PHYSICAL REVIEW A VOLUME 43, NUMBER 4 15 FEBRUARY 1991

Multiplicity of bifurcation in weakly ionized magnetoplasmas

Cheng-Sheng Chern and Lin I
Department of Physics, National Central University, Chungli, Taiwan, Republic of China
(Received 5 June 1990)

We report the experimental study of the fine structure of bifurcation and hysteresis behaviors under two control parameters in a weakly ionized cylindrical rf magnetoplasma system that supports nonlinear ionization drift waves. With increasing rf power up to the onset of the quasiperiodic state, the system shows a cascade of hysteresis loops with an abrupt change of the collective mode for each subcritical bifurcation. Fine structures such as the embedded small hysteresis loops, which correspond to multistability, and the backward supercritical subharmonic and intermittent bifurcations were observed. The effects of changing pressure, for example, the conversion from subcritical to supercritical bifurcation through a tricritical point, are also presented and discussed.

The bifurcation in nonlinear systems has attracted a great deal of attention recently due to its important position in the transition to chaos and the interesting phase-transition-like behavior.[1–8] Although bifurcation through period doubling and intermittency has been observed in low-pressure dc discharge systems,[4,5] the multiplicity features of bifurcation and its spatial behaviors in plasma systems have received much less attention. Recently, our study in a steady-state weakly ionized rf magnetoplasma system demonstrated a transition which roughly followed a quasiperiodic route to spatiotemporal chaos with increasing rf power in the high-rf-power regime.[6] In this paper, by varying two system parameters, rf power and pressure, with improved resolution, we concentrate on the fine structure and multiplicity of the complicated bifurcation and hysteresis behaviors in the above system, especially in the low-rf-power regime up to the onset of the quasiperiodic bifurcation.

According to the bifurcation theory, the state of a nonlinear autonomous dynamical system with a few variables can be described by a set of nonlinear differential equations.[3,7] As the control parameters in the equations are adjusted, the stable solution bifurcates and exhibits phase-transition-like behavior.[3] For a subcritical bifurcation, an S or an inverse-S bifurcation diagram leads to the discovery of two stable steady states and a hysteresis loop.[2,3] The drastic change of the state variable at the bifurcation is similar to the thermodynamic first-order phase transition.[3–5,7,9] As the numbers of the degrees of freedom and control parameters increase, the system becomes more complicated. Controllable multistable states under multi-S-shape bifurcation diagram and variations of the hysteresis diagrams have been observed in chemical systems with multicontrol parameters.[7] This type of phenomenon is less well studied for physical systems, especially for continuous systems described by a set of nonlinear partial differential equations, in which abrupt spatial pattern change usually associates with the subcritical bifurcation.[8] Our weakly ionized magnetoplasma system has simple cylindrical symmetry and supports spontaneously generated azimuthal ionization-drift waves. Its dynamics can be described by reaction-diffusion-type nonlinear partial differential equations,[6] and can be controlled by several parameters such as rf power, pressure, magnetic field, etc.[6,9] It turns out to be a good candidate to study the bifurcation and hysteresis behaviors of the collective spatiotemporal modes of a continuous system with multi-control parameters.

The fundamental principle of ionization-drift waves in the weakly ionized ($<1\%$) low-pressure (~ 10 mTorr) cylindrical rf discharges can be found elsewhere.[6,9] Only electrons are magnetized in the moderate axial B field (~ 100 G). Ions, due to the large mass, only respond to the space-charge field. The rf-induced radial spatial inhomogeneity causes azimuthal electron $E \times B$ and diamagnetic drifts, induces charge separation, and forms the well-known drift wave. The electron-impact ionization is similar to a reaction process and further enhances the electron density fluctuation (the electron-ion volume recombination can be neglected under the low plasma density). Namely, the system shares some common features with a chemical reaction-diffusion system, but with an additional space-charge field and transversed (to the B field) nondissipative spatial coupling due to electron drifts. In our previous study,[6] we varied the rf power that sustained the discharge. The system followed a complex quasiperiodic route with strong nonlinear mode-mode competition, a narrow frequency-locking interval, an unstable third independent frequency to temporal chaos, and finally spatial-temporal chaos.

