Pair annihilation of pointlike topological defects in the ordering process of quenched systems

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The annihilation process of pointlike topological defects moving irreversibly is studied as a model for the growth of order in quenched systems with O(N) symmetry in $D=N$ dimensions. On the basis of the time-dependent Ginzburg-Landau model (TDGL), the defect picture is shown to be valid in any dimensions. We consider defects with pairwise power-type interaction. By a mean-field assumption we find that the total number $N(t)$ of defects decreases as $N(t)\propto t^{-D/(\beta+1)}$, where β is the exponent indicating the separation dependence of the force. We show analytically that this power law is valid for $\beta<D\leq 2\beta$ and confirm the analysis by the simulation of molecular dynamics of defects for various β's in two dimensions. The distribution of defect density is well represented by the binomial distribution function in this parameter region. To keep the distribution uniform is a key of exhibiting mean-field behavior. The TDGL model realizes the mean-field power law only in two dimensions.

I. INTRODUCTION

During recent years, much effort has been devoted to understanding the ordering process of systems quenched below critical temperature.[1,2] It is now thought that topological defects are formed in the early stage after a quench, and then the macroscopic ordering can be characterized by the degrees of freedom of the defects.[3] In such problems, systems with discrete degeneracies have been well studied. The twofold-degenerate (Ising-type) system is known to be described as kink or interface dynamics, the kinetic theories[4,5] of which have been successfully developed and confirmed by both numerical[6] and experimental[7,8] studies. In the last several years, n-fold-degenerate systems such as the Potts model[9] and Clock model[10,11] have been investigated. On the other hand, continuously degenerate systems (CDS) have been less explored. This is partly due to the experimental difficulty of the order growing too fast to observe in many CDS, such as magnetic systems and superfluid ^{4}He. Recently, an example of CDS has been observed in two-dimensional (2D) nematic liquid crystals.[12] One can expect that some liquid crystals can be objects of experimental study. Further, the production and the annihilation processes of defects in the early universe, namely, monopoles, cosmic strings, etc., is an interesting related problem.[13]

Let us consider nonconserved order parameters with O(N) symmetry in D-dimensional space often called model A.[14] If $N>D+1$ systems have no defects, which is analogous to the large-N limit case, the ordering process can be described by an integrodifferential equation for a single variable. It has been studied numerically and theoretically by Mazenko and Zannetti,[15] and by Pasquale and Tartaglia.[16] They have found the structure function in a form scaled by a characteristic length $l(t)$ increasing as $l(t)\propto t^{\mu}$, $\mu=\frac{1}{2}$. Another system investigated so far is the 3D O(2) system, where vortex strings play a crucial role. The statistical theory for the strings[17] has been studied by extensive use of the Ohta-Jasnow-Kawasaki method originally applied to interface dynamics. The correlation function of the string density exhibits a scaling behavior with $\mu=\frac{1}{2}$. Performing the computer simulation of coupled-cell dynamics for a system with the same symmetry, Nishimori and Nukii[18] have observed that random vortex strings are formed in the early stage and then shrink. They found that the length of the strings decreases as $t^{-0.7\pm0.05}$, which is slightly slower than the theoretical prediction. The difference may arise from some simplification in the theory, such as the neglecting of the recombinations of strings and the self-induction approximation for their interaction, and the finite-size effect in the simulation. These results rather confirm the validity of the defect scenario for the ordering process.

In the present paper, we consider the $D=N$ cases, where the topological defects are point singularities.[19] We have already reported a preliminary numerical result for the molecular dynamics of point defects corresponding to 2D O(2) model.[20] That simulation has shown that the number of the defects $N(t)$ decreases as t^{-1}, and the mean separations of pairs of identically signed defects d_{++} and of ones of oppositely signed defects d_{+-} both increase as $t^{1/2}$. Because $N(t)\propto d_{++}^{-2}$ is satisfied, we concluded that the defects with an identical sign are distributed randomly during the growth process. We found also that the ratio d_{++}/d_{+-} has taken a constant value 1.6 ± 0.1 after a transient stage, different significantly from the initial value $d_{++}/d_{+-}=1$. We add in the present paper some detailed information of this growth process. A suggestion for high-dimensional systems $D\geq 3$ is also given.

