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Nonlinear dynamics near the zero-dispersion point in optical fibers

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We analyze the nonlinear dynamics in the normal-dispersion regime near the zero-dispersion point of a single-mode optical fiber using a connection between the nonlinear Schrödinger and Korteweg–de Vries equations. We predict a new type of optical soliton in this region and demonstrate a possibility of transformation of one type of soliton to another.

The possibility of using soliton pulses as information carriers in optical communication systems has attracted considerable attention after it was shown theoretically and experimentally that solitons can propagate in single-mode optical fibers without dispersive broadening.[1,2] In a communication system it is desirable to work near the zero-dispersion (ZD) point,[3–9] where the second-order dispersion is zero, because there the power required for creating bright solitons is significantly lower. Although exact analytical solutions describing the soliton propagation near the point are not available, numerical[4–7] and perturbative[3,8,9] methods have explained the main features of pulse propagation near and at the ZD point in the anomalous-dispersion regime. These results may be summarized as follows. Solitons emerge from arbitrary initial pulses whose central frequencies are exactly at the ZD point.[7] The emerging soliton and an additional dispersive wave are generated simultaneously but with different frequencies: the soliton has a central frequency that is shifted into the anomalous-dispersion region, and the dispersive wave is shifted into the region of normal dispersion. This means that bright solitons may exist near the ZD point but not at it. The effect was described analytically in recent papers[8,9] using different perturbative approaches.

Meanwhile, dark solitons have also drawn the attention of several research groups. They are stable localized excitations of the cw background in the normally dispersive, nonlinear medium (see, e.g., the last chapter of Ref. 2), and these solitons also have been observed experimentally in optical fibers[10–13] (temporal dark solitons), waveguides,[14] and laser beams[15,16] (spatial dark solitons). These solitons may also be useful in optical systems because, in particular, they may be created without a threshold or due to a phase modulation of a background pulse.[17,18] In this paper we analyze the nonlinear dynamics and dark-soliton propagation near the ZD point in the normal-dispersion regime using the analytical approach recently developed in the paper[19] for the small-amplitude case. In particular, we demonstrate that dark solitons may exist near the ZD point and we predict a region of the group-velocity dispersion where a new type of solitary wave in optical fibers, the so-called soliton on a constant background, i.e., a "pedestal," may be observed.

Using the slowly varying envelope approximation (see, e.g., Refs. 2 and 3) we may find that the dimensionless envelope amplitude $u(x,t)$ of the electric field in the neighborhood of the ZD point satisfies the generalized nonlinear Schrödinger (NLS) equation

$$iu_x - \alpha u_{tt} + 2|u|^2 u = i\beta u_{ttt}\,, \tag{1}$$

where the subscripts x and t mean the partial derivatives. Time t in the reference frame moving with the group velocity is measured in units of the pulse duration T, the coordinate x along a fiber is measured in units of $T/|k^{(1)}|$, and also $\alpha = k^{(2)}/2T|k^{(1)}|$, $\beta = k^{(3)}/6T^2|k^{(1)}|$, where k is the propagation wave number, and $k^{(j)} = \partial^i k/\partial\omega^j$, $j=1,2,3$.

In the case $\beta=0$ for $\alpha>0$, Eq. (1) is exactly integrable and it has stable soliton solutions in the form of localized dark pulses propagating on a modulationally stable cw background $|u|=u_0=\text{const}$. The one-soliton dark pulse has the form

$$u(x,t) = u_0 \frac{(\lambda - i\nu)^2 + \exp Z}{1+\exp Z} \exp(2iu_0^2 x)\,, \tag{2}$$

where

$$Z = 2\nu u_0 (t - t_0 - 2\lambda\sqrt{\alpha}u_0 x)/\sqrt{\alpha}, \quad \lambda^2 = 1-\nu^2\,, \tag{3}$$

ν is the soliton parameter, $0<\nu^2<1$, and t_0 is an initial phase. At $\lambda=0$ the soliton (2),(3) describes the so-called fundamental dark soliton

$$u(x,t) = u_0 \tanh[u_0(t-t_0)/\sqrt{\alpha}]\exp(2iu_0^2 x)\,,$$

and for $\nu^2 \ll 1$ it corresponds to the so-called gray (small-amplitude) dark solitons[19]

$$u(x,t) = [u_0 - \tfrac{1}{2}u_0\nu^2 \text{sech}^2(Z/2)]\exp[2iu_0^2 x + i\phi(x,t)]\,,$$

$$\phi(x,t) = -2\nu/(1+\exp Z)\,, \tag{4a}$$

$$Z = 2\nu u_0[t - t_0 \mp u_0\sqrt{\alpha}(2-\nu^2)x]/\sqrt{\alpha}\,, \tag{4b}$$

which propagate in opposite directions.