The experiment was conducted in a cylindrical rf magnetoplasma system, as described elsewhere.[6] Briefly the system consisted of two 30-cm-long concentric cylindrical stainless-steel electrodes. The center one was grounded and the outer one was capacitively coupled to a rf-power amplifier (ENI A-500) which was driven by a Philipps PM 5193 programmable function generator with frequency fixed at 14 MHz. With low-pressure (~ 10 mTorr) argon gas, a steady-state weakly ionized rf discharge confined by a uniform axial magnetic field (50 G) was sustained between the two electrodes. Azimuthal ionization drift waves were spontaneously generated due to the ionization process, radial electric field, and density gradient.[6,9] In this experiment, only the rf power and pressure were changed and the magnetic field was kept at 50 G. The rf power was varied by modulating the amplitude of the output of the function generator V_{rf} (the rf power $W_{rf} \propto V_{rf}^2$ and $W_{rf} = 11$ W at $V_{ef} = 100$ mV). The modulation was computer controlled to study the fine structures of bifurcations. In addition to the two azimuthally displaced side probes monitoring the spatiotemporal behaviors of the waves, as in the previous experiment,[6] the dc part of the ion current, I_{dc}, from a negatively biased plane probe located at the end of the cylinder was used to monitor the steady state of the discharge. The hysteresis diagram of I_{dc} versus V_{rf} was plotted.

Figure 1 shows a cascade of hysteresis loops and the temporal behaviors in different regimes from the stationary state to the onset of the quasiperiodic state with increasing V_{rf} at $P = 13.1$ mTorr. Similar to the previous experiment before the onset of spatial chaos, the signals from the two azimuthally separated probes are almost identical but phase shifted. The spatial behavior can be characterized by the mode number m obtained from the phase shift. Each subcritical bifurcation corresponds to an abrupt generation of a new collective mode with different amplitude. The amplitude then smoothly varies with V_{rf}. The discharge turns off for $V_{rf} < 84$ mV. The discharge is quiescent in the region left of point A and gradually amplifies the 60-Hz noise between points A and B. At point B, the system suddenly jumps through a Hopf bifurcation to a mode with 16.5 kHz oscillation and $m = 0$. The amplitude of oscillation continuously diminishes, and the system becomes stationary again through an inverse supercritical bifurcation (bifurcation with continuous charge of order parameter) at point C. At point D, another subcritical transition from stationary to a new single mode with $f = 240$ kHz and $m = 8$ onsets. It is followed by a subcritical bifurcation to the quasiperiodic state with the appearance of the second mode with an incommensurate frequency $f' = 26.3$ khz and $m' = 1$ at point E. A further increase of V_{rf} leads to a subcritical transition to another quasiperiodic state (beyond point F), followed by frequency-locking windows, and then the spatiotemporal chaos similar to those described in our previous report.[6] Reversing V_{rf} from point F shows a cascade of hysteresis loops. The system returns to the 240-kHz mode after point G and to the 16.5-kHz mode after point H.

Along the upper branch of the leftmost loop, a downward subcritical transition (J-J') from the 16.5-kHz mode ($m = 0$) to an intermediate state with $f = 12.1$ kHz and $m = 0$ was observed [Fig. 2, 13.1-mTorr run and Figs. 3(a) and 3(b)]. A local scan of V_{rf} around the point J shows a nested small hysteresis loop corresponding to the existence of tristable states. At point K, another downward subcritical bifurcation to the stationary state occurs.

The entire hysteresis pattern changes with the operating pressure. As shown in Fig. 2, increasing pressure shifts the first main hysteresis loop to the lower V_{rf} side and reduces the widths of the main and the nested loops. Point J also moves toward point K. At $P = 14.2$ mTorr, stochastic transitions directly to the lowest branch (J-J'') and to the intermediate branch (J-J') were observed. At $P = 14.7$ mTorr, the nested small loop disappears and the transition to the intermediate state changes from subcritical to supercritical bifurcation. At $P = 15.1$ mTorr, points J and K coalesce and the intermediate state disappears. At high system pressure ($P = 18.0$ mTorr), multiple subcritical bifurcations with nested hysteresis loops

