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Plasmon instability in two laser fields under a strong magnetic field

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Plasmon scattering by electrons in two laser fields in the additional presence of a strong (quantizing) magnetic field is discussed. A kinetic equation for the plasmon population is derived, from which the damping rate is calculated. We found that plasma waves may be amplified over a relatively narrow range of plasmon wave numbers in the direction perpendicular to the magnetic-field direction. Furthermore, the joint action of the three external fields results in a very large amplification rate, in contrast to the case where no magnetic field is present.

I. INTRODUCTION

Much attention is focused currently on the interaction of laser fields with plasma,[1-5] mainly regarding thermonuclear fusion.[6] Another interesting aspect of the interacting laser-plasma problem recently treated by some authors[7] is the one in which one considers the changes induced by two laser fields, namely, a strong and a weak field, on the damping of plasma waves due to the electron-plasmon scattering in the presence of these fields. It has been found[7] that the plasmon damping may reverse its signal (amplification) under certain external conditions for the laser fields and that the threshold condition for plasma-wave instability is dependent upon the plasmon wave number k; i.e., there is a selective mechanism for plasmon amplification.

Although the electron cyclotron frequency in these experiments is much smaller than the laser frequencies, the magnetic field probably has little effect on the absorption of laser energy by the electrons but has a major effect on particle confinement. However, a resonance condition, where the laser frequency is equal to the electron cyclotron frequency, may be approached by increasing the magnetic-field strength. It is therefore important to consider the cyclotron resonance absorption of these radiations.

In this paper we study the plasmon instability by electrons in the simultaneous presence of two laser field as in Ref. 7 and include the effects of a strong external (quantizing) magnetic field. Our approach follows closely that of Ref. 7. The plasma is assumed to be infinite and homogeneous. The laser fields are treated as classical plane electromagnetic waves in the dipole approximation. The plasma electrons are described by the solution to the Schrödinger equation for an electron in the laser fields and a uniform static magnetic field. Here, contrary to the method described in Ref. 2, we will make use of a unitary transformation method recently introduced to eliminate the laser field dependences of the kinetic energy term. To be specific, by using a unitary transformation the problem of an electron in three external fields will be reduced to the simple problem of an electron in the presence only of the magnetic field.[8]

II. UNITARY TRANSFORMATION

The procedure to solve the quantum-mechanical problems with the time-dependent Hamiltonian has been discussed in Ref. 8. Here we shall briefly outline the main results.

We begin by writing the Schrödinger equation for an electron in the two laser fields in the presence of a strong magnetic field along the $\hat{\mathbf{z}}$ direction, namely,

$$-\frac{\hbar^2}{2m}\left[\mathbf{p}-\frac{e}{c}\mathbf{A}(t)-\frac{e}{c}\mathbf{A}_0\right]^2\psi(\mathbf{r},t)=-i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t}\ , \tag{1}$$

where $\mathbf{A}(t)$ is the total vector potential of the two laser fields and $\mathbf{A}_0$ is the vector potential of the magnetic field. We now perform a unitary transformation in Eq. (1), namely,[8,9]

$$\psi=U\phi\ , \tag{2}$$

where

$$U=e^{i\boldsymbol{\beta}(t)\cdot\mathbf{p}/\hbar}e^{i\boldsymbol{\alpha}(t)\cdot\mathbf{r}/\hbar}\ . \tag{3}$$

In Eq. (3), the function $\boldsymbol{\beta}(t)$ produces a translation in space and the function $\boldsymbol{\alpha}(t)$ produces a translation in momentum. Under a unitary transformation using the above operator U, the Schrödinger equation for ϕ will have a modified Hamiltonian. Since the functions $\boldsymbol{\beta}(t)$ and $\boldsymbol{\alpha}(t)$ are arbitrary, we can use them to cancel unwanted terms in the modified Schrödinger equation to transform the time-dependent problem into a problem of a particle in the presence only of a static magnetic field.

