Excitation spectrum and quantum inverse problem for an alternative version of the nonlinear Schrödinger equation

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We have formulated the quantum inverse problem for an alternative version of the nonlinear Schrödinger equation (NLS), which is a variant of the derivative NLS (DNLS) equation. While the DNLS equation is nonultralocal and cannot be formulated as a problem of the quantum inverse scattering method, our equation is ultralocal, and hence it is possible to construct the quantum R matrix and the commutation rules for the scattering data. Next we have constructed the algebraic Bethe ansatz, from which the eigenvalue equation for the excited states is derived. This eigenvalue equation admits stringlike configurations, and the density of states corresponding to real and complex eigenvalues is explicitly derived in the thermodynamic limit.

INTRODUCTION

The quantization of nonlinear integrable systems is an important problem that has received[1] serious attention over the last decade. There exist at present two important techniques to formulate the quantization of nonlinear systems. One is the semiclassical approach[2] and the other is the quantum inverse scattering method (QISM) developed by Sklyanin.[3] The difficulty with the second approach is that it cannot be applied to systems with nonultralocal symplectic structure. It was for this reason that the derivative nonlinear Schrödinger (DNLS) problem could not be quantized using QISM. In this paper we discuss the problem of quantizing an alternative equation that actually describes the propagation of Alfven waves in a plasma.[4] This new equation is very similar to the original DNLS equation; however, since it has an ultralocal symplectic structure it is possible to formulate a QISM for this equation. In the following we describe in detail the construction of the QISM and that of the algebraic Bethe ansatz.[5] Then we set up an equation for the determination of the eigenmomenta of the excited states, which admit stringlike solution. Finally, explicit expressions for the density of states of real and complex eigenvalues are obtained.

FORMULATION

The equations under consideration can be written as

$$\begin{aligned} \psi_{1t} &= i\psi_{1xx} - 2c\psi_1\psi_2\psi_{1x} \ , \\ \psi_{2t} &= i\psi_{2xx} - 2c\psi_1\psi_2\psi_{2x} \ . \end{aligned} \tag{1}$$

Equation (1) is an integrable system both in the sense of Painlevé analysis and in having a Lax pair. The Lax pair has a similar form regarding its dependence on the field variables. But with respect to the eigenvalue parameter λ, it resembles the Kaup-Newell problem.[6] The space part of the Lax pair for (1) can be written as

$$\left[\frac{\partial}{\partial x} + \begin{bmatrix} -\frac{1}{2}(c\psi_1\psi_2+\lambda^2) & -\lambda\psi_1\sqrt{c} \\ -\lambda\psi_2\sqrt{c} & \frac{1}{2}(\lambda^2+\psi_1\psi_2 c \end{bmatrix} \right] \psi = 0 \ . \tag{2}$$

At present there exists two approaches to the quantization: (a) the differential equation approach, which has been critized severely by Gutkin,[7] and (b) the space discretization approach, which does not have the difficulties of (a). However, it is still considered to be approximate unless a limit N is properly taken, where N is the number of subdivisions of the interval $(0,L)$ of the real axis in equal subintervals of length Δ. Here we pursue the second approach.

By converting the Lax equation (2) into a Riccati system, we can generate an infinite number of conservation laws C_n. From these it is easy to pick up the Hamiltonian in a given interval (x_0, x_0+L) as given by

$$H = \int_{x_0}^{x_0+L} C_2 dx = \frac{1}{2}\int_{x_0}^{x_0+L} [i(\psi_1\psi_{2x} - \psi_{1x}\psi_{2x}) - \psi_1\psi_2(\psi_1\psi_{2x} - \psi_{1x}\psi_2)]dx \ . \tag{3}$$

This Hamiltonian generates Eq. (1) via

$$\begin{aligned} \frac{\partial\psi_1}{\partial t} &= \{H, \psi_1\}_\alpha \ , \\ \frac{\partial\psi_2}{\partial t} &= \{H, \psi_2\}_\alpha \ , \end{aligned} \tag{4}$$

with α as the constant symplectic operator

$$\mathcal{L} = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \tag{5}$$

in the Poison bracket

$$\{f,g\} = i\int_{x_0}^{x_0+L} \left[\frac{\delta f}{\delta\psi_2}\frac{\delta g}{\delta\psi_1} - \frac{\delta f}{\delta\psi_1}\frac{\delta g}{\delta\psi_2} \right] dx \ . \tag{6}$$

