnetic fields using the zero-range-potential approximation for the electron-atom interaction. This model needs modifications when dealing with the $l \neq 0$ electron in the initial state, since the zero-range potential cannot bind the $l \neq 0$ electron. Such modifications were discussed by Wong, Rau, and Greene[22] for the case of a

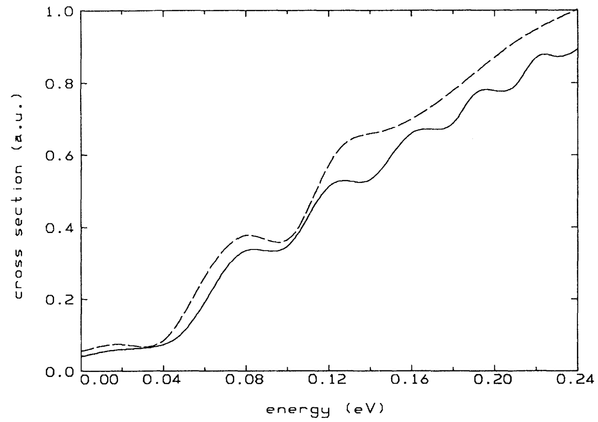

FIG. 7. Comparison of our results (solid curve) for $B=0$, $F=232$ kV/cm with the results of the many-electron, many-photon theory of Mercouris and Nicolaides (Ref. 24) (dashed curve).

pure electric field and by Gurvich and Zil'bermints[7] and Crawford[9] for the case of a pure magnetic field. Another important modification in this case has to do with the Zeeman splitting for the initial state which was taken into account by Blumberg, Itano, and Larson[5] for the process of photodetachment of the S$^-$ ion in a magnetic field.

The limitations of our model also include the neglect of the electron correlations and the strong-laser-field effect. Both effects were taken into account in the many-electron, many-photon theory recently developed by Nicolaides and Mercouris.[23,24] In order to estimate the importance of the electron correlations and the multiphoton effect we present in Fig. 7 comparison of our results with the results of Mercouris and Nicolaides[24] for the one-photon detachment of H$^-$ in an electric field of 232 kV/cm. Mercouris and Nicolaides present the one-photon ionization rate W as a function of the photon frequency. Using their ac field peak intensity $I=3.5\times10^{10}$ W/cm^2 we can calculate the corresponding photodetachment cross section

$$\sigma=\frac{2\hbar\omega}{I}W . \tag{68}$$

The agreement is reasonable, although the reason of the suppression of the electric-field-induced oscillations at higher energies in *ab initio* calculations[24] is unclear. The difference in the absolute magnitudes of the cross sections indicates the role of the electron correlations and the strong-laser-field effect.

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