Ising model in a time-dependent magnetic field

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We report the results of Monte Carlo simulations on a two-dimensional Ising model in a sinusoidally oscillating external magnetic field. We find evidence for a dynamical phase transition, supporting the results of recent mean-field and large-N analyses of this model. We also analyze the hysteresis loops as a function of the amplitude and frequency of the applied field, fitting our data to a proposed areal scaling law.

I. INTRODUCTION

The subject of nonequilibrium effects associated with first-order phase transitions has received considerable attention in recent years, with most of that attention focusing on the quench of a spin system from a disordered to an ordered phase.[1] Yet relatively little attention has been paid to nonequilibrium effects associated with the periodic driving of a spin system between two equivalent ordered phases. There have been two recent theoretical studies of this problem, a mean-field analysis by Tomé and de Oliveira,[2] and a large N (N represents the number of spin components) and Monte Carlo analysis by Rao, Krishnamurthy, and Pandit.[3] Both of these studies have suggested the existence of a dynamical phase transition for this model in the H_0-T phase diagram (see Fig. 1), where H_0 is the amplitude of the external field with sinusoidal time dependence, $H(t)=H_0\sin\omega t$. The phase-transition line separates a phase that is disordered in a time-averaged sense [i.e., the time-dependent magnetization $M(t)$ can be nonzero, but $\langle M(t)\rangle=0$ where $\langle\ \rangle$ denotes a time average], from an ordered phase where $\langle M(t)\rangle\neq 0$. The mean-field analysis of Ref. 2 located a dynamical tricritical point on this phase-transition line separating continuous from first-order transitions. In Ref. 3 an $N=\infty$ solution was analyzed numerically, and a dynamical phase transition was found, however, the order of this transition and the complete phase diagram were not studied. Reference 3 also studied the shape of the hysteresis loops in great detail finding evidence for a scaling law for the area of the hysteresis loop (see Sec. II below).

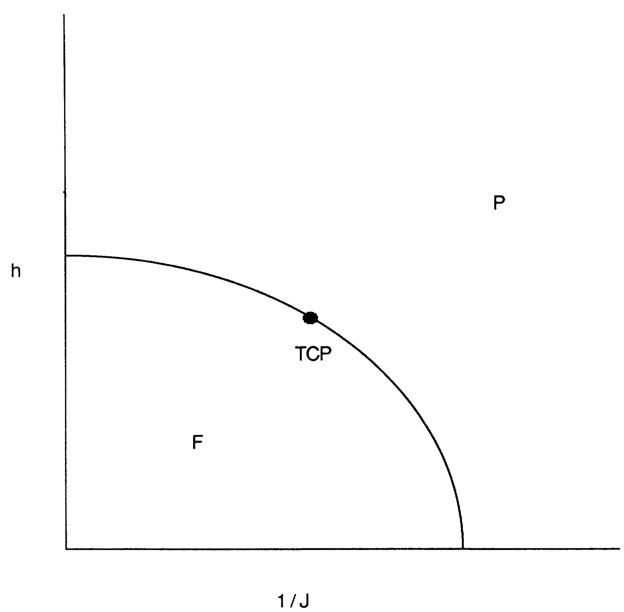

FIG. 1. Mean-field phase diagram for an Ising model in an ac field, from Ref. 2. The external field is given by Eq. (2), and $h\equiv H/J$. The tricritical point (TCP) separates first-order (on the left) from continuous transitions (on the right). The disordered phase ($\langle M(t)\rangle=0$) is denoted by P and the ordered phase [$\langle M(t)\rangle\neq 0$] is denoted by F.

In this paper we report on Monte Carlo simulations on a two-dimensional Ising model in a sinusoidally oscillating magnetic field. The only other simulations done to date on this model are contained in Ref. 3 and were on relatively small systems (50×50) and a detailed study of the phase diagram was not possible due to the authors' limited computational facilities. We report on simulations done on systems 140×140 lattice sites. We find evidence for a dynamical phase transition of the sort described above, though we are unable to say with any confidence whether a tricritical point exists or not. We also consider the hysteretic areal scaling law first proposed in Ref. 3. This law appears to fit our data though with exponents different than those found in Ref. 3.