By substituting the expression for ψ in the Schrödinger equation (1) we obtain the equation for ϕ, namely,

$$i\hbar\frac{\partial\phi}{\partial t}=\tilde{H}\phi\ , \tag{4}$$

where

$$\tilde{H}=\frac{1}{2m}\left[\mathbf{p}+\boldsymbol{\alpha}-\frac{e}{2c}\mathbf{B}\times(\mathbf{r}-\boldsymbol{\beta})-\frac{e}{c}\mathbf{A}(t)\right]^2+\frac{\partial\boldsymbol{\alpha}}{\partial t}\cdot\mathbf{r}+\frac{\partial\boldsymbol{\beta}}{\partial t}\cdot\mathbf{p}\ . \tag{5}$$

In Eq. (5) $\mathbf{B}$ is the static magnetic field. The components of the vector functions $\boldsymbol{\beta}(t)$ and $\boldsymbol{\alpha}(t)$ are chosen to cancel the terms of $\tilde{H}$ which are time dependent and linear in $\mathbf{r}$ or $\mathbf{p}$. Assuming the case of linear polarization for the two laser beams the following relations result:[8]

$$\boldsymbol{\beta}=(\beta_x(t),\beta_y(t),0)\ ,$$

$$\boldsymbol{\alpha}=(\alpha_x(t),\alpha_y(t),0)\ ,$$

$$\beta_x(t)=-\frac{eE_{01}}{m(\omega_c^2-\omega_1^2)}\cos\omega_1 t-\frac{eE_{02}}{m(\omega_c^2-\omega_2^2)}\cos\omega_2 t\ ,$$

$$\beta_y(t)=\frac{eE_{01}\omega_c}{m\omega_1(\omega_c^2-\omega_1^2)}\sin\omega_1 t+\frac{eE_{02}\omega_c}{m(\omega_c^2-\omega_2^2)\omega_2}\cos\omega_2 t\ ,$$

$$\alpha_x(t)=-\frac{eE_{01}\omega_c^2}{2\omega_1(\omega_c^2-\omega_1^2)}\sin\omega_1 t-\frac{eE_{02}\omega_c^2}{2\omega_2(\omega_c^2-\omega_2^2)}\cos\omega_2 t\ ,$$

$$\alpha_y(t)=-\frac{eE_{01}\omega_c}{2(\omega_c^2-\omega_1^2)}\cos\omega_1 t-\frac{eE_{02}\omega_c}{2(\omega_c^2-\omega_2^2)}\cos\omega_2 t\ ,$$

where E_i and ω_i are the laser field amplitudes and frequencies, respectively.

With this choice for $\boldsymbol{\beta}(t)$ and $\boldsymbol{\alpha}(t)$, the modified Hamiltonian becomes

$$\tilde{H}=\frac{1}{2m}\left|\mathbf{p}-\frac{e}{2c}\mathbf{B}\times\mathbf{r}\right|^2\ , \tag{6}$$

which is the Hamiltonian of an electron in a static magnetic field $\mathbf{B}$ whose cyclotron frequency is $\omega_c=eB/2mc$. The solution of Eq. (4) with the Hamiltonian (6) is well known and is given by the Landau wave function.[10]

Therefore, under U the problem of an electron in the presence of the two laser fields and a static magnetic field is reduced to the one of an electron in the presence only of the magnetic field with the original wave function ψ given by

$$\psi_\nu(\mathbf{r},t)=\frac{1}{L}\exp(i\boldsymbol{\beta}\cdot\mathbf{p}/\hbar)\exp(i\boldsymbol{\alpha}\cdot\mathbf{r}/\hbar)\exp(ip_x x/\hbar)\times\exp(ip_z z/\hbar)\exp[(-i/\hbar)\varepsilon_\nu t]\chi_n(\xi-\xi_0)\ , \tag{7}$$

where

$$\varepsilon_\nu=\frac{p_z^2}{2m}+\hbar\omega_c(n+\tfrac{1}{2})\ ,$$

$$\nu\equiv(p_x,p_z,n)\ ,$$

$$\chi_n(\xi-\xi_0)=\frac{1}{(\sqrt{\pi}n!2^n r_c)^{1/2}}\exp[-\tfrac{1}{2}(\xi-\xi_0)^{1/2}]\times H_n(\xi-\xi_0)\ ,$$

with

$$\xi=y/r_c,\quad \xi_0=y_0/r_c\ ,$$

$$r_c=(\hbar/m\omega_c)^{1/2},\quad y_0=p_x/m\omega_c\ .$$

In Eq. (7) $\chi_n(\xi)$ is the harmonic-oscillator wave function.