In Sec. II, we show that the defect scenario can be generally adopted for the ordering process in the model A of $N=D$. We give a simple derivation of the interacting force between a pair of defects first derived by Ostlund.[21] Next, we show that the rate equation for the density of defects leads to a power-growth law under a scaling as-

sumption. Finally, we discuss its applicability in a parameter space (D,β), where β is the exponent indicating that the force between defects depends on the separation r as $r^{-\beta}$. We suggest that the mean-field exponent is not valid for the ordering in $D \geq 3$.

In Sec. III, we report some simulation results for the defect kinetics. The growth exponent $\mu=\frac{1}{2}$ is shown more accurately than in the previous work. We obtain not only the averaged values $N(t)$, d_{++}, d_{+-}, but also the distribution function of the density of defects. Examining cases of $\beta \neq 1$, we test the theoretical prediction for the applicability of the mean-field argument in Sec. II.

II. MEAN-FIELD REGION AND GROWTH EXPONENT

A. Model

The model A for N-component order parameter ψ is given by

$$\frac{\partial \psi}{\partial t}=-\frac{\delta F}{\delta \psi} ,$$
$$F[\psi]=\int d^D r \left[\frac{1}{2} c|\nabla \psi|^2-\frac{1}{2}\gamma|\psi|^2+\frac{1}{4!}g|\psi|^4 \right] . \quad (1)$$

Consider the growth process after the system is brought into the ordered state $\gamma>0$ from the disordered state $\gamma<0$. In the early stage after a quench the amplitude $|\psi|=\Psi$ grows as driven mainly by the local force generated by the second and third terms in $F[\psi]$. The neglect of the nonlocal term $c|\nabla\psi|^2$ yields

$$\frac{\partial \Psi}{\partial t}=\gamma\Psi-\frac{g}{6}\Psi^3 ,$$

the solution of which is given by

$$\Psi(t)=\frac{Ae^{\gamma t}}{[1+(A/\Psi_{\rm eq})^2 e^{2\gamma t}]^{1/2}} ,$$

where $A=\Psi(0)/[1-\Psi(0)^2/\Psi_{\rm eq}^2]^{1/2}$, and $\Psi_{\rm eq}=(6\gamma/g)^{1/2}$ is the equilibrium value of Ψ. Thus the growth time of Ψ is given by γ^{-1}. In the stage $t\sim\gamma^{-1}$, because of the initial fluctuation, all possible orientations of the ordering will be represented among domains. The domain size is estimated as $l_0\sim(c/\gamma)^{1/2}$, because the phase of ψ obeys a diffusion-type equation with the diffusion constant c. This means that topological defects distributed with the mean separation l_0.[22] Thus the late stage can be described by the motion of defects.

The interaction of point defects has been calculated by Ostlund for a typical configuration.[21] Here we will derive the energy by a dimensional consideration. A simple configuration of a defect of winding number $+1$ is written by

$$\psi(r)=\Psi_{\rm eq}(\mathbf{r}-\mathbf{r}_0)/|\mathbf{r}-\mathbf{r}_0|$$

outside the core region. Because the gradient of $\psi(\mathbf{r})$ at $|\mathbf{r}-\mathbf{r}_0|\sim R$ is of order of $\Psi_{\rm eq}/R$, the energy is estimated as $F_d\sim(c/2)\int|\nabla\psi|^2 d^D R\sim c\Psi_{\rm eq}^2 L^{D-2}$, where L denotes the linear size of the system. When two defects of winding numbers $+1$ and -1 are approached each other by $d_\parallel$, $\psi(\mathbf{r})$ is distorted in an area of $d_\parallel d_\perp^{D-1}$, where $d_\perp$ denotes the vertical length of the distortion area. Generally, for finite $d_\parallel$ and $d_\perp$, F_d is given by $F_d\sim d_\parallel d_\perp^{D-3}$. This means that the distortion area can lower the energy by collapsing into a singular line in high dimensions $D\geq 3$. Therefore, we conclude that $F_d\sim d_\parallel$ for $D\geq 3$. For $D=2$, $d_\perp$ is of the same order of $d_\parallel$, and then we have $F_d\sim d_\parallel^0$; that is, F_d depends logarithmically on $d_\parallel$. Accordingly, we have the energy as

$$F=\begin{cases} \sum_{\langle i,j\rangle}\sigma_i\sigma_j|\mathbf{r}_i-\mathbf{r}_j| & \text{for } D\geq 3 , \quad (2) \\ \sum_{\langle i,j\rangle}\sigma_i\sigma_j\ln|\mathbf{r}_i-\mathbf{r}_j| & \text{for } D=2 , \quad (3) \end{cases}$$