II. THE MODEL AND NUMERICAL RESULTS

A. The model

We consider a two-dimensional Ising model on a square lattice with a time-dependent external field. Its Hamiltonian is given by

$$\mathcal{H}=-J_0\sum_{\langle i,j\rangle} s_i s_j - H(t)\sum_i s_i \ , \tag{2.1}$$

where $s_i=\pm 1$, $\langle i,j\rangle$ denotes nearest-neighbor pairs, and the external field $H(t)$ is given by

$$H(t)=H_0\sin\omega t \ . \tag{2.2}$$

We denote the field period by $\tau=2\pi/\omega$. Our Monte Carlo simulations were performed on lattice sizes of 140×140, and we employ periodic boundary conditions.

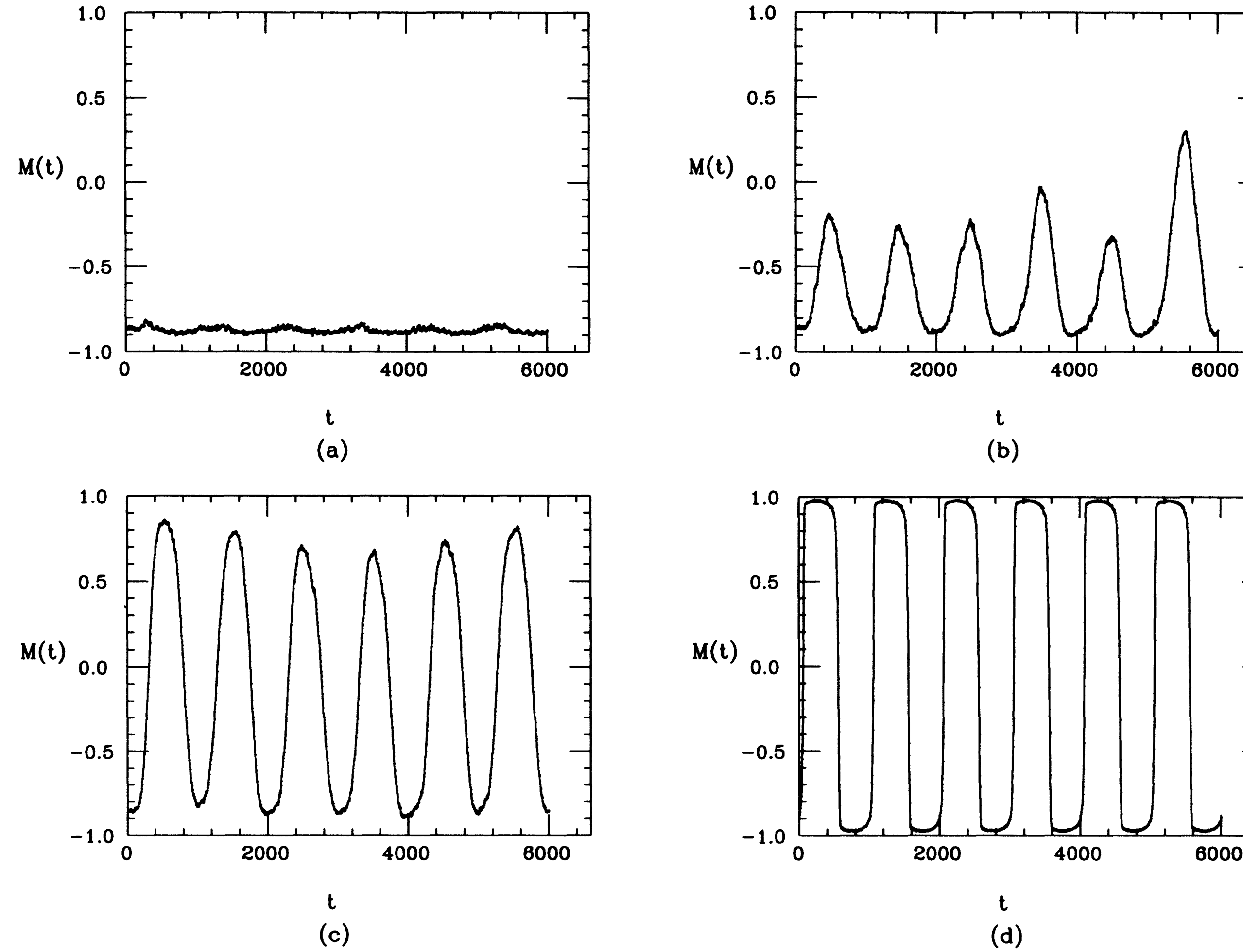

FIG. 2. Plots of the magnetization $M(t)$ vs t in MCS for $J=0.48$, and a field period $\tau=1000$ MCS for different values of H: (a) $H=0.01$, (b) $H=0.03$, (c) $H=0.04$, and (d) $H=0.4$. The phase transition occurs for a value of H in the range 0.03–0.04. Decreasing τ leads to a higher value of the critical field.