Thus we proceed with the discretization of the equations on interval (x_0, x_0+L) and rewrite the Lax Eq. (2) in the form

$$\psi_{n+1}=L_n\psi_n \;, \tag{7}$$

where L_n is defined via

$$L_n \approx 1+\Delta\int_{x_0}^{x_0+L} L\,dx \;, \tag{8}$$

and

$$\psi_{1n}=\int_{x_0}^{x_0+L}\psi_1 dx \;, \qquad \psi_{2n}=\int_{x_0}^{x_0+L}\psi_2 dx \;, \tag{9}$$

The Poisson bracket is defined through

$$[\psi_{1n},\psi_{2n}]=\frac{-i}{\Delta} \tag{10}$$

and

$$L_n=\begin{bmatrix} 1-\frac{i\Delta}{2}(c\psi_{1n}\psi_{2n}+\lambda^2) & -\Delta\lambda\psi_{1n}\sqrt{c} \\ -\Delta\lambda\psi_{2n}\sqrt{c} & 1+\frac{i\Delta}{2}(\psi_{1n}\psi_{2n}c+\lambda^2) \end{bmatrix} . \tag{11}$$

The classical r matrix is defined following Faddeev via the equation

$$\{L(\lambda,x)\overset{\otimes}{,}L(u,y)\}=[r(\lambda,u),L(\lambda,x)\otimes\underline{1}+\underline{1}\otimes L(u,y)] \;, \tag{12}$$

where $\{\overset{\otimes}{,}\}$ denotes the Poisson bracket between the elements of the matrices L and $\otimes$ denotes the direct product. Computing the 16 Poisson brackets on the left-hand side we can easily solve for $r(\lambda,u)$, and

$$r(\lambda,u)=\begin{bmatrix} \frac{-(\lambda^2+u^2)}{2(\lambda^2-u^2)} & 0 & 0 & 0 \\ 0 & 0 & \frac{-\lambda u}{\lambda^2-u^2} & 0 \\ 0 & \frac{-\lambda u}{\lambda^2-u^2} & 0 & 0 \\ 0 & 0 & 0 & \frac{-(\lambda^2+u^2)}{2(\lambda^2-u^2)} \end{bmatrix} . \tag{13}$$

This form of r matrix is distinctly different from that of the NLS model, having poles at two positions $\lambda=\pm u$, but the space-time ultralocal character remains the same.

QUANTUM R MATRIX

For the quantum-mechanical case we start with the discrete form of L, i.e., L_n, and observe that $\prod_n L_n$ can be interpreted as the transition matrix. We now evaluate the equation[8]

$$R(\lambda,u)[L_n(\lambda)\otimes L_n(u)]=[L_n(u)\otimes L_n(\lambda)]R(\lambda,u) \tag{14}$$

using the ordering of ψ_{1n} and ψ_{2n}, and interpreting Eq. (10) as a commutator we obtain

$$R(\lambda,u)=\begin{bmatrix} 1+c\frac{\lambda^2+u^2}{2(\lambda^2-u^2)} & 0 & 0 & 0 \\ 0 & 1 & \frac{c\lambda u}{\lambda^2-u^2} & 0 \\ 0 & \frac{c\lambda u}{\lambda^2-u^2} & 1 & 0 \\ & & & 1+c\frac{(\lambda^2+u^2)}{2(\lambda^2-u^2)} \end{bmatrix} . \tag{15}$$

It is interesting to note that the form of quantum R matrix follows the same rule as that of the NLS equation, i.e., if one thinks of c as proportional to $\hbar$, then Eq. (15) is true to order Δ.

COMMUTATION RULE FOR THE SCATTERING DATA

Since it is written in discrete variable, the space part of the Lax equation becomes

$$\psi_{n+1}=L_n\psi_n \ . \tag{16}$$

One can interpret L_n as the transfer matrix over one lattice site.

We define the transition matrix from site n to m as

$$T(n,m,\lambda)=L_n(\lambda)L_{n-1}(\lambda)\cdots L_m(\lambda) \ , \tag{17}$$

and the monodromy matrix from site l to n as

$$T(\lambda)=L_n(\lambda)L_{n-1}(\lambda)\cdots L_1(\lambda) = \prod_{i=1}^{n} L_i(\lambda) \ . \tag{18}$$

So from the basic relation (14) we get

$$R(\lambda,u)[T(\lambda)\otimes T(u)]=[T(u)\otimes T(\lambda)]R(\lambda,u) \ . \tag{19}$$