where σ_i is an integer denoting the winding number of a defect i, and $\sum_{\langle i,j\rangle}$ represent the sum over all distinct pairs of defects. The logarithmic interaction in 2D is well established in the study of the phase transition and the critical dynamics. For high-dimensional cases, it is not necessarily clear whether the superposition of pair interactions is valid. Equation (3), however, would be a first approximation for the present problems. We then write the equation of motion for defects as

$$\frac{d\mathbf{r}_i}{dt}=-\frac{\partial F}{\partial \mathbf{r}_i}=\sigma_i\sum_j\sigma_j\frac{\mathbf{r}_i-\mathbf{r}_j}{|\mathbf{r}_i-\mathbf{r}_j|^{\beta+1}} , \quad (4)$$

where we introduce an exponent β at a matter of convenience later. In the present model, $\beta=1$ for $D=2$, and $\beta=0$ for $D\geq 3$. Since two oppositely signed defects are attracted to each other and move irreversibly, they vanish eventually when they get close to the core radius ξ.

B. Mean-field exponent

On the basis of the assumption that the ordering process can be scaled by a single length $l(t)$, we estimate the growth exponent. If defects with $\sigma=+1$ and -1 of the same number $N(t)$ are distributed randomly and homogeneously, one can write the rate equation as

$$\frac{dN(t)}{dt}=-\alpha(t)N(t) ,$$

where the coefficient $\alpha(t)$ denotes the inverse of the mean collision time of pairs with opposite signs. Putting the characteristic pair separation by $l(t)$, we obtain with use of Eq. (4)

$$\alpha(t)\sim l(t)^{-(\beta+1)} . \quad (5)$$

Since $N^{-1/D}\sim l(t)$, we then have

$$\frac{dN(t)}{dt}\sim -N(t)^{(D+\beta+1)/D} ,$$

which yields

$$N(t)\sim t^{-D/(\beta+1)},\quad l(t)\sim t^{1/(\beta+1)} . \quad (6)$$

For the real defect systems, this immediately gives $N(t)\sim t^{-1}$ for $D=2$ $(\beta=1)$ and $N(t)\sim t^{-D}$ for $D\geq 3$

($\beta=0$). The dimensional argument for Eq. (4) also gives the relation (6).

C. Estimation of scaling region

If pair annihilations occur locally, the local excesses of defects or antidefects are left, so that the defects and the antidefects are segregated from each other as the annihilation proceeds.[23] On the other hand, the long-range interaction has a role of making the distribution uniform. When both effects are balanced, the mean-field scaling will be satisfied. We investigate the region of that scaling. Introducing the density field $\rho_{\pm}(\mathbf{r})$ for defects of $\sigma=\pm 1$, we can write a force on a defect at $\mathbf{r}$:

$$\mathbf{f}=\pm\int d^D r'\delta\rho(\mathbf{r}')\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^{\beta+1}} , \tag{7}$$

where $\delta\rho(\mathbf{r})=\rho_+(\mathbf{r})-\rho_-(\mathbf{r})$. To know the diffusing property, consider the variance of the force

$$\sigma_f^2=\langle|\mathbf{f}|^2\rangle=\int d^D r'\,d^D r''\langle\delta\rho(\mathbf{r}')\delta\rho(\mathbf{r}'')\rangle \times\frac{(\mathbf{r}-\mathbf{r}')\cdot(\mathbf{r}-\mathbf{r}'')}{|\mathbf{r}-\mathbf{r}'|^{\beta+1}|\mathbf{r}-\mathbf{r}''|^{\beta+1}} . \tag{8}$$

We assume that the distribution at time t is Gaussian similar to the initial distribution:

$$\langle\delta\rho(\mathbf{r}')\delta\rho(\mathbf{r}'')\rangle=A_0\exp\left[-\frac{|\mathbf{r}'-\mathbf{r}''|^2}{2l(t)^2}\right] , \tag{9}$$

where

$$A_0=\langle\delta\rho^2\rangle=2\rho_0^2[1-\langle\rho_+(\mathbf{r})\rho_-(\mathbf{r})\rangle/\rho_0^2] ,$$
$$\rho_0^2=\langle\rho_+^2\rangle=\langle\rho_-^2\rangle .$$