To discuss the dark-soliton dynamics in the neighborhood of the ZD point for the normal-dispersion regime, i.e., $\beta=0$ and $\alpha>0$ in Eq. (1), we look for a solution in the form of small-amplitude excitations of the cw background (see Ref. 19):

$$u(x,t) = [u_0 + a(x,t)]\exp[2iu_0^2 x + i\phi(x,t)] \tag{5}$$

[cf. Eq. (4)]. Substituting Eq. (5) into Eq. (1), we may obtain for the small-amplitude case, when $a^2 \ll u_0^2$, two equations:

$$(a_x - \alpha u_0 \phi_{tt}) - \alpha(2a_t\phi_t + a\phi_{tt}) = \beta(a_{ttt} - 3u_0\phi_t\phi_{tt}), \quad (6a)$$

$$u_0(\phi_x - 4u_0 a) + a\phi_x + \alpha a_{tt} - \alpha u_0(\phi_t)^2 - 6u_0 a^2 = \beta(3a_{tt}\phi_t + 3a_t\phi_{tt} + u_0\phi_{ttt}). \quad (6b)$$

The method to solve the system (6) has been described in Ref. 19. The main basis of this approach is to use the new ("slow") variables

$$\tau = \epsilon(t - Cx), \quad z = \epsilon^3 x, \quad (7)$$

ϵ being an arbitrary small parameter connected with the soliton amplitude v, and to present the wave fields $a(\tau,z)$ and $\phi(\tau,z)$ in the form of the asymptotic series in the same small parameter ϵ:

$$a = \epsilon^2 a_0 + \epsilon^4 a_1 + \cdots, \quad \phi = \epsilon\phi_0 + \epsilon^3\phi_1 + \cdots. \quad (8)$$

The parameter C arising in Eq. (7) is the limit velocity (in the t space) of linear waves propagating on the cw background:

$$C^2 = 4u_0^2\alpha. \quad (9)$$

Substituting Eqs. (8) into Eqs. (6) and using the variables (7), we may reduce Eqs. (6) in the zeroth approximation to the well-known Korteweg–de Vries (KdV) equation for the soliton amplitude a_0 (similar calculations may be found in Ref. 19 for $\beta=0$ and $\alpha=1$):

$$2C(a_0)_z + 24\alpha u_0(1+\beta C/2\alpha^2)a_0(a_0)_\tau - (\alpha^2+2\beta C)(a_0)_{\tau\tau\tau} = 0. \quad (10)$$

Equation (10) at $\alpha=1$ and $\beta=0$ coincides with that obtained in Ref. 19. The sign of the velocity C depends on the propagation direction and, as a result, we obtained two different equations (for $\mathrm{sgn}C=+1$ and for $\mathrm{sgn}C=-1$).

The soliton solution of the KdV equation (10) has the form

$$a_0(\tau,z) = -\frac{[(\alpha^2+2\beta C)/u_0(2\alpha^2+\beta C)]\kappa^2}{\cosh^2\{\kappa[\tau+(2\kappa^2/\alpha C)(\alpha^2+2\beta C)z]/\sqrt{\alpha}\}}, \quad (11)$$

κ being an amplitude parameter of the KdV soliton. Comparison of Eq. (11) and Eq. (8) with Eq. (4) at $\beta=0$ leads to the result $\epsilon\kappa = v u_0$ which demonstrates a relation between the soliton parameter v and the perturbation scale ϵ in the small-amplitude limit (see Ref. 19).

It is important to note that Eq. (10) and the soliton solution (11) depend on the sign of the velocity $C, C=\pm 2u_0\sqrt{\alpha}$. In particular, it means that dark solitons propagating in opposite directions are different, i.e., they have different parameters (intensities) at the same value of the velocity.

A simple analysis of Eq. (11) demonstrates that there are two types of solitons near the ZD point (see Fig. 1). First of all, there are usual dark solitons which in the neighborhood of the ZD point change their shapes:

$$a_0(\tau,z) = -(2\kappa^2/u_0)\mathrm{sech}^2[\kappa(\tau+4\beta\kappa^2 z/\alpha)/\sqrt{\alpha}], \quad \alpha^{3/2} \ll \beta u_0.$$

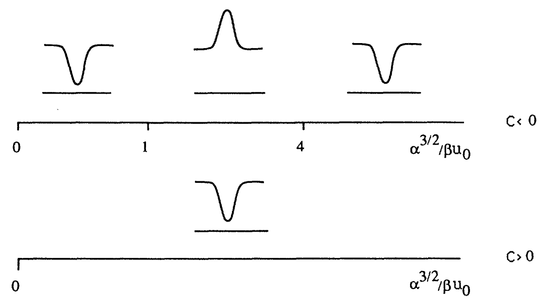

FIG. 1. Regions of the dimensionless parameter $\alpha^{3/2}/\beta u_0$ corresponding to different types of optical solitons on the cw background for $C<0$ and $C>0$.

