Experimental investigation of the collision of Feigenbaum cascades in lasers

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We have experimentally checked on a CO_2 laser with modulated losses the universality law set by Oppo and Politi [Phys. Rev. A **30**, 435 (1984)] for the collision of Feigenbaum cascades. The scaling properties of the first three period-doubling bifurcations are in good agreement with their predictions, but fast passage effects and a limited signal-to-noise ratio impose severe limits on precise quantitative measurements.

The period-doubling cascade has appeared as the most popular scenario of transitions towards chaos since it may be easily identified and occurs in many nonlinear systems.[1] One of the most striking results is that the bifurcation points corresponding to successive bifurcations asymptotically follow a universal rule. Let μ_n be the value of the control parameter at the bifurcation from the regime with period $2^{n-1}T$ to that with period 2^nT. Feigenbaum stated that $(\mu_{n-1}-\mu_n)/(\mu_n-\mu_{n+1})=\delta_n$ asymptotically tends to a universal number $\delta=4.6692\ldots$.

In some dynamical systems, chaotic bands are delimited on both sides by period-doubling cascades and a second parameter allows one to control the width (domain of existence) of these chaotic bands. Oppo and Politi demonstrated that for any sufficiently small but nonzero chaotic window, the *first convergence rates* δ_n stay close to $\delta^{1/2}$ while at larger n's they asymptotically reach δ.[2] By changing a second parameter ν until the chaotic regime disappears ($\nu=\nu_c$), the number of bifurcations showing a rate $\delta^{1/2}$ diverges to infinity. They gave the theoretical function for the universal function $\delta_n(\nu-\nu_c)$ which describes the growth of the convergence rate from $\delta^{1/2}$ to δ versus n and $\nu-\nu_c$. This function which is valid for suitable small $\bar{\nu}>0$ and large enough n is given by

$$\delta_n(\bar{\nu})=\frac{\delta^{1/2}F(\bar{\nu}\delta^{n-1})-F(\bar{\nu}\delta^n)}{F(\bar{\nu}\delta^n)-\delta^{-1/2}F(\bar{\nu}\delta^{n+1})} ,$$

where

$$F(x)=\sqrt{1+x} \text{ and } \bar{\nu}=\nu-\nu_c .$$

At $\nu=\nu_c$, the curve representing the evolution of the control parameters in the (μ,ν) plane is tangential to the chaotic region.

For that purpose, we have used a CO_2 laser containing an amplitude modulation (AM) electro-optic modulator. This system was chosen since it displays colliding Feigenbaum cascades with an easy control of the collision. Moreover the high stability achieved in this system allows one to obtain reliable measurements of at least the first three bifurcation points.

It should be noticed that in spite of its universality, there have been relatively few experimental checks of the geometric convergence that was mostly exhibited on numerical simulations. In the experiments, because of technical noise, it is often very difficult to obtain periodic regimes with a period longer or even equal to 2^4 T, making the asymptotic check impossible.

A modified version of the laser with modulated losses has been used for these experiments.[3,4] An electro-optic modulator together with a ZnSe Brewster angle plate have been inserted in the laser cavity. The polarization state of the laser radiation changes according to the voltage applied to the electro-optic modulator and the Brewster plate acts as an output coupler for the part of the laser field perpendicular to the incidence plane.[5] The amplitudes of the dc and ac voltages applied to the modulator are the two control parameters.[6] Bifurcation diagrams have been recorded with the dc bias of the modulator as a control parameter while the modulation amplitude acts as the second parameter which controls the collision of the two cascades. The dc bias plays a double role: it increases the cavity losses but, as the response of the modulator is nonlinear, it also alters the sensitivity to ac modulation. The modulation frequency is chosen coincident with the resonance of the device of 640 kHz and periodic sampling of the laser output intensity at that frequency has been used to obtain bifurcation diagrams such as those reported in Fig. 1. In this kind of diagram, a periodic response synchronous to the modulation appears as a single value branch. When the laser responds with a period equal to n times the modulation period T, n different branches are obtained.

The presence of the $16T$ periodic regimes is an indication of the good stability of the laser, which is a necessary condition for experiments such as those considered here. By varying the modulation amplitude, it has been possible to draw a two-dimensional parameter space diagram of the dynamics of the laser as shown in Fig. 2. Bifurcation diagrams similar to those of Fig. 1 correspond to straight horizontal lines in this diagram. For a modulation amplitude V_{ac} larger than critical value $V_c=2.94$ V, chaos may be observed and for the largest modulation voltages used in our experiments, the two period-doubling cascades are decoupled. Experiments in which the bifurcation diagrams are recorded versus V_{ac} as the (main) control parameter would correspond to the