We define

$$T(\lambda)=\begin{bmatrix} a(\lambda) & \bar{b}(\lambda) \\ b(\lambda) & -\bar{a}(\lambda) \end{bmatrix} \ . \tag{20}$$

So forming (19) we obtain

$$\begin{aligned} &a(\lambda)a(u)=a(u)a(\lambda) \ , \\ &\bar{b}(\lambda)b(u)=\bar{b}(u)\bar{b}(\lambda) \ , \\ &b(\lambda)b(u)=b(u)b(\lambda) \ , \\ &\bar{a}(\lambda)\bar{a}(u)=\bar{a}(u)\bar{a}(\lambda) \ , \\ &a(u)\bar{b}(\lambda)=\alpha(\lambda,u)\bar{b}(\lambda)a(u)-\beta(\lambda,u)\bar{b}(u)a(\lambda) \ , \\ &b(\lambda)a(u)=\alpha(\lambda,u)a(u)b(\lambda)-\beta(\lambda,u)a(\lambda)b(u) \ , \\ &b(u)\bar{a}(\lambda)=\alpha(\lambda,u)\bar{a}(\lambda)b(u)-\beta(\lambda,u)\bar{a}(u)b(\lambda) \ , \\ &\bar{a}(\lambda)\bar{b}(u)=\alpha(\lambda,u)\bar{b}(u)\bar{a}(\lambda)-\beta(\lambda,u)\bar{b}(\lambda)\bar{a}(u) \ , \end{aligned} \tag{21}$$

where

$$\alpha(\lambda,u)=1-\frac{c(\lambda^2+u^2)}{a(\lambda^2-u^2)} \ ,$$

$$\beta(\lambda,u)=\frac{c\lambda u}{\lambda^2-u^2} \ .$$

REFINEMENT OF THE YANG-BAXTER RELATION

The above calculations are confined to the periodic case $n\to -L$ to $+L$. But for $n\to\infty$ the commutation relations get changed due to the modified Yang-Baxter relation.

The expectation value of $L_n(\lambda)\otimes L_n(u)$ between the vacuum states are[9]

$$W(\lambda,u)=\langle 0|L_n(\lambda)\otimes L_n(u)|0\rangle \ .$$

Thus

$$W(\lambda,u)=\begin{bmatrix} 1-\frac{i\Delta}{2}(\lambda^2+m^2) & 0 & 0 & 0 \\ 0 & 1-\frac{i\Delta}{2}(\lambda^2-n^2) & -ic\lambda u\Delta & 0 \\ 0 & 0 & 1+\frac{i\Delta}{2}(\lambda^2-u^2) & 0 \\ 0 & 0 & 0 & 1+\frac{i\Delta}{2}(\lambda^2+u^2) \end{bmatrix} \ . \tag{22}$$

The normalized monodromy matrix $T(\lambda)$ is defined by

$$T(\lambda)=\lim_{N\to\infty} v(\lambda)^{-N}L_n(\lambda)\cdots L_{-N+1}(\lambda)v(\lambda)^{-N} \ ,$$

where

$$v(\lambda)=\begin{bmatrix} w_1 & 0 \\ 0 & w_2 \end{bmatrix} \ ,$$

with

$$w_1=1-\frac{i\Delta}{2}\lambda^2 \ , \qquad w_2=1+\frac{i\Delta}{2}\lambda^2 \ . \tag{23}$$

The Yang-Baxter relation for $T(\lambda)$ is modified to

$$R_1(\lambda,u)[T(\lambda)\otimes T(u)]=[T(u)\otimes T(\lambda)]R_2(\lambda,u) \ , \tag{24}$$

where

$$R_1(\lambda,u)=U_1(u\lambda)^{-1}R(\lambda,u)U_1(\lambda,u) \ ,$$

$$R_2(\lambda,u)=U_2(u,\lambda)R(\lambda,u)U_2(\lambda,u)^{-1} \ ,$$

$$U_1(\lambda,u)=\lim_{N\to\infty} W(\lambda,u)^{-N}[v(\lambda)^n\otimes v(u)^N] \ ,$$

$$U_2(\lambda,u)=\lim_{N\to\infty} [v(\lambda)^N\otimes v(u)^N]W(\lambda,u)^{-N} \ ,$$