III. TRANSITION PROBABILITY

The probability amplitude for the electron transition from the initial state i with quantum number $\nu=(p_x,p_z,n)$ to the final state $\nu'=(p_x+q_x,p_z+q_z,n')$ due to a collision with a plasmon of momentum $\hbar\mathbf{q}$ is given by

$$a(\nu\to\nu';\mathbf{q})=-\frac{i}{\hbar}\int\int_{-\tau/2}^{+\tau/2}d^3r\,dt\,\psi_{\nu'}^{*}V(\mathbf{q})e^{i(\mathbf{q}\cdot\mathbf{r}-\omega_q t)}\psi_\nu\ , \tag{8}$$

where $|V(\mathbf{q})|^2=2\pi e^2\hbar\omega_q/\Omega q^2$ is the electron-plasmon vertex,[11] ω_q is the plasmon dispersion relation, and Ω is the normalization volume. By substituting Eq. (7) and Eq. (8) and performing the integrations over x and z, we obtain

$$a(\nu\to\nu';\mathbf{q})=-\frac{i}{\hbar}V(\mathbf{q})(2\pi)^2\int dy\,\chi_{n'}(\xi-\xi_0)e^{iq_y y/\hbar}\chi_n(\xi-\xi_0)\delta(p_x'-p_x-q_x)\delta(p_z'-p_z-q_z)\times\int_{-\tau/2}^{\tau/2}dt\exp\left[-i\frac{\lambda_1}{\hbar\omega_1}\cos\omega_1 t-i\frac{\lambda_2}{\hbar\omega_2}\cos\omega_2 t-i\hbar\omega_q t+\left[\frac{i}{\hbar}\right](\varepsilon_{\nu'}-\varepsilon_\nu)t\right]\ , \tag{9}$$

where $\lambda_i=e\mathbf{E}_{0i}\cdot\Delta\mathbf{p}\omega_i/m(\omega_i^2-\omega_c^2)$ $(i=1,2)$ is the field parameter. Integrals in the y variable similar to the one in Eq. (9) may be found elsewhere.[11,12] The integral over t in Eq. (9) may be performed after expanding the exponentials $\exp[(-i\lambda_i/\hbar\omega_i)\cos\omega_i t]$ in the usual form

$$e^{-i(\lambda_i/\hbar\omega_i)\cos\omega_i t}=\sum_{j=-\infty}^{+\infty}(-i)^j J_j\left[\frac{\lambda_i}{\hbar\omega_i}\right]e^{-ij\hbar\omega_i t}\ .$$

Then Eq. (9) may be written

$$a(\nu\to\nu';\mathbf{q})=-\frac{i}{\hbar}V(\mathbf{q})(2\pi)^3F(n,n',\rho)\delta(p_x'-p_x-q_x)\delta(p_z'-p_z-q_z)\times\sum_{l=-\infty}^{+\infty}(-i)^l J_l\left[\frac{\lambda_1}{\hbar\omega_1}\right]\sum_{m=-\infty}^{+\infty}(-i)^m J_m\left[\frac{\lambda_2}{\hbar\omega_2}\right]\delta(\varepsilon_{\nu'}-\varepsilon_\nu-\hbar\omega_q-l\hbar\omega_1-m\hbar\omega_2)\ . \tag{10}$$