Because $\langle\rho_+\rho_-\rangle/\rho_0^2$ should be independent of time for the self-similar growth, we have

$$A_0\sim N(t)^2\sim l(t)^{-2D} . \tag{10}$$

Substituting Eq. (9) into Eq. (8), we then execute the integration over a region $\xi\le|\mathbf{r}'|\le L$, $\xi\le|\mathbf{r}''|\le L$, and $\xi\le|\mathbf{r}'-\mathbf{r}''|\le L$, and estimate the dominant term for $\xi\to 0$ and $L\to\infty$, where ξ and L are the core radius and the linear size of the system, respectively. Since the calculation is straightforward but somewhat complicated, we present the detail in the Appendix. The result is

$$\sigma_f^2\sim\begin{cases} L^{D-2\beta}l(t)^{-D} & \text{for } D>2\beta , \quad (11a)\\ l(t)^{-2\beta} & \text{for } \beta<D\le 2\beta , \quad (11b)\\ \xi^{2(D-\beta)}l(t)^{-2D} & \text{for } D\le\beta . \quad (11c)\end{cases}$$

We specify the regions in the parameter space (D,β) corresponding to Eqs. (11a)–(11c) by (a)–(c), respectively. The excess density is diffused by this force fluctuation. The diffusion length is given by $\eta_D(t)\sim\sigma_f\alpha(t)^{-1}$. From Eq. (5) we have $\eta_D(t)\sim l(t)$ for the region (b), which means that the diffusion length is proportional to the mean separation of nearest-neighbor pairs. Hence the self-similar growth will be observed in this region. One should note also that σ_f does not depend on the upper or lower sizes only in (b). This implies that the assumption of self-similarity in growth cannot be justified in the outside regions (a) and (c).

On the border lines, σ_f depends logarithmically on L or ξ, where the initial condition may be essentially relevant to whether the mean-field scaling is satisfied. For a case where the defects are randomly distributed in the initial state, which is the most plausible initial condition for quenched systems, the sign of equality should be attached as seen in Eqs. (11a)–(11c). This is because on $D=2\beta$, the long-range force can make the distribution appropriate to the self-similar growth whereas on the short-range boundary $D=\beta$, defects will be annihilated locally more rapidly than the distribution is rearranged.

III. SIMULATION OF DEFECT-ANTIDEFECT ANNIHILATION

We perform a simulation for the system described by Eq. (4). In the present simulation the winding number σ_i of a defect is constrained to be $\sigma_i=+1$ or -1, which we call here defect and antidefect, respectively. Although higher-order defects are formed in the early stage of evolution, they are annihilated faster than unit defects because (i) the interaction is proportional to the winding number and (ii) the collision of a pair with different winding numbers yields a lower-order defect. Therefore, the late stage of evolution is described by a system of unit defects.

If all the pair interactions are taken into account, the calculation time increases proportional to N_0^2, where N_0 denotes the initial defect number. To simulate a system as large as possible, we truncate the interaction range as follows. Dividing the system into square cells $\{(i,j)\}$ with the same size, we sum the force on a defect in (i,j) cell from the ones in the neighbor cells $\{(i\pm k,j\pm l)|0\le k,l\le i_c\}$. This is reasonable because the forces from far defects would be canceled out on account of the randomness of the distribution.[23] This approximation reduces the calculation time to the order N_0. We assume the periodic boundary condition.[24]

First, we randomly distribute defects and antidefects of

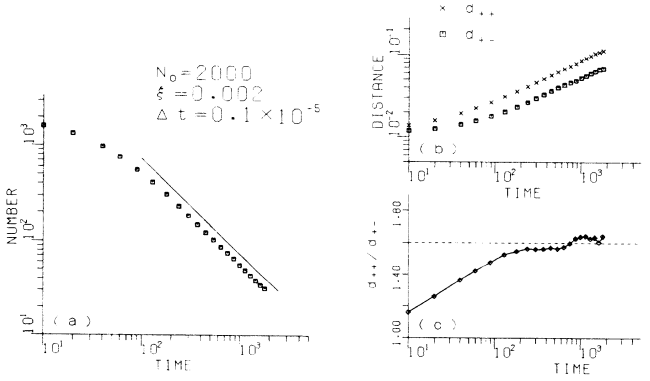

FIG. 1. (a) The number of defects, (b) the mean separations d_{++} and d_{+-} of nearest-neighbor pairs of defect-defect and defect-antidefect, and (c) d_{++}/d_{+-} vs time for the $\beta=1$ case. The solid lines represent the power law expected by the mean-field argument (6). Time is indicated in units of Δt.