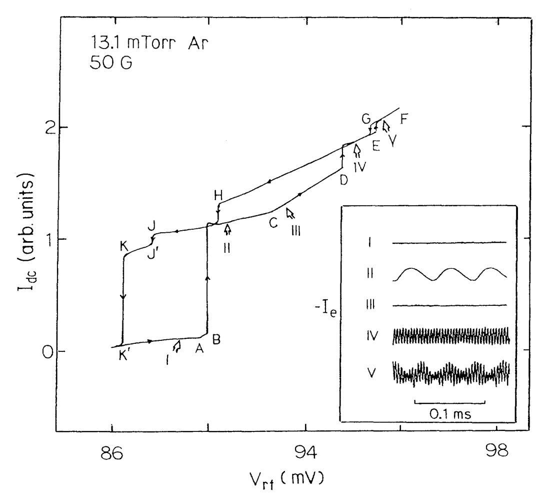

FIG. 1. A cascade of hysteresis loops up to the onset of the quasiperiodic state. The inset shows the temporal behaviors in different regimes as rf power increases.

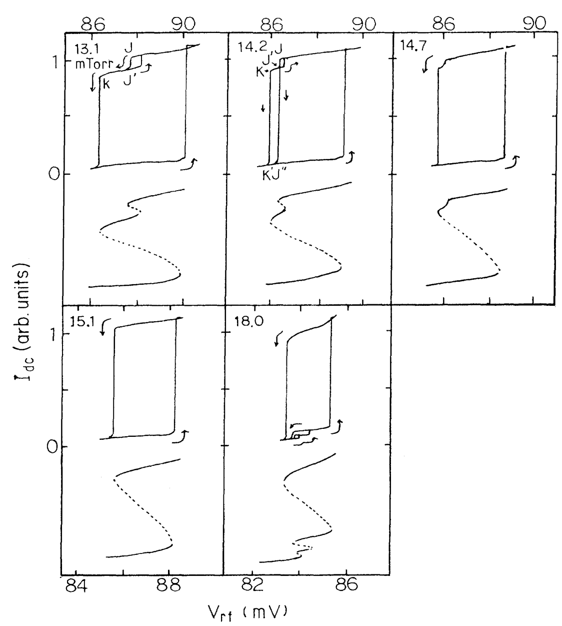

FIG. 2. The evolution of the first hysteresis loop and the corresponding bifurcation diagram under different system pressures.

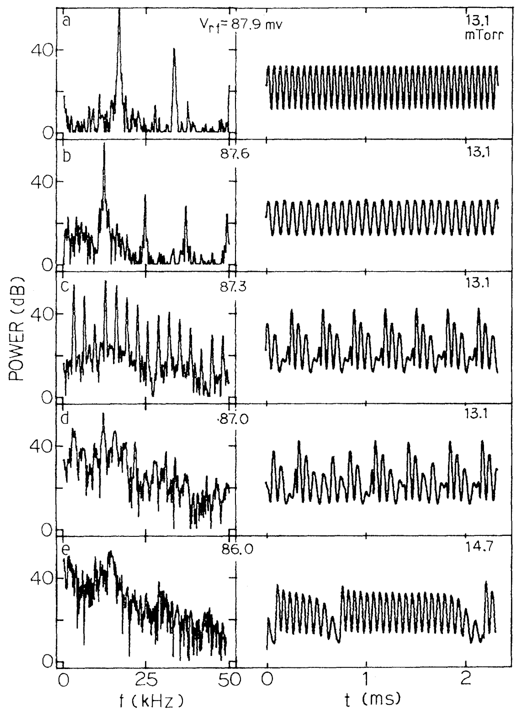

FIG. 3. The power spectra and temporal oscillations of the inverse supercritical bifurcations in the intermediate state (J' to K) of the first major loop. (a) and (b) States right before and after the downward J-J' bifurcation respectively. (c) and (d) P_4 and $C_{i,j}$ states, respectively. (e) The intermittent state [$P = 13.1$ mTorr for (a)–(d) and 14.7 mTorr for (e)].

were also observed in the lowest branch of the first main loop. It again manifests the existence of the multistable state.