In Eq. (10), $J_j(x)$ is the Bessel function of order j and argument x. Also in Eq. (10) $F(n,n',\rho)$ is given by the following expression:

$$F(n,n',\rho)=\delta_{n',n}e^{-\rho/2}L_{n'}(\rho)+\left[\Theta(n-n')\left[\frac{n'!}{n!}\right]^{1/2}e^{i(n-n')\phi}e^{-\rho/2}\rho^{(n-n')/2}\right]L_{n'}^{n-n'}(\rho)$$
$$+\left[\Theta(n'-n)\left[\frac{n!}{n'!}\right]^{1/2}e^{i(n'-n)\phi}e^{-\rho/2}\rho^{(n'-n)/2}\right]L_{n}^{(n'-n)}(\rho)\ , \tag{11}$$

$$\phi=\tan^{-1}(q_y/q_x),\quad \rho=\hbar^2(q_x^2+q_y^2)/2m\hbar\omega_c\ . \tag{12}$$

In Eq. (11) $L_n^{n'-n}(\rho)$ is the Laguerre polynomial. Equation (10) is now squared to obtain the transition probability per unit time, namely,[13]

$$\frac{|a(\nu\to\nu';\mathbf{q})|^2}{\tau}=\sum_{l=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}T(l,m;\nu\to\nu';\mathbf{q})\ ,$$
$$T(l_1 m;\nu\to\nu';\mathbf{q})=\left[\frac{2\pi}{\hbar}\right]|V(\mathbf{q})|^2J_l^2(\lambda_1/\hbar\omega_1)J_m^2(\lambda_2/\hbar\omega_2)|F(n,n',\rho_0)|^2\delta(\varepsilon_{\nu'}-\varepsilon_\nu-\hbar\omega_q-l\hbar\omega_1-m\hbar\omega_2)\ , \tag{13}$$

where ρ_0 is given by Eq. (12) with $\mathbf{q}_0=(p_x'-p_x,q_y,p_z'-p_z)$ in place of $\mathbf{q}$. It followed from the δ function of Eq. (13) that the transitions are induced between Landau levels n and n' due to a collision with a plasmon $\mathbf{q}$ with the absorption $(l,m>0)$ or emission $(l,m<0)$ of $|l|$ and $|m|$ photons of the two laser fields.

IV. KINETIC EQUATION

The change in N_q, the number of plasmons of wave number $\mathbf{q}$, may be written schematically as in Fig. 1. As usual,[14] we may convert this schematic equation into a mathematical one by substituting the transition probability. One has

$$\frac{dN_q}{dt}=\gamma_q N_q\ , \tag{14}$$

where

$$\gamma_q=\left[\frac{2\pi}{\hbar}\right]\sum_{l=-\infty}^{+\infty}\sum_{m=-\infty}^{+\infty}\sum_{n,p_x,p_z}\sum_{s=-\infty}^{+\infty}J_l^2(\lambda_1/\hbar\omega_1)J_m^2(\lambda_2/\hbar\omega_2)|V(\mathbf{q})|^2|F(n,n+s,\rho_0)|^2[f(\varepsilon_{\nu'})-f(\varepsilon_\nu)]$$
$$\times\delta(\varepsilon_{\nu'}-\varepsilon_\nu-\hbar\omega_q-l\hbar\omega_1-m\hbar\omega_2)\ . \tag{15}$$

In Eq. (15) and in Fig. 1 we have introduced for convenience a new label s such that $s=n'-n$. Also in Eq. (15) $f(\varepsilon_\nu)$ is the electron distribution function. We proceed now to evaluate the sums in the kinetic equation (14) from which the damping (amplification) rate γ_q is evaluated. From the beginning we have assumed the laser fields to be linearly polarized plane waves

$$E_i=E_{0i}\mathbf{e}_x\sin\omega_i t\quad(i=1,2)$$

so that the field parameters λ_i appearing in the arguments of the Bessel functions in Eq. (15) depend on the laser field strengths E_{0i} $(i=1,2)$, the laser frequencies ω_i, and the electron cyclotron frequency ω_c. The case $\omega_c\ll\omega_i$ is essentially the problem considered in a previous paper.[7] We consider here only the interesting case $\omega_c=\omega_i$. We also consider the case where one of the two laser fields, say, $i=1$, is a weak laser field and $i=2$ is the strong pumping field. In the latter case $\lambda_2\gg\hbar\omega_2$ and the argument of the Bessel function $J_m(\lambda_2/\hbar\omega_2)$ is large. For large values of the argument, in the Bessel function J_m is small except when the order m is equal to the argument. The sum over m in Eq. (15) may be written approximately[7]