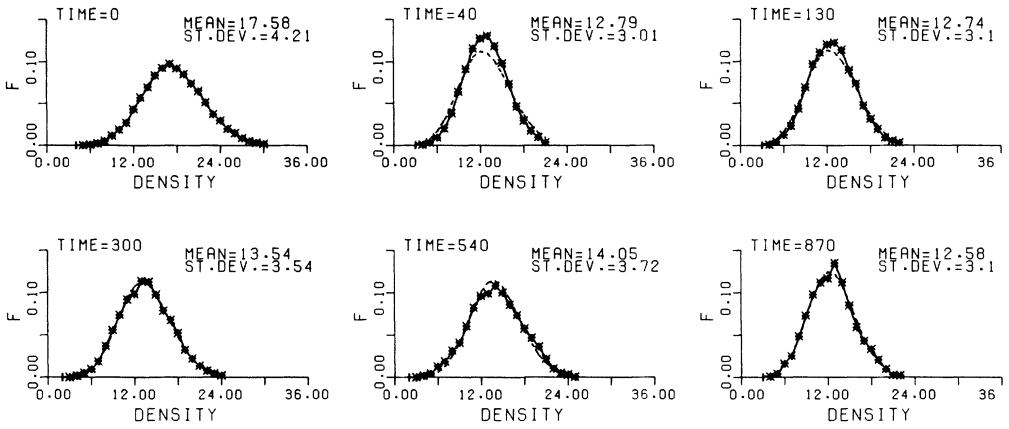

FIG. 2. Distribution function of the number of defects in square cells for the $\beta=1$ case. The dashed lines indicate the binominal distributions determined by the total number and the cell size.

the same number N_0 in a square of unit area. Then we update the defect positions by the Euler difference method with a unit step Δt. On the evolving process, we annihilate the pairs of defect-antidefect coming within the core radius ξ. The cell number (i,j) for each defect is reassigned step by step, and the cell size is changed so that the average number of defects per cell, n_c, is kept to be constant. Note that the whole procedure needs calculation time proportional to N_0 at most. The following results have been obtained under the parameter values $i_c=3$ and $n_c=2$.

Figure 1 shows a result for $\beta=1$: (a) the number of defect $N(t)$ and (b) the mean separations of nearest-neighbor defect-defect and defect-antidefect pairs defined by

$$d_{\substack{++\\+-}}(t)=\frac{1}{N(t)}\sum_{i=1}^{N(t)}\min[\{|\mathbf{r}_i^{(+)}-\mathbf{r}_j^{(\pm)}|:\ 1\le j\le N(t)\}]\ ,$$

where $(+)$ and $(-)$ denote defect and antidefect, respectively, and the $i=j$ term is excepted for d_{+-}. Twenty runs with different initial distributions were done for averaging. After a transient stage, the system exhibits $N\propto t^{-1}$, $d_{++}\propto d_{+-}\propto t^{0.5}$, which agrees with the mean-field argument (6). The averaged value of d_{++}/d_{+-} for over 20 runs is found to be initially 0.99 ± 0.02 as expected from random distributions, while in the scaling region the ratio becomes $d_{++}/d_{+-}\approx 1.6$ [Fig. 1(c)]. This value is characteristic of the correlation between defects and antidefects.

Next we count the number of defects m in 9 cells centered at (i,j) for each (i,j)'s and compute its relative frequency $F(m)$ representing the density distribution function. The result is shown in Fig. 2, which is in good agreement with the binomial distribution function

$$F_B(m)=\{N!/[m!(N-m)!]\}\langle m\rangle^m(1-\langle m\rangle/N)^{N-m}\ , \tag{12}$$

where $\langle m\rangle$ denotes the averaged m. This indicates that the spatial distribution of defects continues to be random during the evolution as well as the initial state.

Figures 3 and 4 show respective results for $\beta=0.5$ and

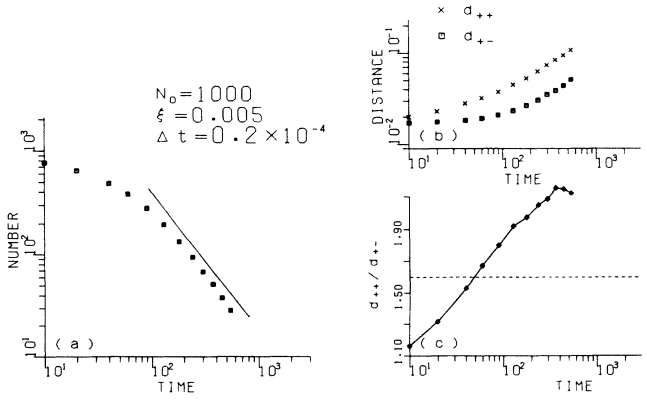

FIG. 3. (a) The number of defects, and (b) the mean separations of nearest-neighbor pairs d_{++} and d_{+-}, and (c) d_{++}/d_{+-} vs time for the $\beta=0.5$ case. The solid line has a slope of $-\frac{4}{3}$ given by Eq. (6). The dashed line in (c) is the asymptotic ratio of the $\beta=1$ case.