and where $R_1(\lambda,u)$ and $R_2(\lambda,u)$ are given by

$$R_1(\lambda,u)=\begin{bmatrix} 1+\dfrac{c(\lambda^2+u^2)}{2(\lambda^2-n^2)} & 0 & 0 & 0 \\ 0 & 1-f(\lambda,u) & \dfrac{c\lambda u}{(\lambda^2-n^2)}[1+h(\lambda,u)] & 0 \\ 0 & \dfrac{c\lambda u}{(\lambda^2-u^2)} & 1-g(\lambda,u) & 0 \\ 0 & 0 & 0 & 1+\dfrac{c(\lambda^2+u^2)}{2(\lambda^2-n^2)} \end{bmatrix},$$

$$R_2(\lambda,u)=\begin{bmatrix} 1+\dfrac{c(\lambda^2+u^2)}{2(\lambda^2-u^2)} & 0 & 0 & 0 \\ 0 & 1-g(\lambda,u) & \dfrac{c\lambda u}{(\lambda^2-n^2)}[1+h(\lambda,u)] & 0 \\ 0 & \dfrac{c\lambda u}{(\lambda^2-u^2)} & 1-f(\lambda,u) & 0 \\ 0 & 0 & 0 & 1+\dfrac{c(\lambda^2+u^2)}{2(\lambda^2-n^2)} \end{bmatrix} \quad (25)$$

with

$$f(\lambda,u)=\left[\frac{c\lambda u}{\lambda^2-u^2}\right]^2 \times[1+2\pi i\delta(\lambda^2-u^2)(\lambda^2-u^2+i0)]\ ,$$

$$g(\lambda,u)=\left[\frac{c\lambda u}{\lambda^2-u^2}\right]^2 \times[1-2\pi i\delta(\lambda^2-u^2)(\lambda^2-u^2-i0)]\ ,$$

$$h(\lambda,u)=-\left[\frac{2c\lambda u}{\lambda^2-u^2}\right]\left[1-\left[\frac{c\lambda u}{\lambda^2-u^2}\right]^2\right] \times[1-i\pi\delta(\lambda^2-u^2)]\ .$$

One can now substitute $R_1(\lambda,u)$ and $R_2(\lambda,u)$ in Eq. (24) to calculate the refined commutation rules of the scattering data and construct the eigenstates.

CONSTRUCTION OF THE EIGENSTATE

Before we proceed to the actual construction of the eigenstates it is very important to realize that our R matrix is a function of λ and u, yet it does not depend solely on $\lambda-u$. To remedy this difficulty we set

$$\ln\lambda=\theta_1\ ,\quad \ln u=\theta_2\ ,$$

whence we observe that

$$\alpha(\theta_1,\theta_2)=1+\frac{\hbar\alpha^2}{2}\frac{\cosh(\theta_1-\theta_2)}{\sinh(\theta_1-\theta_2)}\ , \quad (26)$$

$$\beta(\theta_1,\theta_2)=\frac{\hbar\alpha^2}{2\sinh(\theta_1-\theta_2)}\ , \quad (27)$$

which exhibits a dependence on $\theta_1-\theta_2$.

Then the eigenstates $\Omega_1(\theta_1)$, $\Omega_2(\theta_1,\theta_2)$, $\Omega_3(\theta_1,\theta_2,\theta_3)$, etc., of the quantized system can be constructed by starting with a postulated vacuum. Let us designate the vacuum state by

$$\psi_{2n}|0\rangle=0\ , \quad (28)$$

which shows that the action of L_n on $|0\rangle$ yields a triangular matrix, and due to the property of triangular matrices, we find the action of $T(\theta)$.

If we write

$$T(\theta)|0\rangle=\begin{bmatrix} a(\theta) & \bar{b}(\theta) \\ b(\theta) & \bar{a}(\theta)\end{bmatrix}|0\rangle\ , \quad (29)$$

then

$$a(\theta)|0\rangle=\left[1-\frac{h\alpha^2}{2}-\frac{i\Delta\theta^2}{2}\right]^n|0\rangle\ ,$$

$$\bar{a}(\theta)|0\rangle=+\left[1+\frac{h\alpha^2}{2}+\frac{i\Delta\theta^2}{2}\right]^n|0\rangle\ , \quad (30)$$

$$\bar{b}(\theta)|0\rangle=0\ ,$$

and the ith excited state

$$b(\theta)|0\rangle=|i\rangle\ .$$

Let us designate the one-, two-, and three-particle states as

$$\Omega_1(\theta_1)=b(\theta_1)|0\rangle\ ,$$

$$\Omega_2(\theta_1,\theta_2)=b(\theta_1)b(\theta_2)|0\rangle\ , \quad (31)$$

$$\Omega_3(\theta_1,\theta_2,\theta_3)=b(\theta_1)b(\theta_2)b(\theta_3)|0\rangle\ ,$$

and similarly for n-particle states.