$$\sum_{\substack{m=-\infty\\ m\neq 0}}^{+\infty}J_m^2(\lambda_2/\hbar\omega_2)\delta(E-m\hbar\omega_2)\cong\tfrac{1}{2}[\delta(E-\lambda_2)+\delta(E+\lambda_2)]\ .$$

where $E\equiv\varepsilon_{\nu'}-\varepsilon_\nu-\hbar\omega_q-l\hbar\omega_1$. The first δ function corresponds to the emission and the second to the absorption of $\lambda_2/\hbar\omega_2$ photons. Since $\lambda_2/\hbar\omega_2\gg 1$, only multiphoton processes are significant. The damping rate then becomes

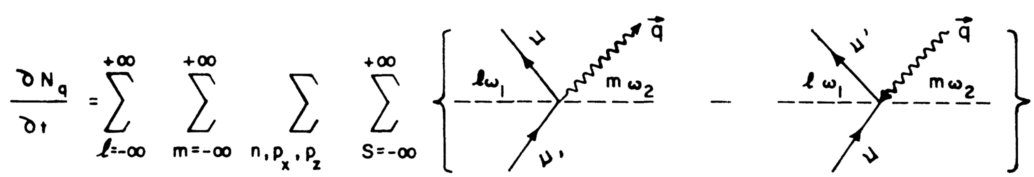

FIG. 1. Change in N_q, the number of plasmons of wave number q. The wavy lines represent plasmons; the dashed lines, photons. Solid lines are for electrons.

$$\gamma_q=\frac{\pi}{\hbar}\sum_{n,p_x,p_z}\sum_{l=-\infty}^{+\infty}\sum_{\substack{s=-\infty\\ s\neq 0}}^{+\infty} J_l^2(\lambda_1/\hbar\omega_1)|V(\mathbf{q})|^2|F(n,n+s,\rho_0)|^2[f(\varepsilon_{\nu'})-f(\varepsilon_\nu)]$$
$$\times[\delta(\varepsilon_{\nu'}-\varepsilon_\nu-\hbar\omega_q-l\hbar\omega_1-\lambda_2)+\delta(\varepsilon_{\nu'}-\varepsilon_\nu-\hbar\omega_q-l\hbar\omega_1+\lambda_2)]\ . \tag{16}$$

We now assume a Maxwellian distribution for the plasma electrons and in the case in which laser 1 is a weak laser field, $\lambda_1 \ll \hbar\omega_1$, the Bessel function J_l^2 appearing in Eq. (16) may be written approximately

$$J_l^2(\lambda_1/\hbar\omega_1)\cong\frac{1}{(l!)^2}\left[\frac{1}{2}\frac{\lambda_1}{\hbar\omega_1}\right]^{2|l|}$$

and, consequently, only the $l=\pm 1$ terms should be retained, i.e., in the weak-field regime of laser 1 only single-photon processes are significant. Under the foregoing assumptions, Eq. (16) then becomes

$$\gamma_q=\frac{\pi}{\hbar}\left[\frac{1}{2}\frac{\lambda_1}{\hbar\omega_1}\right]^2|V(\mathbf{q})|^2\sum_{n,p_x,p_z}\sum_{\substack{s=-\infty\\ s\neq 0}}^{+\infty}|F(n,n+s,\rho_0)|^2[f(\varepsilon_\nu)(e^{-(\lambda_2+\hbar\omega_q+\hbar\omega_1)/k_BT}-1)\delta(\varepsilon_{\nu'}-\varepsilon_\nu-\hbar\omega_q-\hbar\omega_1-\lambda_2)$$
$$+f(\varepsilon_\nu)(e^{-(\lambda_2+\hbar\omega_q-\hbar\omega_1)/k_BT}-1)\delta(\epsilon_{\nu^1}-\varepsilon_\nu-\hbar\omega_q+\hbar\omega_1-\lambda_2)$$
$$+f(\varepsilon_\nu)(e^{(\lambda_2-\hbar\omega_q-\hbar\omega_1)/k_BT}-1\delta(\varepsilon_{\nu'}-\varepsilon_\nu-\hbar\omega_q-\hbar\omega_1+\lambda_2)$$
$$+f(\varepsilon_\nu)(e^{(\lambda_2-\hbar\omega_q+\hbar\omega_1)/k_BT}-1)\delta(\varepsilon_{\nu'}-\varepsilon_\nu-\hbar\omega_q+\hbar\omega_1+\lambda_2)]\ . \tag{17}$$