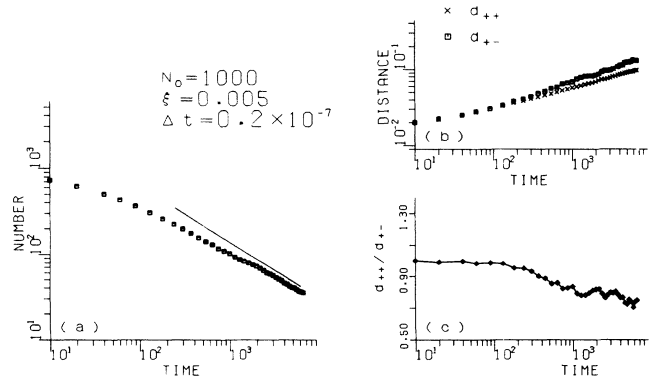

FIG. 4. (a) The number of defects, and (b) the mean separations of nearest-neighbor pairs d_{++} and d_{+-}, and (c) d_{++}/d_{+-} vs time for the $\beta=2$ case. The solid line has a slope of $-\frac{2}{3}$ given by Eq. (6).

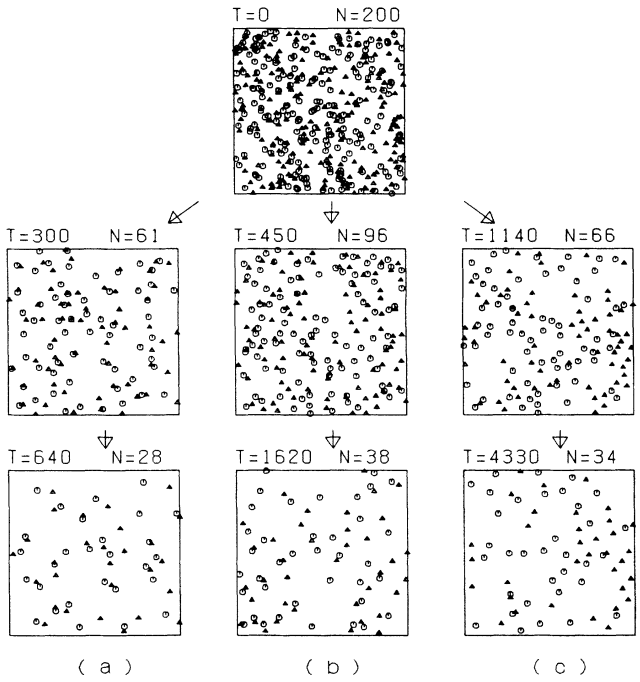

FIG. 5. Evolutions of the distribution of defects for (a) $\beta=0.5$, (b) $\beta=1$, and (c) $\beta=2$ from the same initial configuration. Defects and antidefects are shown by circles and triangles, respectively, and the time T is indicated in units of Δt. (a) $\Delta t=0.2\times10^{-4}$, (b) $\Delta t=0.1\times10^{-5}$, and (c) $\Delta t=0.2\times10^{-7}$.

2. For the $\beta=2$ case, we obtained $N(t)\propto t^{-0.54}$. This exponent is less than the theoretical value. In the $\beta=0.5$ case, the density decreases faster than the expected one. These deviations are understood qualitatively as follows. Figure 5 shows the three cases started from a single distribution, where one can see that (i) the mean separations of defect-antidefect decrease with the range of force and (ii) the excess regions of defects or antidefects appear solely in the case of $\beta=2$.[25] These circumstances agree with the theoretical expectation in Sec. II C. Figure 6 represents the density fluctuation. The short-range case ($\beta=2$) exhibits the broadening of $F(m)$ corresponding to the segregation in Fig. 5(c). On the other hand, in the long-range case ($\beta=0.5$), $F(m)$ narrows because the long-range force flattens the density fluctuation.