The algebraic Bethe ansatz can now be formulated by operating with $a(\theta)+\bar{a}(\theta)$ on $\Omega_i(\theta_1,\theta_2,\ldots,\theta_i)$ and requiring that it be an eigenstate, which is equivalent to the imposition of the condition that the unwanted terms generated due to the commutation rules (21) must vanish. For n-particle states this condition turns out to be[10]

$$\sigma\left[\frac{\sinh(\theta_i+\gamma/2-\delta/2)}{\sinh(\theta_1-\gamma/2+\delta/2)}\right]^n=\prod_{\substack{i,j\\ i\neq j}}\frac{\sinh(\theta_j-\theta_i-\delta)}{\sinh(\theta_j-\theta_i+\delta)}\ , \tag{32}$$

where σ, γ, and δ are constants determined in terms of the parameters of the systems.

For convenience let us set

$$\phi(z,c)=\ln\frac{\sinh(z+ic)}{\sinh(z-ic)}\ . \tag{33}$$

Since Eq. (32) is, in general, very difficult to solve for each θ_j, we consider the density of such states given by

$$d(\theta_j)=\operatorname*{Lim}_{N\to\infty}\frac{1}{N(\theta_{j+1}-\theta_j)}(-\pi/2\le\theta_j\le\pi/2)$$

and $\rho(\theta)=Nd(\theta)$.

The solutions of Eq. (32) can be of two kinds: real θ_i, $i=1,\ldots,M$, and complex $Z_L=\sigma_L+i\xi_L$ $(L=1,\ldots,N)$. An important observation is that when Z_i is a solution, so is $\bar{Z}_i=\sigma_i-i\xi_i$. Then Eq. (32) splits into two parts:

$$\left[\frac{\sinh(\theta_j+i\gamma/2)}{\sinh(\theta_j-i\gamma/2)}\right]^N=-\prod_{i=1}^{M}\frac{\sinh(\theta_j-\theta_i+i\gamma)}{\sinh(\theta_j-\theta_i+i\gamma)}\times\prod_L\frac{\sinh(\theta_j-Z_L+i\gamma)}{\sinh(\theta_j-Z_L-i\gamma)}\times\frac{\sinh(\theta_j-\bar{Z}_L+i\gamma)}{\sinh(\theta_j-\bar{Z}_L-i\gamma)} \tag{32'}$$

and

$$\left[\frac{\sinh(Z_j+i\gamma/2)}{\sinh(Z_j-i\gamma/2)}\right]^N=-\prod_{i=1}^{N}\frac{\sinh(Z_j-\theta_i+i\gamma)}{\sinh(Z_j-\theta_i+i\gamma)}\times\prod_L\frac{\sinh(Z_j-Z_L+i\gamma)}{\sinh(Z_j-Z_L-i\gamma)}\times\frac{\sinh(Z_j-\bar{Z}_L+i\gamma)}{\sinh(Z_j-\bar{Z}_L-i\gamma)}\ . \tag{32''}$$

In the thermodynamic limit $N\to\infty$ these equations become integral equations.[11]

Taking the logarithm of Eq. (32) we obtain

$$N\ln\frac{\sinh(\theta_j+i\gamma/2)}{\sinh(\theta_j-i\gamma/2)}=-\sum_{i=1}^{M}\ln\frac{\sinh(\theta_j-\theta_i+i\gamma)}{\sinh(\theta_j-\theta_i-i\gamma)}+\sum_L\ln\frac{\sinh(\theta_j-Z_L+i\gamma)}{\sinh(\theta_j-Z_L-i\gamma)}+\sum_L\ln\frac{\sinh(\theta_j-\bar{Z}_L+i\gamma)}{\sinh(\theta_j-\bar{Z}_L-i\gamma)}\ , \tag{34}$$

or using the definition (33), that is,

$$\phi=\ln\frac{\sinh(\theta_j-\theta_i-\delta)}{\sinh(\theta_j-\theta_i+\delta)}$$

we obtain

$$N\phi(\theta_j,\gamma/2)=\sum_{i=1}^{M}\phi(\theta_j-\theta_i;\gamma)+\sum_L[\phi(\theta_j-Z_L,\gamma)+\phi(\theta_j-\bar{Z}_L,\gamma)]+2\pi I_j\ . \tag{35a}$$

Actually these complex roots form a stringlike configuration, which in the limit of $N\to\infty$ will become a continuous distribution. In the ground state of the system the integers $\{I_j\}$ form a monotonic sequence and $I_{j+1}-I_j=1$.