Although the bifurcation behaviors look quite complicated, they have been partially observed in other nonlinear experimental systems. For example, the hysteresis diagram of a low-pressure ($P \lesssim 10^{-4}$ Torr) dc discharge system supporting drift waves also shows a transition to an intermediate state.[10] Unfortunately, no detailed studies have been conducted in that region. In general, a simple S-shape bifurcation diagram with an unstable middle branch connecting the stable upper and lower branches can induce a simple hysteresis loop with bistable states. The cascade of hysteresis loops can be described by a cascade of S-shaped bifurcation curves. If one of the two stable branches in a simple loop becomes S shaped and bifurcates, nested small-loop and multistable states occur. In our system, varying a second control parameter such as pressure changes the shape of the bifurcation diagram and in turn the hysteresis diagram (Fig. 2). In the chemical reaction experiment by Harold and Luss,[7] the temperature of the pellet was measured, on which ethane, carbon monoxide, or both reactants were oxidized. Hysteresis diagrams of the pellet temperature versus the gas feeding temperature were plotted. Although they did not study the spatial behavior and observe the stochastic routes to different states, hysteresis diagrams with multiple stable states were observed. Varying the second control parameter (reactant concentration) in their experiment also affects the hysteresis pattern.

The change from subcritical to supercritical bifurcation with increasing pressure and the observation of the stochastic subcritical routes to different states in certain pressure windows (Fig. 2) are very interesting. The former is similar to the change from a first-order to a second-order thermodynamic phase transition through a tricritical point in a two-dimensional parameter space using Landau's approach.[11] The bifurcation with stochastic transitions to two different states has been observed in other dc discharge experiments.[5] Similar to the free energy in Landau's approach, we can construct a variable U (e.g., the integrand of the Lyapunov functional[8,11]) as a function of the order parameter A (e.g., I_{dc}). The multistable state and the stochastic transitions manifest the coexistence of multiminima in the $U(A)$ diagram. The stochastic transitions occur while the old state loses its stability and the two neighboring minima are equally accessible and stable.

It should be pointed out that the intermediate branch J'-K has some fine structure. It exhibits a series of backward supercritical bifurcations. Depending on the pressure, the bifurcation basically follows a period-n subharmonic or intermittent route (Fig. 3). Similar to those observed in the Belousov-Zhabotinskii reaction system,[1] windows with $\mathcal{P}_n$ and $C_{i,j}$ states and increasing amplitude were found as V_{rf} decreased at $P = 13.1$ mTorr [Figs. 3(c) and 3(d)]. $\mathcal{P}_n$ is a period-n oscillation, and $C_{i,j}$ is a chaotic state with a combination of randomly distributed period-i and -j oscillations. Depending on the pressure, up to the $\mathcal{P}_{11}$ mode has been observed. The amplitude of oscillation increases with the decreasing V_{rf}. It increases the electron-loss rate through particle-wave interaction,[12] and forces the system to return to a quiescent state at small V_{rf} (point K). As the pressure increases (e.g., to $P = 14.7$ mTorr), the transition shifts from the period-n route to the intermittent route [Fig. 3(e)]. A detailed study of the backward supercritical bifurcation will be reported in the near future.

Our system is a complicated open dissipative system. With insufficient available energy at the low-rf-power end, only single-mode oscillation is excited and a one-dimensional map is sufficient to describe the period-n and intermittent-type transitions. In the high-power region, a higher number of degrees of freedom can be excited and the system manifests a quasiperiodic route to turbulence. It should be noted that in the computer simulations for a drift wave system, period doubling and quasiperiodic transitions were found when low- and high-order independent-variable truncations were used, respectively.[13,14]

Similar to other nonequilibrium continuous systems described by Landau-Ginzburg-type equations, the subcritical bifurcation is associated with an abrupt pattern change, and it also occurs in our system.[3,8] The integer mode numbers of all the spatiotemporal modes are due to the periodic boundary condition of our cylindrical system. The supercritical period-n bifurcation which reduces the mode number to m/n can only occur for the mode with $m=0$ if m is restricted to the small mode number due to the low-input rf power.

In conclusion, we have investigated the complicated cascaded bifurcations and their fine structures in our rf magnetron plasma system in the low-rf-power regime up the onset of the quasiperiodic state. An abrupt mode change was found for each subcritical transition of the cascaded hysteresis loops. In the low-rf-power regime, nested hysteresis loops corresponding to the multistable states in conjunction with inverse period-n and intermittent transitions have been observed. The pressure plays an important role as a second control parameter. Varying pressure causes the conversion from the subcritical to supercritical bifurcation through a tricritical point, and the occurrence of the stochastic transitions.

This work is supported by the Department of Physics of the National Central University and the National Science Council of the Republic of China.

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