Backward-stimulated emission and nonreciprocity in optics

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Conditions for enhanced backward-stimulated emission are formulated for a thin layer of gain medium. A comparison is made between the enhanced backward-stimulated emission and the reflection from a dielectric film. A nonreciprocal reflective structure using a gain layer and a dielectric film is described. Such a device can be used to construct ring lasers with controllable unidirectional operation.

I. INTRODUCTION

Generally, stimulated emission refers to the radiation that is copropagating in phase with the incident stimulating laser beam. However, within the framework of semiclassical and classical theories, the backward-stimulated emission exists also, although usually very weak. Except for the case of a thin layer of gain medium, or for a periodic gain structure with a spacing of $\lambda/2$, destructive interferences make the backward-stimulated emission negligible as compared to the forward emission. When such a gain structure is used in a ring laser, the backward- and forward-stimulated emissions are comparable,[1,2] and therefore the conventional equations for the ring laser dynamics are no longer applicable.[1]

From the classical point of view, the backward-stimulated emission is simply a reflection from the interface between a dielectric and a gain medium, due to the mismatch in the imaginary parts of their indices of refraction. Consequently, the "reflection" from a gain layer and that from a dielectric film differ by a phase of $\pi/2$. A nonreciprocal device is created when a gain layer and a dielectric film are located $\lambda/8$ apart. A beam incident from one side of the device experiences a constructive interference in reflection, while the beam incident from the other side observes a destructive interference in reflection. This feature of the device should have potential application in ring lasers for unidirectional operation.

Unidirectional operation of ring lasers is desirable for various practical applications as well as for fundamental research purposes. More power can be extracted from a unidirectional homogeneously broadened laser, because of the absence of mutual saturation. Unidirectional lasers are needed to study the dynamics of a single mode ring laser,[3] and to perform intracavity investigation of the interaction of a traveling wave with matter. Other applications include the development of active optical bistable devices.[4,5]. Simplicity, efficiency, and compactness of structure are the key advantages of the structure proposed here, over the various schemes for unidirectional operation of ring lasers that have been reported [6-9].

II. BACKWARD-STIMULATED EMISSION

The conventional concept of the stimulated emission being copropagating with the stimulating radiation results from the averaging of dipole radiation over a macroscopic volume. According to the semiclassical theory of electrodynamics, the stimulated emission from an atom in the excited state has the pattern of an electric-dipole moment with its axis lined up with the driving field.[10] Let the incident beam propagating along axis z and polarized along x be $\mathbf{E} = \tilde{\mathcal{E}} e^{i\omega t - kz}\hat{\mathbf{x}}$. The electric field radiated at the position $\mathbf{r}$ by an atomic dipole at $\mathbf{r}'$ is given by[11]

$$\tilde{\mathcal{E}}_s = \tilde{\mathcal{E}}(r)\hat{\mathbf{r}} \times \hat{\mathbf{x}} \times \hat{\mathbf{r}} e^{-ik(\hat{\mathbf{z}}-\hat{\mathbf{r}})\cdot\mathbf{r}'} \tag{1}$$

where $\tilde{\mathcal{E}}(r)$ is a function of $r \equiv |\mathbf{r} - \mathbf{r}'|$, and $\hat{\mathbf{r}} \equiv (\mathbf{r} - \mathbf{r}')/r$. When a large number of atoms is located within a thin layer $|\mathbf{r}' * \hat{\mathbf{z}}| < d$, the total radiation, as a result of interference, is then only along the forward and the backward direction:

$$\mathbf{E}_s = \sum_m \tilde{\mathcal{E}}_m e^{i(\omega t - kz)} + \sum_m \tilde{\mathcal{E}}_m e^{-2ikz_m} e^{i(\omega t + kz)} , \tag{2}$$

where m refers to the mth atomic dipole.

It is obvious from the above Eq. (2) that the backward-stimulated emission is negligible if the atoms are distributed uniformly over a layer with thickness $d \gg \lambda$. This is consistent with the observations under common experimental conditions. At the limit of $d \ll \lambda$, however, the backward- and forward-stimulated electric fields are of equal magnitude.

Another approach is that of the classical optical theory, in which a medium with a gain constant α has a complex index of refraction $\tilde{n} = n_0(1 - i\alpha/2k)$. Let us consider a structure as shown in Fig. 1, in which a thin layer of such a gain medium is embedded in a dielectric with an index of refraction n_0.