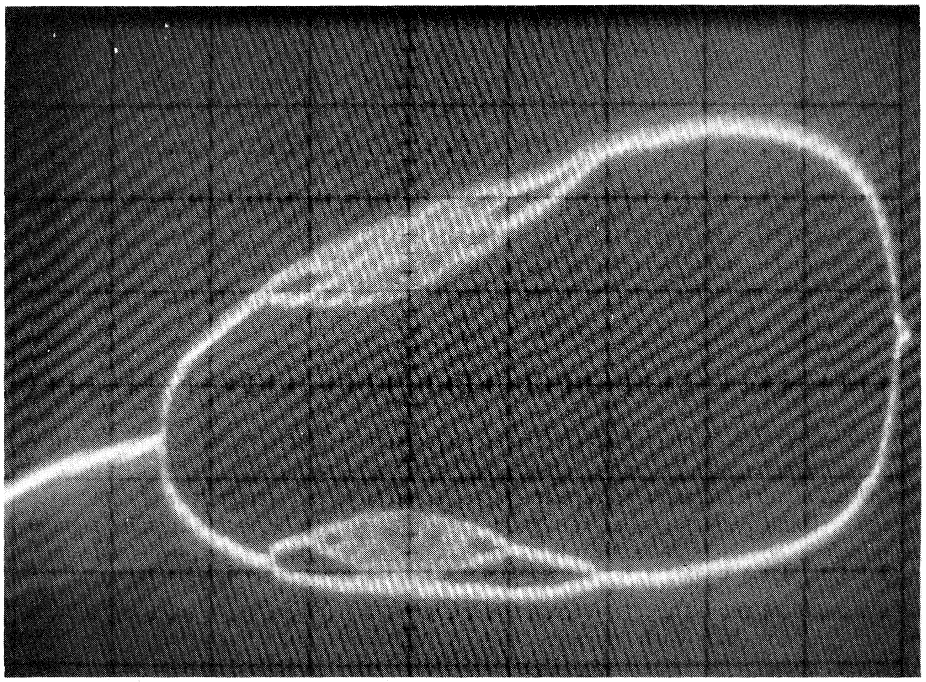

FIG. 1. Bifurcation diagram of the laser with modulated losses with the dc bias as control parameter ($60 < V_{dc} < 460$ V) and a fixed modulation ($V_{ac} \approx 3$ V).

straight vertical lines and would not lead to colliding cascades.

From bifurcation diagrams such as that of Fig. 1, it is very easy to extract the values V_n of the bias voltage corresponding to the bifurcation from 2^{n-1} to 2^nT periodic regimes and to deduce from sets of bifurcation voltages a value of δ_n. Unfortunately, in experiments it is hardly possible to explore more than three or four values of V_n, of which one or two values of δ_n can be extracted. Obviously the evolution of δ_n versus n cannot be considered on the basis of experimental data. Consequently, to check Oppo and Politi's theory, we have plotted δ_n versus $(V_{ac}-V_c)/V_c$ which is somehow equivalent to $\bar{\nu}$ of their theory and measures the offset from criticality. Figure 3 reports the comparison between the values of δ_n deduced following this procedure and those provided by the theory. The horizontal scale has been adjusted to fit the data since the proportionality between $\bar{\nu}$ and $(V_{ac}-V_c)/V_c$ is unknown. It is remarkable that at criticality ($\bar{\nu}=0$), the experimental value of δ_n is close to the

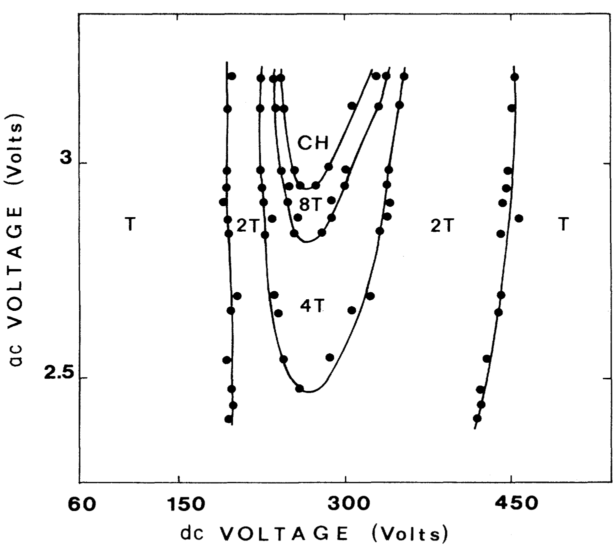

FIG. 2. Parameter space diagram for the laser with internal modulation. Bifurcation diagrams as reported in Fig. 1 correspond to horizontal straight lines. They intersect the chaotic region for V_{ac} larger than 2.94 V.