We use the standard Metropolis Monte Carlo algorithm.[4,5] We define a dimensionless coupling $J \equiv J_0/k_B T$. From the exact solution of the two-dimensional Ising model[6] we know that the critical coupling at $H=0$ is given by

$$J_c = \frac{J_0}{k_B T_c} = -\tfrac{1}{2}\ln(1-\sqrt{2}) \approx 0.440\,686\,8 \ . \tag{2.3}$$

We also define a dimensionless field amplitude $H \equiv \beta H_0$.

B. Results of the Monte Carlo simulations

At time $t=0$ we equilibrate our system in the absence of the external field $H(t)$. We work at temperatures $T<T_c$, so we begin with a configuration with all spins down and sweep through the lattice following the Monte Carlo algorithm. One sweep constitutes one Monte Carlo step (MCS). After we establish equilibrium at the temperature T we introduce the sinusoidal field $H(t)$ changing the field after one complete sweep through the lattice. We then monitor the magnetization as a function of time. Typical data for a fixed temperature is shown in Fig. 2 where we plot $M(t)$ versus t for several different field amplitudes H. A dynamical phase transition when $H_c \approx 0.035$ is apparent. For $H<H_c$ the magnetization fluctuates about a *nonzero* average value whereas for $H>H_c$ the average magnetization is zero. We have

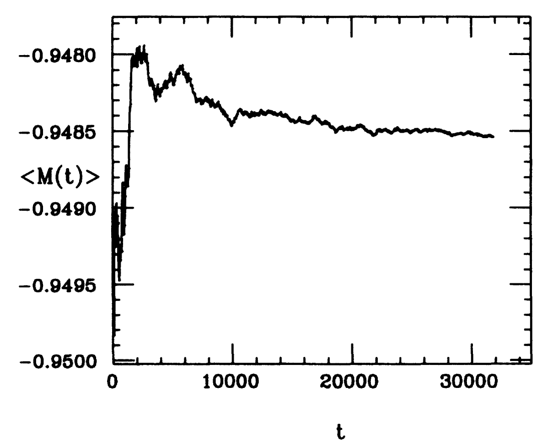

FIG. 3. Time-averaged magnetization, $\langle M(t)\rangle$, for $\tau=50$ MCS; $J=0.55$ and $H=0.09$. Note the scale of the magnetization.

checked this assertion by running the simulations up to 650 periods of the field (for $\tau=50$ MCS) and the average magnetization is essentially time independent (with a standard deviation less than 10^{-4}) (see Fig. 3). In that sense the large fluctuations about the mean evident in Figs. 2(b) and 2(c) do *not* represent an upward trend of $\langle M(t)\rangle$; rather they are merely large fluctuations. From Fig. 3 it is apparent that the transient period after the introduction of the field is approximately 200 field periods.

As first discussed by Rao, Krishnamurthy, and Pandit,[3] the hysteresis loops will look qualitatively different in different regions of the $H-T$ phase diagram. In Fig. 4 we show examples of these loops at a fixed temperature given by $J=0.48$ and a $\tau=1000$ MCS. For large enough fields [Figs. 4(c)–4(e)], it is meaningful to calculate the area of this loop. In Ref. 3 the large-N calculation produced a scaling law for this area of the form

$$A \propto H^{\alpha'}\omega^{\beta'} , \tag{2.4}$$

where $\alpha'=0.66\pm0.05$ and $\beta'=0.33\pm0.03$ independent of temperature. In Fig. 5 we plot on a log-log scale the area versus H [Fig. 5(a)] and the area versus $\tau=2\pi/\omega$ [Fig. 5(b)]. From these plots we conclude on the basis of least-squares fitting that

$$\alpha' \approx 0.46\pm0.05 , \tag{2.5a}$$

$$\beta' \approx 0.36\pm0.06 , \tag{2.5b}$$

if we neglect the roundoff at small H (where the statistics of the area calculation are poor), and the roundoff at large ω, where we are presumably out of the asymptotic regime. The value of β' is in good agreement with that found in Ref. 3, while α' is substantially different, suggesting that α' may have N dependence while β' does not. To further check this scaling law we have replotted the data of Fig. 5 onto a single plot (Fig. 6), plotting area versus the scaling variable $H^{0.46}\omega^{0.36}$. The fit to a straight line is quite good, except for some points at low fields, where again our statistics on the area are poor [see Fig. 2(a)]. Finally we have attempted to search for a tricritical point but at this time our data is inconclusive and we are unable to assess the order of the transition. The