Assuming — as is the case in most practical situations — that $\alpha/2k \ll 1$, the reflection coefficient from the interface is

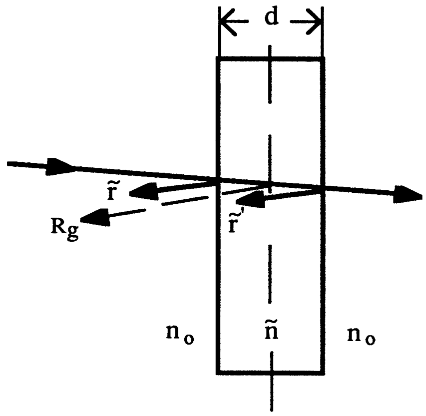

FIG. 1. Thin gain layer embedded in a dielectric medium of index n_0.

$$\tilde{r} = \frac{n_0 - \tilde{n}}{n_0 + \tilde{n}} \approx \frac{i\alpha}{4k} = -\tilde{r}' \ . \tag{3}$$

Assuming that $kd \ll \pi$, the total reflection coefficient R from the structure is given by

$$R \approx \tilde{r} + \tilde{r}' e^{2ik(1-i\alpha/2k)d} \approx \tilde{r}(1 - e^{2ikd}) \approx -2i\tilde{r}kde^{ikd} = R_g e^{ikd} \ , \tag{4}$$

where

$$R_g \equiv \frac{\alpha d}{2} \tag{5}$$

is the effective reflection coefficient localized at the plane of symmetry of the layer. The transmission coefficient is given by

$$T = tt' e^{ik(1-i\alpha/2k)d} \approx e^{ikd}(1 + \alpha d/2) \ , \tag{6}$$

where we have used $tt' = 1 - r^2 \approx 1$.[12] The forward-stimulated emission, $T - e^{ikd}$, is indeed equal to the backward-stimulated emission, R, under the conditions of $\alpha/2k \ll 1$ and $kd \ll \pi$.

Let us now compare the previous situation to that of a thin (thickness $D \ll \lambda$) *dielectric layer* of refractive index n embedded in a dielectric of index n_0. Within the approximation of $|n_0 - n| \ll n_0$ and $kD \ll \pi$,

$$R \approx r + r' e^{2ikD} \approx -2irkDe^{ikD} = R_d e^{ikD} \ , \tag{7}$$

where $r = (n_0 - n)/(n_0 + n) = -r'$ is a real number, and $R_d \equiv -2irkD$ is the effective reflection coefficient at the plane of symmetry of the dielectric film. An essential distinction between the two cases considered above is that the effective reflection coefficient of a dielectric layer is imaginary, while that of a gain layer is real (with the reflection plane localized at the plane of symmetry of the corresponding layer). The conclusion for the dielectric film still holds for a film with arbitrary thickness.

III. OPTICAL NONRECIPROCITY

Let us consider the device depicted in Fig. 2, with a spacing of $l = \lambda/8 + m\lambda/2$ between a gain layer and a dielectric film.

The total reflection coefficient of the device for a beam incident from the left is given by

$$R_+ \approx R_g + R_d e^{2ikl} = R_g + iR_d \approx \frac{\alpha d}{2} + 2rkD \ , \tag{8}$$

while the total reflection coefficient from the right is given by

$$R_- \approx R_g + R_d e^{-2ikl} = R_g - iR_d \approx \frac{\alpha d}{2} - 2rkD \ . \tag{9}$$

In Eqs. (8) and (9), the plane of symmetry of the gain layer is chosen as reference. Equation (8) expresses the enhancement of the total reflection from one side of the device, through constructive interference of the reflection from the two layers. In Eq. (9) on the other hand, the total reflection from the other side of the device is reduced through destructive interference. This very elementary device is thus optically nonreciprocal. This statement remains true when the gain layer is replaced by an absorptive layer. A more detailed expression can be readily derived for R_+ and R_- by treating the film and the device as Fabry-Pérot étalons.

The preceding evaluation of single pairs of layers can obviously be extended to multilayer structures. The device of Fig. 1, or the pair of dielectric-gain layer considered above, can be the "unit cell" of a structure with periodicity of $\lambda/2$. In both cases, the reflection coefficient of either Eq. (5) or (8) or (9) is increased proportionally to the number of the layers. In the case of the structure of Fig. 1, the backward radiation can be increased by a periodic structure, and the restriction on the thickness of each layer, $kd \ll \pi$, does not set a limit to the amplitude

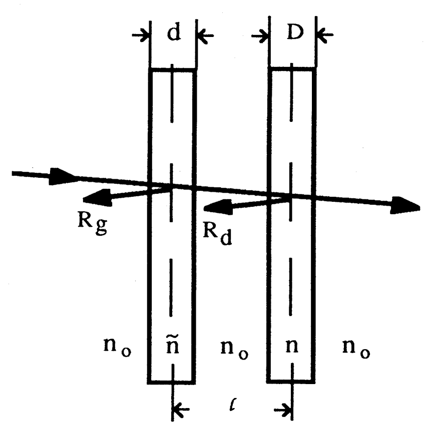

FIG. 2. Optically nonreciprocal structure consisting of a thin gain medium and a dielectric reflector.

of the backward-stimulated emission that can be generated. The length of the structure to be used is limited by the linewidth of the incident beam.