Second, for $C<0$ there is a region

$$1 < \alpha^{3/2}/\beta u_0 < 4, \quad (12)$$

where the soliton (11) changes the sign of its amplitude and then is actually a bright soliton on a pedestal (see Fig. 1); the pedestal is the cw background which is stable in this case, too.

The above result means that in the region of the dispersion parameters (12) two different types of optical solitons may coexist together. Solitons moving to the right are dark, but solitons moving to the left are bright, and these solitons may propagate on the same cw background. Therefore, it is possible to observe interactions between different types of optical solitons; probably, this situation is the only one when dark and bright solitons may interact directly but for the nonzero cw background pulse. We do not believe that this interaction will be elastic because the system (1) is not integrable, the latter has been demonstrated in Ref. 9 for the reverse sign of the second-order dispersion α.

As follows from our analysis, there are regions in the dimensionless parameter $\delta=\alpha^{3/2}/\beta u_0$ where coefficients in Eq. (10) may change their signs. It means that for variable dispersion properties of fibers new interesting effects may be predicted. The much more important case is that in which either coefficient $(1+\beta C/2\alpha^2)$ or $(\alpha^2+2\beta C)$ in Eq. (10) vanishes at some $z=z_0$. A similar situation for two-layer shallow liquid with a changing depth is well known (see, e.g., references in Ref. 20). In the case when the coefficient in front of the nonlinear term $a_0(a_0)_\tau$ in Eq. (10) vanishes as the result of the KdV soliton transformation induced by a change of the sign of this term has been studied numerically by Helfrich, Melville, and Miles[21] and analytically by Malomed and Shrira (see Ref. 20, p. 874). The analytical approach may be directly applied to the problem under consideration and the main results of this analysis are as follows. Because of the vanishing of the coefficient $(1+\beta C/2\alpha^2)$, when the fiber's parameters are slowly changing passing the point $\delta=\alpha^{3/2}/\beta u_0=1$, e.g., from the value $\delta_1<1$ to the value $4>\delta_2>1$, the soliton amplitude varies in space, and, simultaneously, the dark soliton gives rise to a long small-amplitude shelf. The duration of this shelf is directly proportional to the distance L to the critical point $z=z_0$. After the change of sign of the coefficient $(1+\beta C/2\alpha^2)$ in Eq. (10), the shelf decays into new (bright) solitons which may be regarded as dark solitons of opposite signs of amplitude. The total number N of these new solitons can be found analytically [see Eq. (8.43) of Ref. 20]; for example, the value N^2 is directly proportional to $L^{1/2}$ and to the limit value of the coefficient $(1+\beta C/2\alpha^2)$, i.e., in fact, to the value $(1+\delta_2)/\delta_2$, where δ_2 has been defined above. The same transformation is possible when the changing coefficient $(1+\beta C/2\alpha^2)$ will cross the critical point in the opposite direction. As a result, the changing in the dispersion properties of a fiber gives rise to transformations of one type of optical soliton to another. The effect does not depend sufficiently on higher-order nonlinear terms, e.g., $a_0^2(a_0)_\tau$, which also may be taken into account near the ZD point (see Ref. 20, p. 875). It is important that these transformations do not occur in the vicinity of the second critical point $\delta=\alpha^{3/2}/\beta u_0=4$, where the linear dispersion coefficient in Eq. (10) changes its sign. As was demonstrated by Malomed and Shrira (see Ref. 20, p. 875), in that case the shelf generated by the primary soliton cannot produce secondary solitons at all, and the soliton simply decays passing the second critical point.

In conclusion, using the small-amplitude approximation we have studied the nonlinear dynamics and dark-soliton propagation near the zero of the second-order dispersion in single-mode optical fibers for the normal-dispersion regime. We have demonstrated that dark solitons may exist as stable excitations near the ZD point and, similar to bright solitons, they do not exist exactly at this point. We have also predicted a new type of optical soliton, the so-called bright soliton on a pedestal, which may exist in the region $1<\alpha^{3/2}/\beta u_0<4$, where α and β are coefficients of the second- and third-order dispersion, respectively, and u_0 is the amplitude of the background pulse. In this region it is possible to observe interactions between two optical solitons of different types. Probably, this is the only example when the direct interaction between dark and bright solitons is possible. In a fiber with changing parameters, when passing through the critical point $\alpha^{3/2}/\beta u_0=1$ is possible, transformations of one type of solitons to another may be observed. In particular, a dark soliton crossing the critical point has to generate a set of bright solitons on a pedestal.

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