IV. DISCUSSION

In this paper we have considered the irreversible pair-annihilation process of point defects as a model for the late-stage ordering in quenched systems. By considering the fluctuation of force causing effectively the diffusion of excess density of defects or antidefects, we have shown that the temporal behavior can be classified into three regions in the space of relevant parameters (D,β). In the region (b) of $\beta<D\leq2\beta$, the diffusion length in the mean collision time grows in proportion to the mean separation of defects, where the mean-field behavior is exhibited. The system in the region (a) of long-range force $(\beta<D/2)$ exhibits anomalous diffusion, where the density fluctuation flattens the initial random distribution. In the region (c) of short-range force $(D\leq\beta)$, the local excesses of defect or antidefect density leave without diffusion. From this consideration we have concluded that the (2,2) system obeys the mean-field power law, but the high-dimensional system does not. Thus the dimensional (mean-field) argument for ordering dynamics is not generally valid, although it is successful in deriving the power laws in scalar order-parameter systems.

The simulation leaves two main questions. First, what function of time describes the change of the defect density in the regions (a) and (c) (the non-mean-field exponent have not been derived theoretically), and second, do systems on the border $(\beta=D)$ between (b) and (c) exhibit the mean-field behavior if the initial distribution is appropriate? These are interesting kinetic problems even if apart from ordering dynamics. From the equation of motion given in Sec. II B, we concluded that the high-dimensional cases $D\geq3$ belong to the region (a), where the growth process would be faster than the mean-field power law. Although we have performed a simulation for $(D,\beta)=(3,0)$, the system is numerically unstable because of extremely long-range force, so that a definite re-

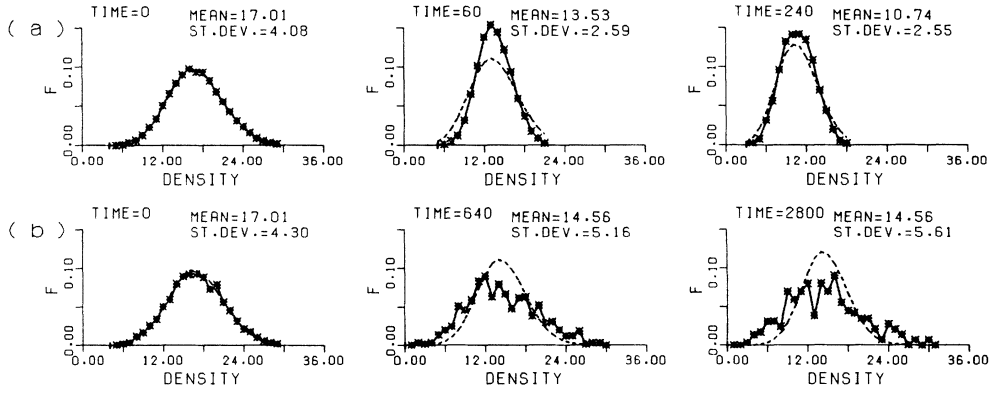

FIG. 6. Distribution function of the number of defects in some temporal stages for (a) $\beta=0.5$ and (b) $\beta=2$. The dashed lines are the binominal distribution functions defined by Eq. (12).

sult has not been obtained. The numerical unstableness corresponds to the argument that σ_f diverges as $L \to \infty$.

We left a further basic question as to whether the interaction energy can be decomposed to pairwise terms. Nonlinearity arising from high dimensionality makes it complicated. Nevertheless, it would be meaningful to describe the ordering process in three dimensions by the kinetics of topological defects, because (i) recently defects were found numerically to play a crucial role in the phase transition of the 3D Heisenberg model,[26] and (ii) the ordering in the 2D system is successfully described by the model of defects with pairwise interaction as well as the phase transition, though the interaction of topological excitations intrinsically cannot be superposed even in low-dimensional systems. No information is available at present on the nonlinearity modifies the pairwise expression of energy.

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APPENDIX

We derive Eqs. (11a)–(11c) from Eqs. (8)–(10). Substituting (9) into (8) and transforming variables $\mathbf{r}'$ and $\mathbf{r}''$ as $\mathbf{r}-\mathbf{r}'=\mathbf{R}_1$ and $\mathbf{r}''-\mathbf{r}'=\mathbf{R}_2$, we have

$$\sigma_f^2 = A_0 \int dR_1\, dR_2\, d\Omega_1\, d\Omega_2 R_1^{D-1} R_2^{D-1} \times \frac{\exp[-R_2^2/2l(t)^2] R_1 (R_1 - R_2\cos\theta)}{R_1^{\beta+1}(R_1^2+R_2^2-2R_1R_2\cos\theta)^{(\beta+1)/2}} ,$$