Taking the difference between the two forms of Eq. (35), which use $j=K+1$ and K,

$$N[\phi(\theta_{k+1},\gamma/2)-\phi(\theta_k,\gamma/2)]=-\left[\sum_i\phi(\theta_{k+1}-\theta_i,\gamma)\right]-\sum_i\phi(\theta_k-\theta_i,\gamma)+\sum_L[\phi(\theta_{k+1}-Z_L,\gamma)-\phi(\theta_k-Z_L,\gamma)+\phi(\theta_{k+1}-\bar{Z}_L,\gamma)-\phi(\theta_k-\bar{Z}_L,\gamma)]+2\pi(I_{k+1}-I_k)\ . \tag{35b}$$

Also

$$\lim_{N\to\infty}\sum_k f(\theta_k)=\int_{-\pi/2}^{\pi/2}f(\theta)\rho(\theta)d\theta$$

and

$$\lim_{N\to\infty}\frac{I_{k+1}-I_k}{N(\theta_{k+1}-\theta_k)}=-\frac{1}{N}\rho(\theta)-\frac{1}{N}\sum_{h=1}^{N_h}\delta(\theta-\theta_h)\ .$$

Dividing (35b) by $N(\theta_{k+1}-\theta_k)$ and passing to the limit $N\to\infty$, we obtain

$$\int_{-\pi/2}^{\pi/2} dx\, \rho(x)\phi(\theta-x,\gamma) = 2\rho(\theta)+N\phi'(\theta,\gamma/2) - \sum_L [\phi'(\theta-Z_L,\gamma)+\phi'(\theta-\bar{Z}_L,\gamma)] + 2\pi \sum_{h=1}^{N_h} \delta(\theta-\theta_h)\ . \quad (36)$$

To solve this equation we apply the method of Fourier transform, let us set

$$\rho(\theta)=\sum_{m=-\infty}^{\infty} \hat{\rho}_{(m)} e^{2im\theta}\ ,$$
$$\phi'(x+iy,\gamma)=\sum_{m=-\infty}^{\infty} C_m(y)e^{2imx}\ , \quad (37)$$

where

$$C_n(y)=-2i\, sgn(y+z)\exp[-2/m(y+z)]\ .$$

Substituting in Eq. (37) we obtain for the vacuum sector

$$\bar{\rho}_{\text{vac}}(m)=\frac{Ne^{im\gamma}}{2\pi\cos(m\delta)}\ ,$$
$$\bar{\rho}_{\text{holes}}(m)=\frac{-e^{+}m\delta}{2\pi\cos(m\delta)}\sum_{h=1}^{N_h} e^{-2im\theta_h}\ . \quad (38)$$

The density of states corresponding to the complex roots can be classified into two parts; wide pairs and close pairs. And for these two cases we obtain

$$\bar{\rho}_{\text{close}}(m)=\frac{1}{2\pi}\frac{e^{m\delta}}{\cos(m\delta)} \times \sum_l e^{-2im\sigma_l}[e^{-2im(\delta-\xi_l)}+e^{-2im(\delta+\xi_l)}]\ , \quad (39)$$

$$\bar{\rho}_{\text{wide}}(m)=-\frac{1}{2\pi}\frac{e^{m\delta}}{\cos(m\delta)} \times \sum_p e^{-2im\sigma_p}[e^{-2im(\delta-\xi_p)}+e^{-2im(\delta+\xi_p)}]\ . \quad (40)$$

DISCUSSION

In our above analysis we discussed the QISM formulation of a class of NLS equations, which is similar in form to the usual DNLS problem, but with an ultralocal symplectic structure. The role of the nonlinear Schrödinger-like system is really very important in the analysis of various nonlinear problems. The original NLS equation or its analogs have been deduced by many authors from various systems in plasma or hydrodynamic problems. Such systems in the presence of a NLS-type equation have also been stressed by Calogero and Sabatier.[12] So a quantum-mechanical study of such a nonlinear equation is of utmost relevance. It is furthermore interesting to note that the algebraic Bethe ansatz can be explicitly formulated and the eigenvalues can be explicitly obtained in the thermodynamic limit. It will now be very interesting to consider a finite N correction to the density-of-state integral equation and compare the conformal invariance of the system. Such computations are under consideration and will be reported in the future.

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