We now take the classical limit of Eq. (17) by letting[1,7,14]

$$\hbar\to 0 \quad\text{and}\quad n\to\infty\ , \tag{18}$$

such that

$$n\hbar\omega_c\to\tfrac{1}{2}mv_\perp^2\ , \tag{19}$$

$$\sum_{n,p_x,p_z}(\cdots)f(\varepsilon_{np_xp_z})\to\Omega\int d^3v(\cdots)f(\mathbf{v})\ . \tag{20}$$

Hence expanding Eq. (17) in powers of h and retaining only the lowest-order terms (because $\hbar\to 0$), one has

$$\gamma_q=\frac{\pi^2}{2}\left[\frac{e\mathbf{E}_{01}\cdot\mathbf{q}}{m(\omega_1^2-\omega_c^2)}\right]^2\frac{e^2\omega_q}{q^2k_BT}\sum_{\substack{s=-\infty\\ s\neq 0}}^{+\infty}\int d^3v\, J_s^2\left[\frac{q_\perp v_\perp}{\omega_c}\right]f(\mathbf{v})[-(v_{02}q_\perp+\omega_q+\omega_1)\delta(s\omega_c+v_zq_z-\omega_q-\omega_1-v_{02}q_\perp)$$
$$-(v_{02}q_\perp+\omega_q-\omega_1)\delta(s\omega_c+v_zq_z-\omega_q+\omega_1-v_{02}q_\perp)$$
$$+(v_{02}q_\perp-\omega_q-\omega_1)\delta(s\omega_c+v_zq_z-\omega_q-\omega_1+v_{02}q_\perp)$$
$$+(v_{02}q_\perp-\omega_q+\omega_1)\delta(s\omega_c+v_zq_z-\omega_q+\omega_1+v_{02}q_\perp)]\ , \tag{21}$$

where we have written λ_2 as $\hbar q_\perp v_{02}$, with $v_{02}=(eE_{02}/m\omega_2)(1-\omega_c^2/\omega_2^2)$, and replaced $|F(n,n+s,\rho_0)|^2$ by its classical limit, namely, $|F(n,n+s,\rho_0)|^2_{\rm cl}\to J_s^2(q_\perp v_\perp/\omega_c)$. Here $q_\perp$ stands for q perpendicular to the magnetic-field direction. Replacing $f(\mathbf{v})$ by $n_0(\pi v_T^2)^{-3/2}\exp(-v^2/v_T^2)$, where $v_T^2=2k_BT/m$, and performing the integration over v_z using the δ function, Eq. (21) reduces to

$$\gamma_q=\frac{\pi^{1/2}}{2}\frac{e^4E_{01}^2n_0\omega_q^2}{m^3(\omega_1^2-\omega_c^2)^2v_T^3q}\sum_{\substack{s=-\infty\\ s\neq 0}}^{+\infty}I_s(q_\perp v_T/\omega_c)G_s(\alpha,\beta,a)\ ,$$

where

$$I_s(q_\perp v_T/\omega_c)=\int_0^\infty dx\, e^{-x}J_s^2(q_\perp v_T\sqrt{x}\,/\omega_c)\ , \tag{22}$$

$$G_s(\alpha,\beta,a)=\exp\{-[\beta^2+(1-sb)^2]a^2\}(\{\beta\tanh[2\beta a^2(1-sb)]-1\}\cosh[2\beta a^2(1-sb)]$$
$$+\exp[-a^2(\alpha^2-\beta^2)]\{\alpha\tanh[2\alpha a^2(1-sb)]-1\}\cosh[2\alpha a^2(1-sb)])\ , \tag{23}$$

and

$$\alpha=(v_{02}q_\perp-\omega_1)/\omega_q, \quad \beta=(v_{02}q_\perp+\omega_1)/\omega_q ,$$
$$a=\omega_q/q_\perp v_T ,$$
$$b=\omega_c/\omega_q .$$

In arriving at Eq. (22) we have assumed that the plasmons are propagating parallel to both $\mathbf{E}_1$ and $\mathbf{E}_2$, the laser field amplitudes.