where $R_i = |\mathbf{R}_i|$, Ω_i is a solid angle, and θ the angle between $\mathbf{R}_1$ and $\mathbf{R}_2$. Integration with respect to Ω_1 and Ω_2 and the partial integration for R_1 yield

$$\sigma_f^2 = \frac{A_0 K_d^2}{2(1-\beta)(3-\beta)} [P_L - P_\xi - (D-\beta-1)Q] ,$$

where

$$P_x = x^{D-\beta-2} \int dR\, R^{D-2} u(R,x) \exp[-R^2/2l(t)^2] ,$$

$$Q = \int\int dR_1\, dR_2\, R_1^{D-\beta-3} R_2^{D-2} u(R_1,R_2) \times \exp[-R_2^2/2l(t)^2] ,$$

$$u(x,y) = (x+y)^{3-\beta} - |x-y|^{3-\beta} ,$$

and $K_d = 2\pi^{D/2}/\Gamma(D/2)$ is the element of solid angle. Using the power-series expansion for $u(x,y)$,

$$u(x,y) = 2x^{3-\beta} \sum_{n=0}^{\infty} a_n \left(\frac{y}{x}\right)^{2n+1} \quad \text{for } x > y+\xi ,$$

where

$$a_n = \frac{\prod_{m=0}^{2n} (3-\beta-m)}{(2n+1)!} ,$$

and exchanging x and y for $x < y-\xi$, and performing the integration, we obtain

$$P_L = 2L^{D-2\beta} \sum_{n=0}^{\infty} a_n L^{-2n} g_{D+2n-1} ,$$

$$P_\xi = 2\xi^{D-\beta-1} \sum_{n=0}^{\infty} a_n \xi^{2n} g_{D-\beta-2n} ,$$

$$Q = 2 \sum_{n=0}^{\infty} a_n \left[\frac{L^{D-2\beta-2n}}{D-2\beta-2n} g_{D-1+2n} - \frac{\xi^{D-\beta+2n-1}}{D-\beta+2n-1} g_{D-\beta-2n} - \frac{\beta+4n-1}{(D-2\beta-2n)(D-\beta+2n-1)} g_{2D-2\beta-1} \right] ,$$

where we define

$$g_\delta = \int_\xi^L dx\, x^\delta \exp[-x^2/2l(t)^2] .$$

The asymptotic behavior for $L \to \infty$ and $\xi \to 0$ is estimated as

$$g_\delta \sim \begin{cases} l(t)^{\delta+1} & \text{for } \delta > -1 , \\ \xi^{\delta+1} & \text{for } \delta \le -1 , \end{cases}$$

which yields

$$\sigma_f^2 \sim \begin{cases} A_0 L^{D-2\beta} l(t)^D & \text{for } D > 2\beta , \\ A_0 l(t)^{2(D-\beta)} & \text{for } \beta < D \le 2\beta , \\ A_0 \xi^{2(D-\beta)} & \text{for } D \le \beta . \end{cases} \tag{A1}$$

Substituting Eq. (10) into Eq. (A1), we obtain Eqs. (11a)–(11c).

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[22]One should note that l_0 represents also the order of the core radius defined by the region of $\Psi \ll \Psi_{eq}$ [see, for example, A. Onuki, J. Phys. C **15**, L1089 (1982)]. If the core radius is larger than l_0, the defect picture would not make sense. This is related to the fact that, by rescaling variables as $\psi(\mathbf{r},t)/\Psi_{eq}=\psi_R(\mathbf{x},\tau)$, $\gamma t=\tau$, and $(c/\gamma)^{1/2}\mathbf{r}=\mathbf{x}$, one can reduce Eq. (1) to the parameter-free equation $\partial\psi_R/\partial\tau=\Delta_x\psi_R+(\psi_R-|\psi_R|^2\psi_R)$.

[23]In Ref. 20, we confirmed the validity of this approximation by comparing the version of full calculation of interactions with approximate one.

[24]In the previous simulation (Ref. 20), we adopt the absorption-type boundary with mirror image effect as corresponding to supefluid ^{4}He, while in the present trial we choose the periodic one. Although the former is rather realistic, the latter has the advantage of eliminating the finite-size effect, so that we would know the scaling property of the temporal evolution with accuracy. Owing to the absorption at the boundary, the previous one has resulted in an exponent about 5% larger than expected theoretically. Such difference does not appear in the present simulation.

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