IV. UNIDIRECTIONAL RING LASER

When the device in Fig. 2 is placed in a ring laser cavity, the equations of motion of the laser fields for the clockwise (cw) and counterclockwise (ccw) modes are, respectively,[13]

$$\dot{E}_+ = \frac{\alpha}{2}E_+ - i\Omega_+E_+ - \beta(I_+ + 2I_-)E_+ + R_-E_- - \frac{\gamma}{2}E_+ , \quad (10)$$

$$\dot{E}_- = \frac{\alpha}{2}E_- - i\Omega_-E_- - \beta(I_- + 2I_+)E_- + R_+E_+ - \frac{\gamma}{2}E_- , \quad (11)$$

where $\alpha/2$ is the total gain coefficient, including the gain from the device and that from an additional conventional gain medium, Ω_+ and Ω_- are the intrinsic frequencies of the cw and ccw laser modes, respectively, $\gamma/2$ is the loss coefficient of the laser cavity, and β is the self-saturation coefficient. It should be noticed that the device has the same transmission in both sides, despite the difference in reflection. This ensures the validity of using the same gain and loss coefficients for the oppositely propagating waves. (See Ref. 1 for more complete expressions of the saturation terms.)

If R_+ and R_- are real, as was the case in Eqs. (8) and (9), Eqs. (10) and (11) can be rewritten as

$$\dot{I}_+ = \alpha I_+ - 2\beta(I_+ + 2I_-)I_+ + 2R_-\sqrt{I_+I_-}\cos\Psi - \gamma I_+ , \quad (12)$$

$$\dot{I}_- = \alpha I_- - 2\beta(I_- + 2I_+)I_- + 2R_+\sqrt{I_+I_-}\cos\Psi - \gamma I_- , \quad (13)$$

$$\dot{\Psi} = \Omega_+ - \Omega_- + \left(R_-\sqrt{\frac{I_-}{I_+}} + R_+\sqrt{\frac{I_+}{I_-}}\right)\sin\Psi , \quad (14)$$

where we have used $E_+ = \sqrt{I_+}e^{i\phi_+}$, $E_- = \sqrt{I_-}e^{i\phi_-}$, and $\Psi \equiv \phi_+ - \phi_-$ for the phase difference between the waves at the gain layer. When $|\Omega_+ - \Omega_-| \ll |R_-\sqrt{I_-/I_+} + R\sqrt{I_+/I_-}|$, one can expect that, according to the standard theory of ring lasers,[13] the following are true.

(1) The two modes lock to each other, i.e., $\dot{\Psi}$ and $\Psi \approx 0$.

(2) The laser lases unidirectionally or is dominant at the mode with higher gain, $\alpha + 2R_-\sqrt{I_-/I_+}$ or $\alpha + 2R_+\sqrt{I_+/I_-}$. In other words, if $R_- \gg R_+$, then $I_+ \gg I_-$.

It can also be seen from Fig. 2 that changing the spacing between the gain layer and the dielectric film from $\lambda/8 + m\lambda/2$ to $-\lambda/8 + m\lambda/2$ results in a switch of directionality of the device. The side that experienced constructive interferences will thereafter experience destructive interferences, and vice versa. The reflections R_+ and R_- can thus be modulated by tuning the spacing l in Fig. 2. The direction of operation of a ring laser can be controlled and switched by using this multilayer element as laser gain medium, as can be investigated through Eqs. (10) and (11).

In summary, we have shown that enhanced backward-stimulated emission can be obtained by a thin layer of gain medium or a periodic gain structure with spacing of $\lambda/2$. A type of optically nonreciprocal device has been proposed, which uses the interference between the enhanced backward-stimulated emission from the layered gain and the reflection from a dielectric film. Such a device can be used in a ring laser to control or modulate its unidirectional operation.

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[1]M. Lai and J.-C. Diels, Phys. Rev. A **42**, 536 (1990).
[2]M. O. Scully, Phys. Rev. A **35**, 452 (1987).
[3]B. McNamara, K. Wiesenfeld, and R. Roy, Phys. Rev. Lett. **60**, 2626 (1988).
[4]W. W. Rigrod and T. J. Bridges, IEEE J. Quantum Electron. **QE-1**, 298 (1965).
[5]J. L. Boulnois, G. Bret, P. Cottin, A. Van Lergerghe, and G. P. Agrawal, Opt. Lett. **10**, 7 (1985).
[6]C. L. Tang, H. Statz, and G. deMars, Appl. Phys. Lett. **2**, 222 (1963).
[7]F. R. Faxvog and A. D. Gara, Appl. Phys. Lett. **25**, 306 (1974).
[8]V. I. Sardyko and A. Ya. Smirnov, Opt. Spektrosk. **52**, 713 (1982) [Opt. Spectrosc. (U.S.S.R.) **52**, 424 (1982)].
[9]R. Roy, P. A. Schulz, and A. Walther, Opt. Lett. **12**, 672 (1987).
[10]W. Heitler, *The Quantum Theory of Radiation* (Dover, New York, 1984).
[11]J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1976).
[12]M. Born and E. Wolf, *Principles of Optics* (Pergamon, New York, 1975).
[13]M. Sargent III, M. O. Scully, and W. E. Lamb, *Laser Physics* (Addison-Wesley, Reading, MA, 1974).