V. DISCUSSION AND CONCLUSIONS

Equation (22) is the expression for the plasmon damping under two laser fields in the simultaneous presence of a strong (quantizing) magnetic field we want to discuss. It tells us that if γ_q is positive, the plasmon population grows with time, whereas if γ_q is negative, it is damped.

We first notice by looking at Eq. (22) that for $\omega_2=\omega_c$ (i.e., $v_{02}\rightarrow\infty$), ω_2 being the strong laser field frequency, γ_q vanishes. Physically this result may be interpreted as follows. Consider the problem of one electron in the electromagnetic field of the strong laser described by $\mathbf{A}_2(t)$ and moving in the potential V (the plasmon field). We have

$$H=\tfrac{1}{2}mv_{\parallel 2}^2+\tfrac{1}{2}mv_{02}^2+V ,$$

where $\frac{1}{2}mv_\parallel^2$ and $\frac{1}{2}mv_{02}^2$ are the longitudinal and transverse energies on the electron, respectively. For $\omega_2=\omega_c$ ($v_{02}\rightarrow\infty$), the transverse energy is much larger than V, and the electron interaction is "frozen." This results in a vanishing γ_q. The interesting case is, however, the one in which ω_2 is near ω_c but not necessarily at resonance. In this case the plasmon population may in principle grow (amplification) with time provided G as given by Eq. (23) be positive. On the other hand, for $B\rightarrow 0$ the argument of the Bessel function in Eq. (21) is large, so that as before we will reproduce results of previous work,[7] namely, the expression for the plasmon damping if we had only the two laser fields.[7]

Finally, the expression for G_s is, in general, quite involved. A detailed analysis of it, however, indicates that it is more favorable for G_s to be positive when $\omega_1\gg v_{02}q_\perp$ and $2\beta a^2(1-sb)\ll 1$. Then $\beta=-\alpha=\omega_1/\omega_q$ and Eq. (23) reduces to

$$G_s\simeq 8e^{-a^2(1-sb)}e^{-x^2}(x^2-\tfrac{1}{2}) , \quad x\equiv\omega_1/q_\perp v_T , \tag{24}$$

provided $\omega_2\ll q_\perp v_{02}\ll\omega_1$, and $v_{0i}<v_T$ $(i=1,2)$. It follows from Eq. (24) that as in the previous case,[7] namely, the zero magnetic field case, the threshold condition for plasmon amplification is also dependent upon the value of q, namely, the values of q in the direction perpendicular to the magnetic field. This is seen from Eq. (24), which becomes positive for $x>1/\sqrt{2}$ (or $q_\perp<\omega_1\sqrt{2}/v_T$), has a maximum at $x=\sqrt{3/2}$ and then decreases quite rapidly with increasing x. In other words, in the simultaneous presence of a weak laser, a strong laser, and a strong magnetic field, the plasmon population in a relatively narrow range of q in the direction perpendicular to the z axis may become unstable, i.e., there is a very selective mechanism for plasmon amplification

By comparing the expression for the damping (amplification) in the $B\neq 0$ case with that in the absence of the magnetic field[7] we notice the presence in the former of a resonance factor $(\omega_1^2-\omega_c^2)$, which can be made very large whenever the resonance condition $\omega_1=\omega_c$ is reached, ω_1 being the weak laser field frequency.

In closing, we have proposed in this paper the amplification of plasma waves by electrons in two laser fields in the additional presence of a strong magnetic field. We have shown that the joint action of the two laser beams plus the magnetic field results in a very large amplification rate whenever $\omega_1=\omega_c$ (the resonance condition) in contrast to the case where no magnetic field is present.

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