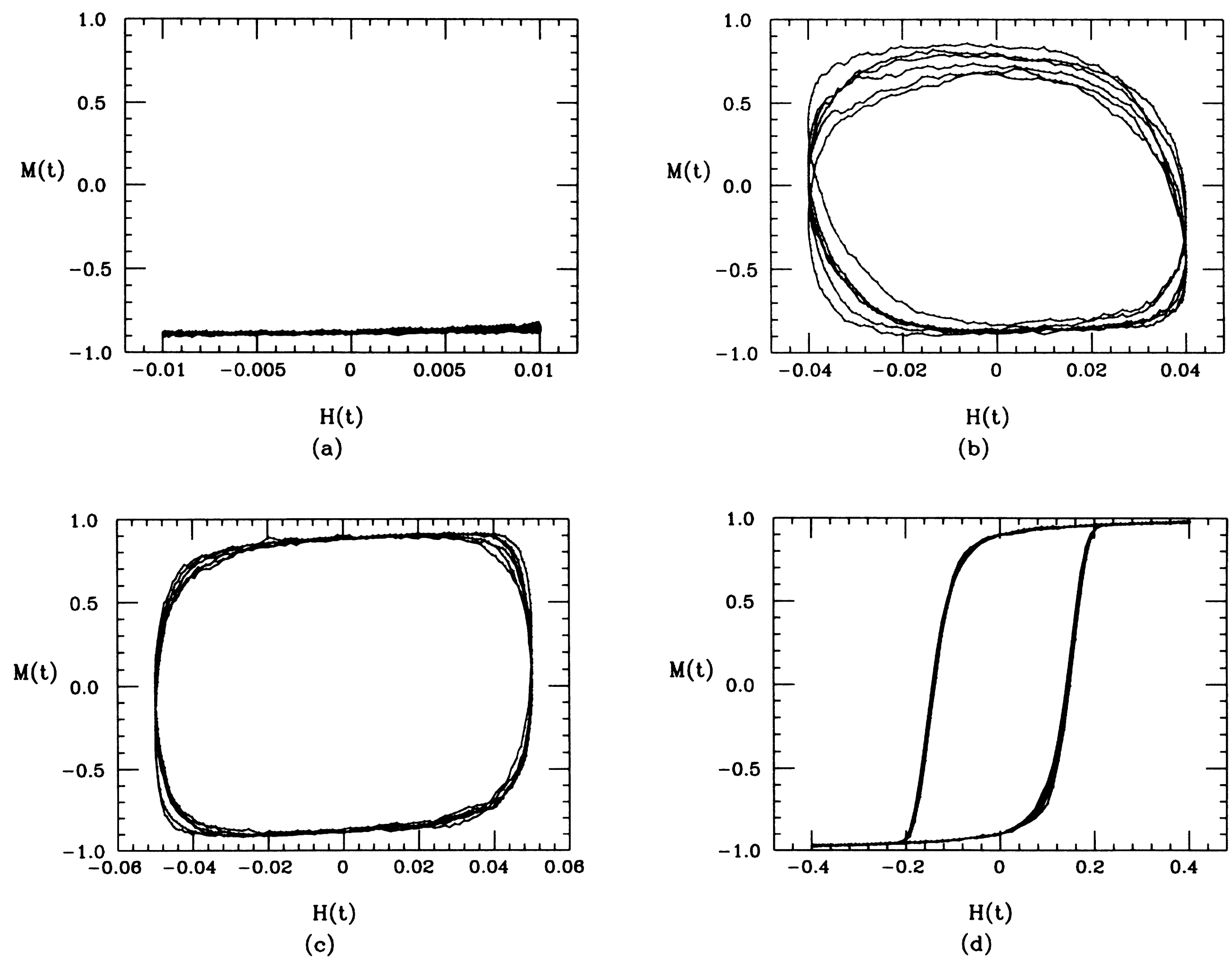

FIG. 4. Examples of the different types of hysteresis loops for $\tau=1000$ MCS, $J=0.48$ for the following values of H: (a) $H=0.01$, (b) $H=0.03$, (c) $H=0.05$, and (d) $H=0.4$. Only six cycles of the field are plotted to aid visualization. (b) would probably become smoother if we were to average the results over many seeds of the random number generator.