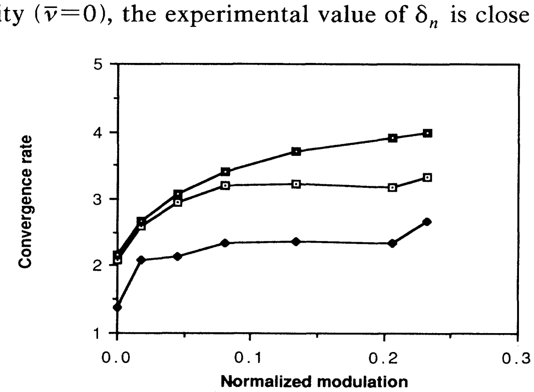

FIG. 3. Plot of the ratios δ_2 of the widths of the first three periodic domains vs the modulation voltage in reduced units for the direct cascade (□) and for the reverse cascade (♦). The solid squares (■) refer to the theory of Oppo and Politi.

theory $\delta^{1/2}=2.16\ldots$. Similarly, in the limit of well-separated cascades, we find a value close to that of the Feigenbaum constant.

The accuracy in the experimental measurements of the convergence rate is limited by several factors. (i) The rule for the variation of δ is valid asymptotically and the experimental measures could be made only for the first three bifurcations. (ii) As a consequence of this, the control parameter was varied on a finite range in which its effect on the laser parameters is highly nonlinear. This finiteness of the changes in the control parameter together with the nonlinear correspondence between the control voltage and the laser losses induces systematic deviations from Oppo and Politi's predictions. Eventually (iii) the postponement of the bifurcation points induced by the sweep of the control parameter is reduced if very slow sweeps are used.[7,8] However, this requirement becomes more and more stringent as higher-order bifurcations are examined. Although sweeps as slow as 15 sec, i.e., $10^7 T$, were used, some postponement of the last bifurcation could not be avoided. As a result of this, the evolution of the dynamics is different for the two sides of the chaotic region and this is responsible for the difference of the δ values for the two sides. Faster sweeps indicate that this could explain that our experimental values are downshifted by a factor of about 10–20 %. Slower sweeps did not prove helpful in providing better data because the stability of the laser parameters gets worse as the sweep duration becomes longer.

The theory of Oppo and Politi on the collision of Feigenbaum cascades appears to describe qualitatively well the experimental observation on a CO_2 laser with modulated losses although the asymptotic limits $n\rightarrow\infty$ and $\bar{\nu}\rightarrow 0$ are far away from the domain that can be reached in experiments. Some quantitative discrepancies can be explained either by these experimental limits or by fast passage effects. The results reported here also provide the first experimental observation and characterization of the collision of period-doubling cascades in a nonlinear optical system. Other such collisions should also be obtained in other systems such as the laser with a saturable absorber[9] and corresponding experiments would be helpful to support by additional experimental results the universality of their theory.

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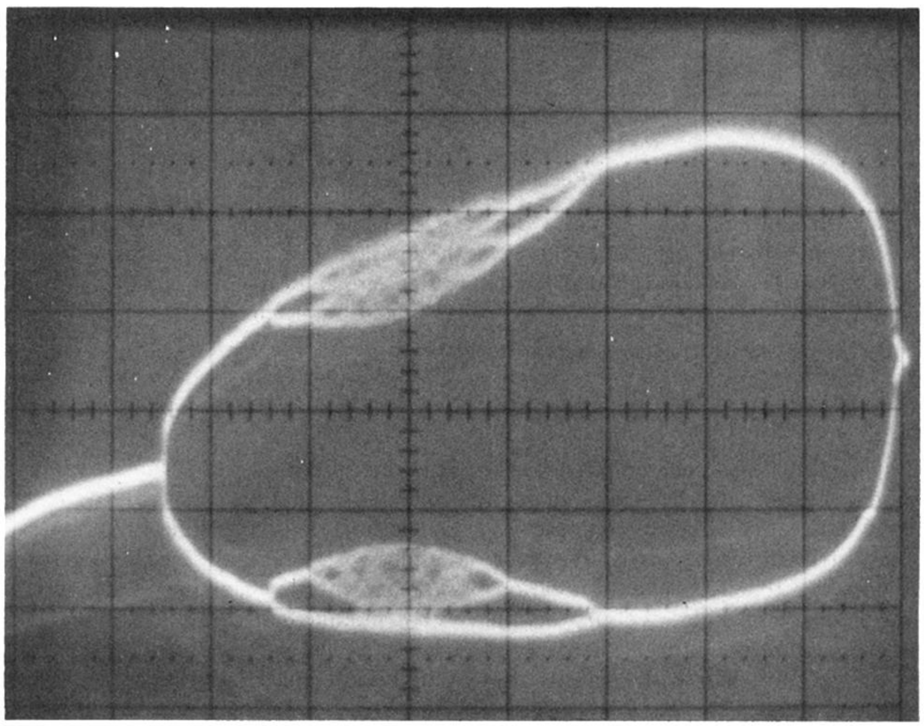

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