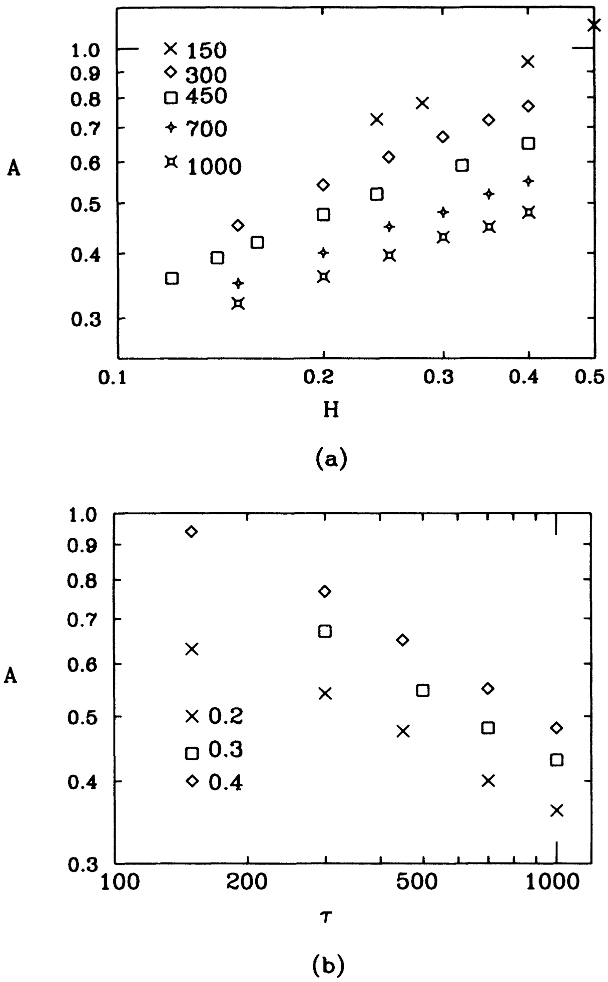

FIG. 5. Scaling of the area A of the hysteresis loops. (a) A vs H for various values of τ in units of MCS, (b) A vs τ for various values of H. Least-squares fitting yields the exponents of Eq. (2.5).

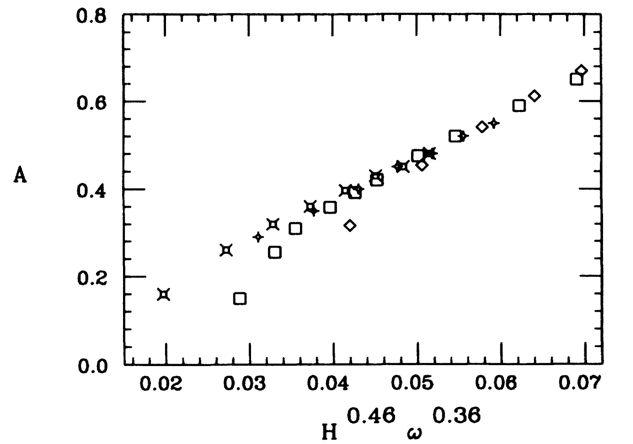

FIG. 6. Scaling of the area A of the hysteresis loops. The data of Fig. 5 is replotted as a function of the scaling variable $H^{0.46}\omega^{0.36}$. The symbols correspond to the values of τ indicated in Fig. 5(a).

oscillations in the time-averaged magnetization are relatively large near the transition, making it difficult to probe the critical regime accurately. To provide an idea of how much computer time is needed, we note that Fig. 3 required 2 h of CPU time on an IBM 3090. Near the transition considerably more time would be required.

III. CONCLUSIONS

We have performed Monte Carlo simulations on a two-dimensional Ising model in an ac field with the aim of studying a dynamical phase transition and the scaling of hysteresis loops. Our results are in at least qualitative agreement with the mean-field and large N calculations of Refs. 2 and 3, suggesting that the transition is not destroyed by either thermal fluctuations or a finite number of spin components. Our results are inconclusive regarding the existence of the tricritical point found in the mean-field theory.[2] We hope to perform more simulations in the future to study this point.

Another item of interest which we have not touched on here is the nucleation process and the structure of the droplets of minority spins. When $H > H_c$, the critical field $M(t)$ changes sign, and we are confronted with a nucleation problem in the presence of a time-dependent field. Once $M(t)$ has changed sign, there are still droplets of spins of opposite sign, and they are especially prolific in number when $H \gtrsim H_c$. We have performed some preliminary simulations on the droplet size distribution function which appear to suggest a power-law behavior. However, our statistics are not very good at this time and further work is needed.

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